EFTs: Formalism, tools and LNV beyond Weinberg operator

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Astroneutrino Theory Workshop

Prage, Czech Republic

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- Matching
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- Tools

Second part

- Why LNV operators?
- LNV operators in SMEFT
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- Conclusions

Introduction

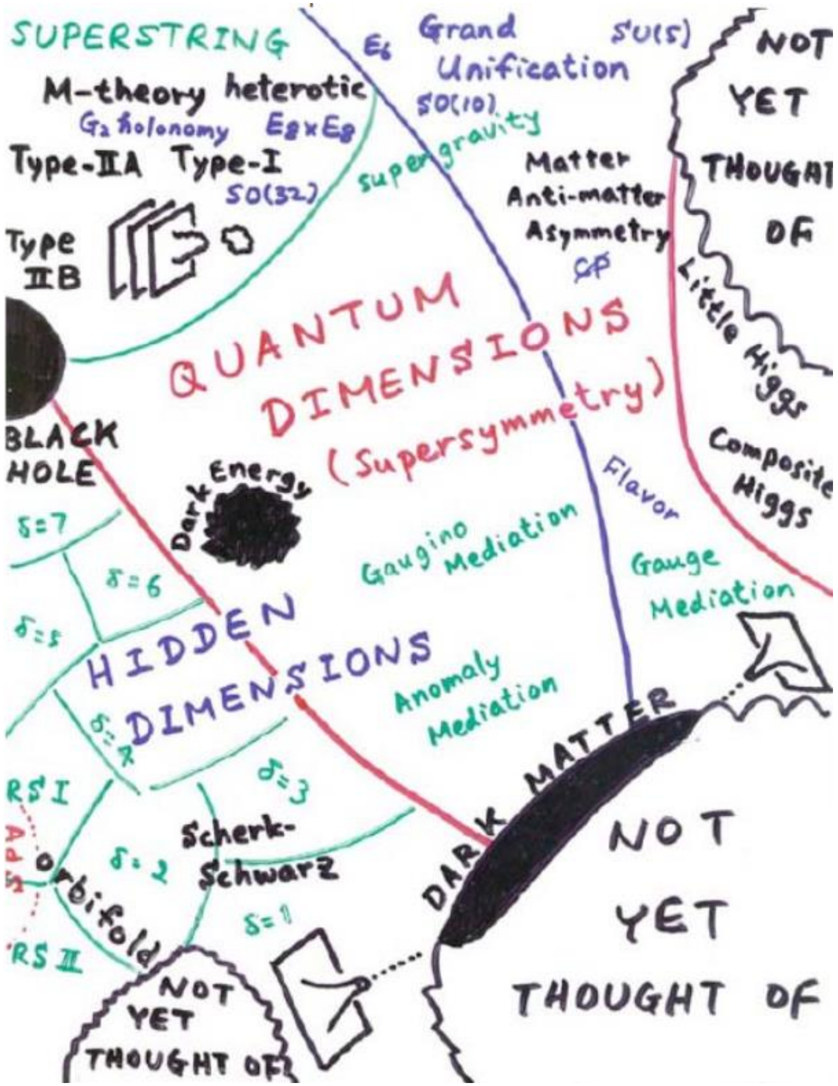
Advantages:

- Simplifying heavy dofs
- Model independent observables
- Easy to relate to experimental observables

Construction of an EFT:

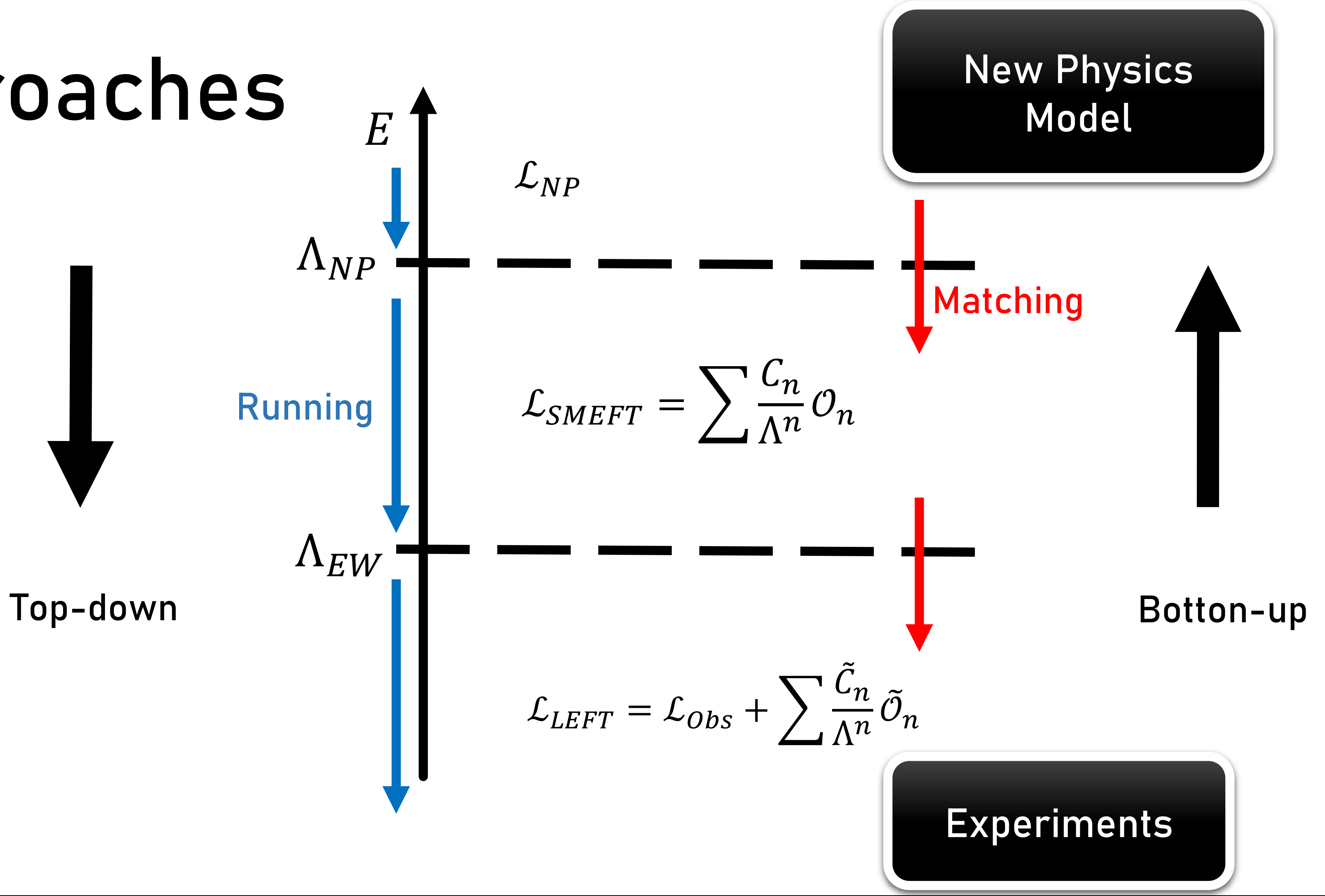
- Energy scale
- Relevant degrees of freedom
- Symmetries

Good tool for hunting new physics!



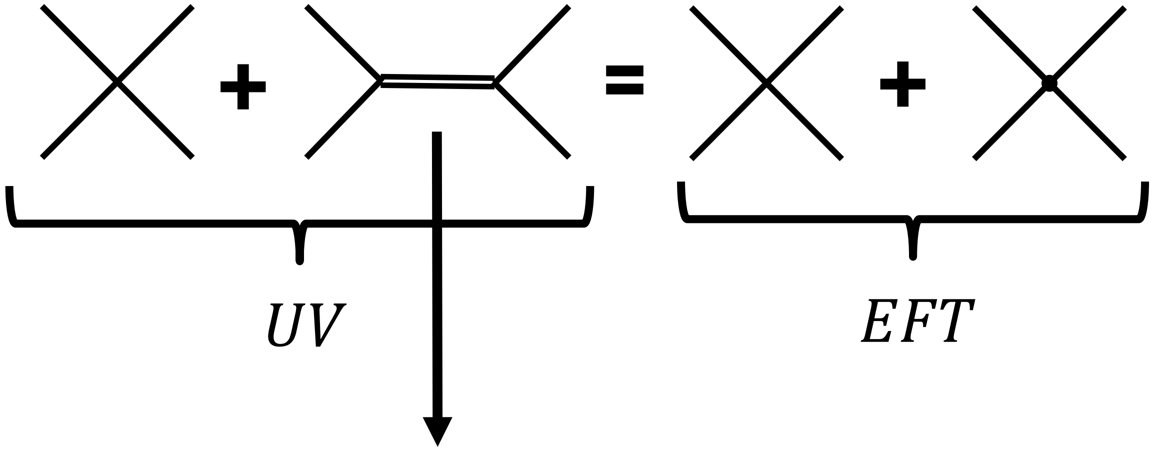
[Image: H. Murayama]

Approaches


New Physics Model
E
$\mathcal{L}_{NP}$
Λ_{NP}
Matching
Running
$\mathcal{L}_{SMEFT} = \sum \frac{C_n}{\Lambda^n} \mathcal{O}_n$
Λ_{EW}
Top-down
Botton-up
$\mathcal{L}_{LEFT} = \mathcal{L}_{Obs} + \sum \frac{\tilde{C}_n}{\Lambda^n} \tilde{\mathcal{O}}_n$
Experiments

Matching

Diagrammatic approach:



$$\frac{1}{p^2 - M^2} = \frac{1}{\mathrm{M}^2}\left(1 - \frac{\mathrm{p}^2}{\mathrm{M}^2} + \mathcal{O}(M^{-4})\right)$$

Path integral approach:

$$\int \mathcal{D}\phi\mathcal{D}\Phi \; e^{i\int d^4x \, \mathcal{L}_{UV}(\phi,\Phi)}$$

$$\downarrow$$

$$\int \mathcal{D}\phi \; e^{i\int d^4x \, \mathcal{L}_{EFT}(\phi)}$$

Functional method

[Fuentes et al. '21]

$\eta \to$ Quantum fluctuation
$\hat{\eta} \to$ Background field configuration

$$O_{ij} = \delta_{ij}\Delta_i^{-1} - X_{ij}$$

$$\mathcal{L}_{UV}[\eta + \hat{\eta}] = \underbrace{\mathcal{L}_{UV}[\hat{\eta}]}_{\text{Tree level}} + \underbrace{\frac{1}{2}\eta_i \frac{\delta^2 \mathcal{L}_{UV}}{\delta\eta_j \delta\eta_i}\eta_j \Big|_{\eta=\hat{\eta}}}_{\text{One-loop}} + \mathcal{O}(\eta^3)$$

Tree level

One-loop

Functional method

[Fuentes et al. '21]

Tree level

$$\mathcal{L}_{UV}^{tree\ level}[\hat{\eta}_L, \hat{\eta}_H(\hat{\eta}_L)] = \mathcal{L}_{EFT}^{tree\ level}$$

One-loop

$$e^{i\Gamma_{UV}^{(1)}} = \int \mathcal{D}\eta\, e^{i\int d^dx \frac{1}{2}\eta O \eta} = (SDet O)^{-\frac{1}{2}} \longrightarrow \Gamma_{UV}^{(1)} = \frac{i}{2} STr \ln O$$

Logarithm properties
Taylor expansion
Method of regions

$$\int \mathrm{d}^d x \mathcal{L}_{EFT}^{(1)} = \frac{i}{2} \mathrm{STr}[\ln \Delta^{-1}]\Big|_{\mathrm{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}[(\Delta X)^n]\Big|_{\mathrm{hard}}$$

Functional method

[Fuentes et al. '21]

One-loop

$$\int \mathrm{d}^d x \mathcal{L}_{EFT}^{(1)} = \frac{i}{2}\mathrm{STr}[\ln\Delta^{-1}]\Big|_{\text{hard}} - \frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}[(\Delta X)^n]\Big|_{\text{hard}}$$

Covariant Derivative Expansion (CDE)

$$\mathrm{STr}[\ln\Delta_{\eta_H}^{-1}]\Big|_{\text{hard}} = \pm\int \mathrm{d}^d x\int\frac{\mathrm{d}^d p}{(2\pi)^d}\mathrm{tr}\left\{\int_{\infty}^{1}\mathrm{d}\xi\frac{\mathrm{d}(\Delta_{\eta_H}^{\xi})^{-1}}{\mathrm{d}\xi}\Delta_{\eta_H}^{\xi}\sum_{n=1}^{\infty}(\mathcal{G}_{\eta_H}\Delta_{\eta_H}^{\xi})^n\right\}$$

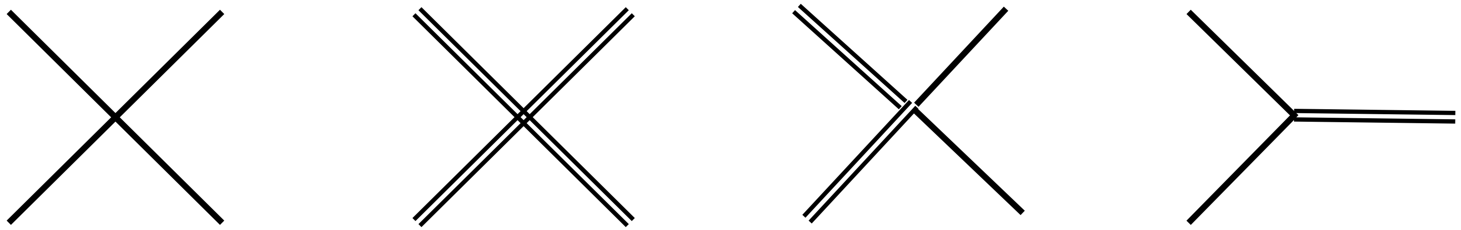
$$\mathrm{STr}\left[(\Delta X)^n\right]\big|_{\text{hard}} = \pm\int \mathrm{d}^d x\int\frac{\mathrm{d}^d p}{(2\pi)^d}\mathrm{tr}\left\{[\Delta\sum_{m=0}^{\infty}(\mathcal{G}\Delta)^m\tilde{X}]^n\right\}$$

Toy model

- Two scalars:

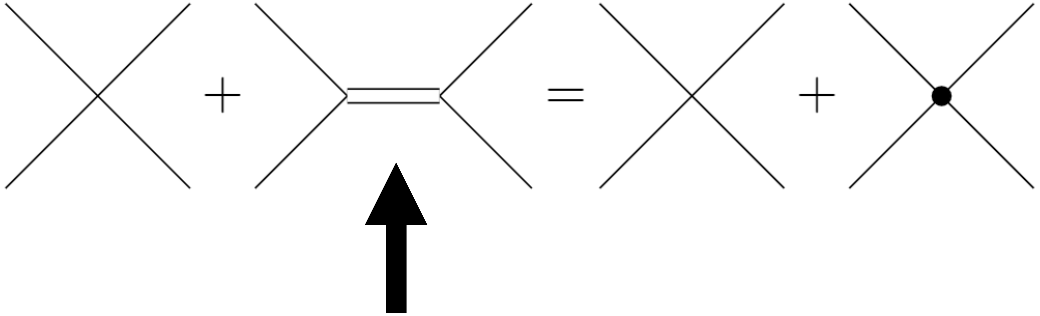
$$\mathcal{L}_{UV} = \frac{1}{2}\left(\partial_\mu \phi\right)^2 - \frac{1}{2}m^2\phi^2 + \frac{1}{2}\left(\partial_\mu \Phi\right)^2 - \frac{1}{2}M^2\Phi^2 - \frac{\lambda_\phi}{4!}\phi^4 - \frac{\lambda_\Phi}{4!}\Phi^4 - \frac{\lambda_{\phi\Phi}}{4}\phi^2\Phi^2 - \frac{k}{2}\phi^2\Phi$$

- Possible interactions:



Tree level

- Diagrammatic approach



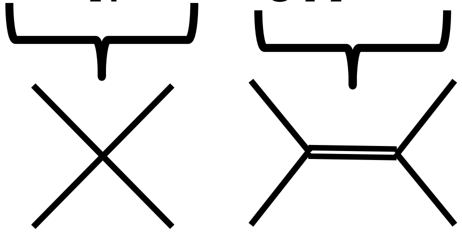
3 channels

- Path integral approach

$$\Phi = [-\partial^2 - M^2]^{-1} \frac{k}{2}\phi^2 \approx -\frac{1}{M^2}\frac{k}{2}\phi^2$$

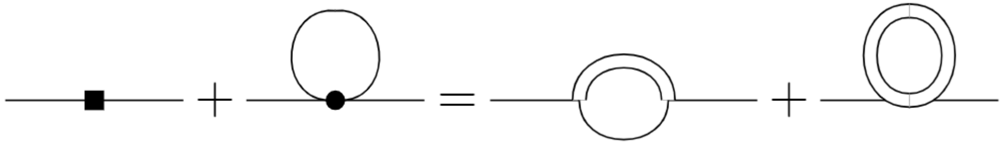
Plug into $\mathcal{L}_{UV}$

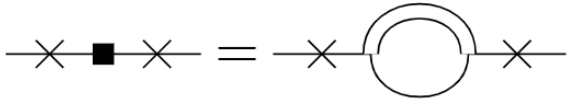
$$\mathcal{L}_{EFT} \supset -\frac{\lambda_\phi}{4!}\phi^4 + \frac{1}{8}\frac{k^2}{M^2}\phi^4 + \mathcal{O}(M^{-4})$$

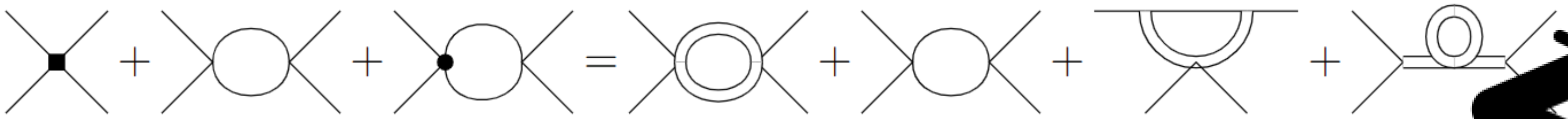


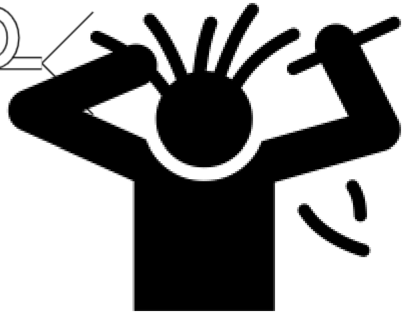
One loop diagrammatic approach

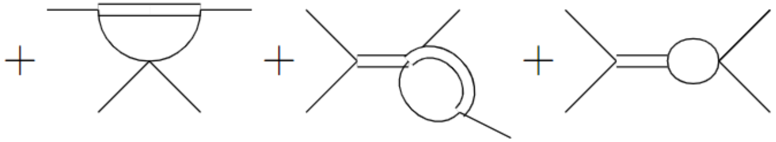
- Diagrammatic approach





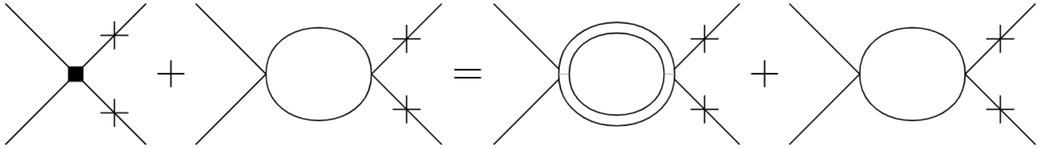


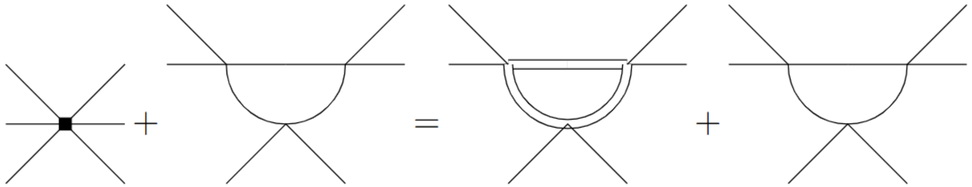




+ Channels!

+ Symmetry factors!





One loop functional approach

- Functional approach

$$\Delta^{-1} = \begin{pmatrix} -\partial^2 - M^2 & 0 \\ 0 & -\partial^2 - m^2 \end{pmatrix}$$

$$X = \begin{pmatrix} \frac{\lambda_\Phi}{2}\Phi^2 + \frac{\lambda_{\phi\Phi}}{2}\phi^2 & \lambda_{\phi\Phi}\phi\Phi + k\phi \\ \lambda_{\phi\Phi}\phi\Phi + k\phi & \frac{\lambda_\phi}{2}\phi^2 + \frac{\lambda_{\phi\Phi}}{2}\Phi^2 + k\Phi \end{pmatrix}$$

$$\int \mathrm{d}^d x \mathcal{L}_{EFT}^{(1)} = -\frac{i}{2}\sum_{n=1}^{\infty}\frac{1}{n}\mathrm{STr}[(\Delta X)^n]\Big|_{\mathrm{hard}}$$

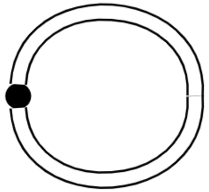
where

$$\mathrm{STr}[(\Delta X)^n] = \int \mathrm{d}^d x \int \frac{\mathrm{d}^d p}{(2\pi)^d}\mathrm{tr}[(\Delta X)^n]$$

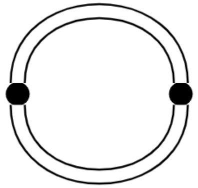
Loop integrals!

Friendly notation!

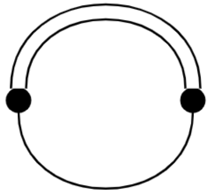
$\Delta_\Phi X_{\Phi\Phi}$



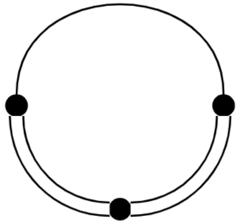
$\Delta_\Phi X_{\Phi\Phi} \Delta_\Phi X_{\Phi\Phi}$



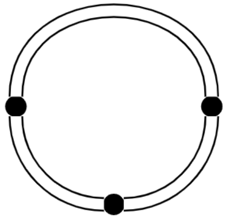
$\Delta_\Phi X_{\Phi\phi} \Delta_\phi X_{\phi\Phi}$



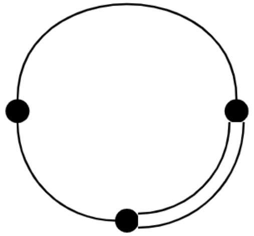
$\Delta_\Phi X_{\Phi\Phi} \Delta_\Phi X_{\Phi\phi} \Delta_\phi X_{\phi\Phi}$



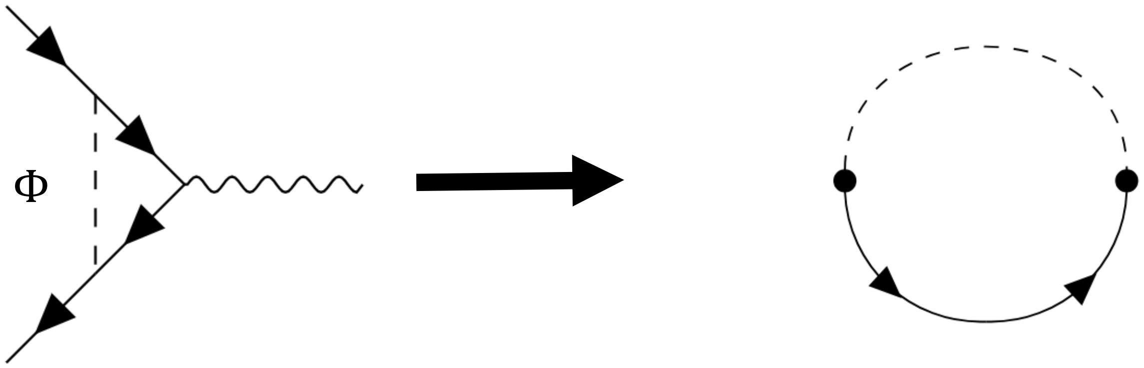
$\Delta_\Phi X_{\Phi\Phi} \Delta_\Phi X_{\Phi\Phi} \Delta_\Phi X_{\Phi\Phi}$



$\Delta_\Phi X_{\Phi\phi} \Delta_\phi X_{\phi\phi} \Delta_\phi X_{\phi\Phi}$



And actually useful!



$$\left\{ \begin{array}{l} \Delta_l X_{\bar{l}\Phi} \Delta_\Phi X_{\bar{\Phi} l} \\ \Delta_e X_{\bar{e}\Phi} \Delta_\Phi X_{\bar{\Phi} e} \end{array} \right.$$

Construction of basis

Matching

Running

Sym2Int

[Fonseca '17]

AutoEFT

[Harlander et al. '23]

Matchete

[Fuentes et al. '23]

MatchMakerEFT+SOLD

[Carmona et al. '22]

DsixTools

[Fuentes et al. '20]

AMONG OTHERS!

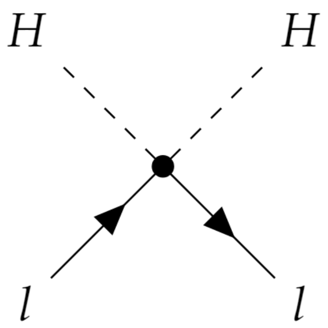
[2307.08745]

Why LNV operators?

[See lectures of Michal Malinský!]

Lepton number is an accidental global symmetry of SM

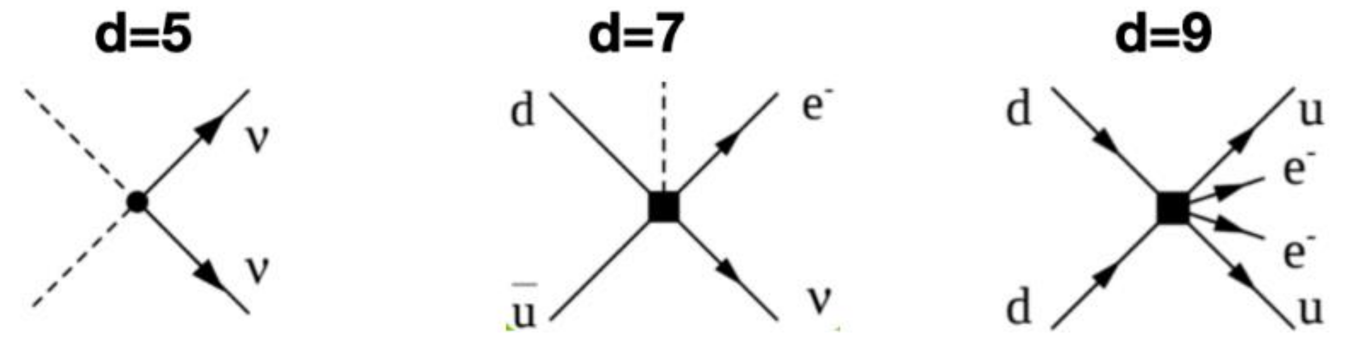
Smoking gun for New Physics!



- Neutrino mass mechanisms: Majorana vs Dirac

LNV operators in SMEFT

$$\mathcal{L} = \mathcal{L}_{\rm SM} + C_5\mathcal{O}_5 + \sum_i C_7^i\mathcal{O}_7^i + \sum_i C_9^i\mathcal{O}_9^i + \ldots .$$



. . .

[Weinberg '79]

[Babu,Leung '79]
[de Gouvea,Jenkins '08]
[Lehman '14]

[Graesser '16]
[Liao, Ma '20]

➢ LNV in odd dimensional operators

Dim 7 LNV operators

[Fridell et al. '23]

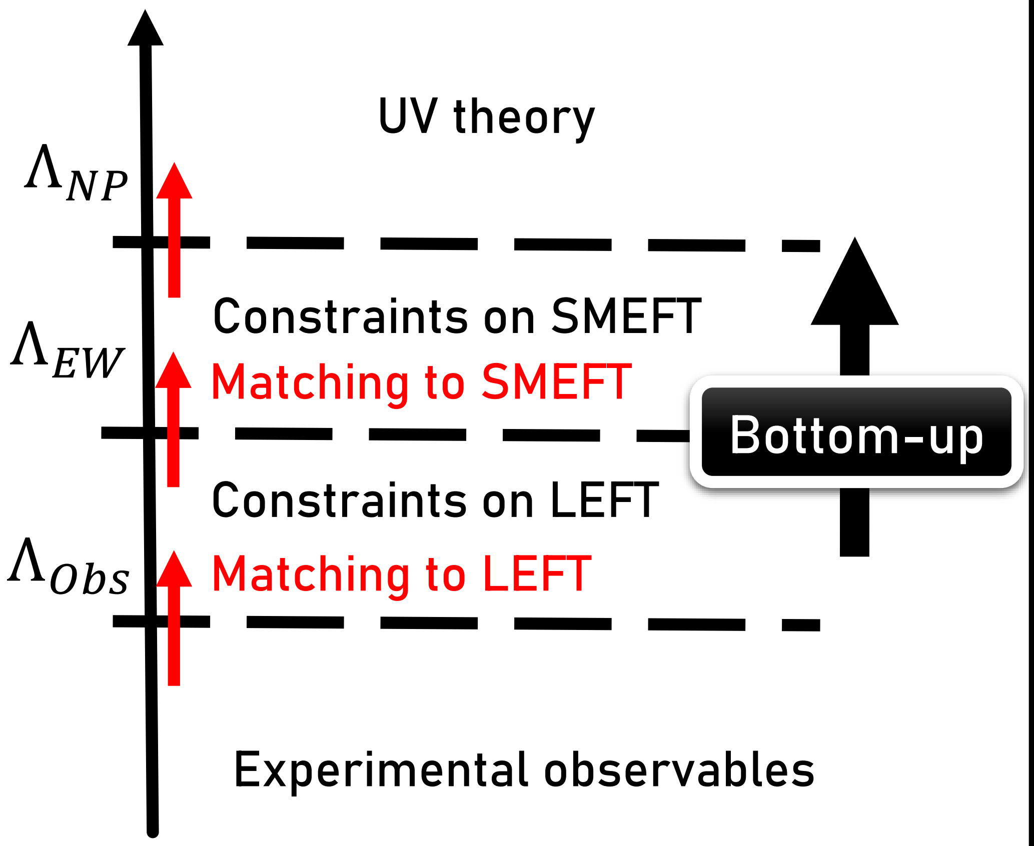
Type	$\mathcal{O}$	Operator
$\Psi^2 H^4$	$\mathcal{O}_{LH}^{pr}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{ci}} L_r^m\right) H^j H^n \left(H^\dagger H\right)$
$\Psi^2 H^3 D$	$\mathcal{O}_{LeHD}^{pr}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{ci}}\gamma_\mu e_r\right) H^j \left(H^m iD^\mu H^n\right)$
$\Psi^2 H^2 D^2$	$\mathcal{O}_{LHD1}^{pr}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{ci}} D_\mu L_r^j\right)\left(H^m D^\mu H^n\right)$
	$\mathcal{O}_{LHD2}^{pr}$	$\epsilon_{im}\epsilon_{jn}\left(\overline{L_p^{ci}} D_\mu L_r^j\right)\left(H^m D^\mu H^n\right)$
$\Psi^2 H^2 X$	$\mathcal{O}_{LHB}^{pr}$	$g\epsilon_{ij}\epsilon_{mn}\left(\overline{L_p^{ci}}\sigma_{\mu\nu} L_r^m\right) H^j H^n B^{\mu\nu}$
	$\mathcal{O}_{LHW}^{pr}$	$g'\epsilon_{ij}\left(\epsilon\tau^I\right)_{mn}\left(\overline{L_p^{ci}}\sigma_{\mu\nu} L_r^m\right) H^j H^n W^{I\mu\nu}$
$\Psi^4 D$	$\mathcal{O}_{\bar{d}uLLD}^{prst}$	$\epsilon_{ij}\left(\overline{d_p}\gamma_\mu u_r\right)\left(\overline{L_s^{ci}} iD^\mu L_t^j\right)$
$\Psi^4 H$	$\mathcal{O}_{\bar{e}LLLH}^{prst}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{e_p} L_r^i\right)\left(\overline{L_s^{cj}} L_t^m\right) H^n$
	$\mathcal{O}_{\bar{d}LueH}^{prst}$	$\epsilon_{ij}\left(\overline{d_p} L_r^i\right)\left(\overline{u_s^c} e_t\right) H^j$
	$\mathcal{O}_{\bar{d}LQLH1}^{prst}$	$\epsilon_{ij}\epsilon_{mn}\left(\overline{d_p} L_r^i\right)\left(\overline{Q_s^{cj}} L_t^m\right) H^n$
	$\mathcal{O}_{\bar{d}LQLH2}^{prst}$	$\epsilon_{im}\epsilon_{jn}\left(\overline{d_p} L_r^i\right)\left(\overline{Q_s^{cj}} L_t^m\right) H^n$
	$\mathcal{O}_{\bar{Q}uLLH}^{prst}$	$\epsilon_{ij}\left(\overline{Q_p} u_r\right)\left(\overline{L_s^c} L_t^i\right) H^j$

- 12 independent operators with $\mathbf{\Delta L = 2}$

Matching from SMEFT to LEFT

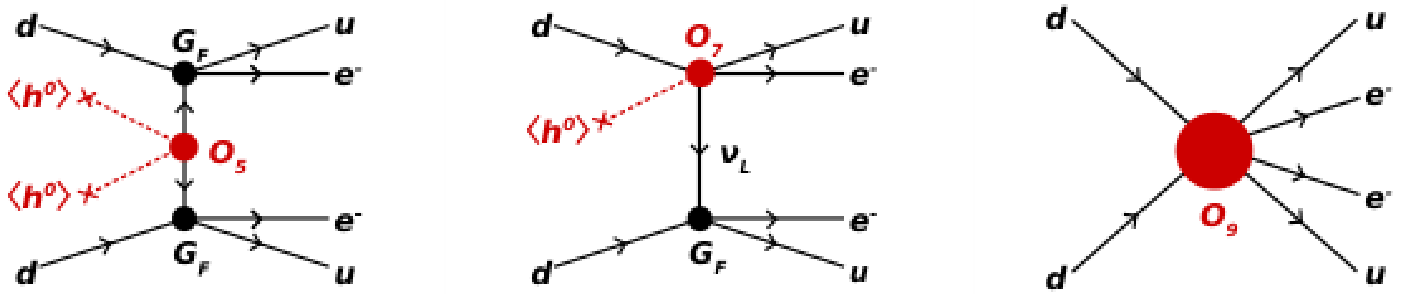
[Fridell et al. '23]

O	Operator	Matching
$O^{S,prst}_{e\nu;LL}$	$(\overline{e_{Rp}}e_{Lr})(\overline{\nu^c_s}\nu_t)$	$\frac{4G_F}{\sqrt{2}}c^{S,prst}_{e\nu;LL} = -\frac{\sqrt{2}v}{8}(2C^{prst}_{\bar{e}LLLH} + C^{psrt}_{\bar{e}LLLH} + s \leftrightarrow t)$
$O^{S,prst}_{e\nu;RL}$	$(\overline{e_{Lp}}e_{Rr})(\overline{\nu^c_s}\nu_t)$	$\frac{4G_F}{\sqrt{2}}c^{S,prst}_{e\nu;RL} = -\frac{\sqrt{2}v}{2}(C^{sr}_{LeHD}\delta^{tp} + C^{tr}_{LeHD}\delta^{sp})$
$O^{T,prst}_{e\nu;LL}$	$(\overline{e_{Rp}}\sigma_{\mu\nu}e_{Lr})(\overline{\nu^c_s}\sigma^{\mu\nu}\nu_t)$	$\frac{4G_F}{\sqrt{2}}c^{T,prst}_{e\nu;LL} = +\frac{\sqrt{2}v}{32}(C^{psrt}_{\bar{e}LLLH} - C^{ptrs}_{\bar{e}LLLH})$
$O^{S,prst}_{d\nu;LL}$	$(\overline{d_{Rp}}d_{Lr})(\overline{\nu^c_s}\nu_t)$	$\frac{4G_F}{\sqrt{2}}c^{S,prst}_{d\nu;LL} = -\frac{\sqrt{2}v}{8}V_{xr}(C^{ptxs}_{\bar{d}LQLH1} + C^{psxt}_{\bar{d}LQLH1})$
$O^{T,prst}_{d\nu;LL}$	$(\overline{d_{Rp}}\sigma_{\mu\nu}d_{Lr})(\overline{\nu^c_s}\sigma^{\mu\nu}\nu_t)$	$\frac{4G_F}{\sqrt{2}}c^{T,prst}_{d\nu;LL} = -\frac{\sqrt{2}v}{32}V_{xr}(C^{ptxs}_{\bar{d}LQLH1} - C^{psxt}_{\bar{d}LQLH1})$
$O^{S,prst}_{u\nu;RL}$	$(\overline{u_{Lp}}u_{Rr})(\overline{\nu^c_s}\nu_t)$	$\frac{4G_F}{\sqrt{2}}c^{S,prst}_{u\nu;RL} = +\frac{\sqrt{2}v}{4}(C^{prst}_{\bar{Q}uLLH} + C^{prts}_{\bar{Q}uLLH})$
$O^{S,prst}_{du\nu e;LL}$	$(\overline{d_{Rp}}u_{Lr})(\overline{\nu^c_s}e_{Lt})$	$\frac{4G_F}{\sqrt{2}}c^{S,prst}_{du\nu e;LL} = +\frac{\sqrt{2}v}{8}(2C^{ptrs}_{\bar{d}LQLH1} + C^{ptrs}_{\bar{d}LQLH2} - C^{psrt}_{\bar{d}LQLH2})$
$O^{S,prst}_{du\nu e;RL}$	$(\overline{d_{Lp}}u_{Rr})(\overline{\nu^c_s}e_{Lt})$	$\frac{4G_F}{\sqrt{2}}c^{S,prst}_{du\nu e;RL} = +\frac{\sqrt{2}v}{2}V^*_{xp}C^{xrts}_{\bar{Q}uLLH}$
$O^{T,prst}_{du\nu e;LL}$	$(\overline{d_{Rp}}\sigma_{\mu\nu}u_{Lr})(\overline{\nu^c_s}\sigma^{\mu\nu}e_{Lt})$	$\frac{4G_F}{\sqrt{2}}c^{T,prst}_{du\nu e;LL} = +\frac{\sqrt{2}v}{32}(2C^{ptrs}_{\bar{d}LQLH1} + C^{ptrs}_{\bar{d}LQLH2} + C^{psrt}_{\bar{d}LQLH2})$
$O^{V,prst}_{du\nu e;LR}$	$(\overline{d_{Lp}}\gamma_\mu u_{Lr})(\overline{\nu^c_s}\gamma^\mu e_{Rt})$	$\frac{4G_F}{\sqrt{2}}c^{V,prst}_{du\nu e;LR} = +\frac{\sqrt{2}v}{2}V^*_{rp}C^{st}_{LeHD}$
$O^{V,prst}_{du\nu e;RR}$	$(\overline{d_{Rp}}\gamma_\mu u_{Rr})(\overline{\nu^c_s}\gamma^\mu e_{Rt})$	$\frac{4G_F}{\sqrt{2}}c^{V,prst}_{du\nu e;RR} = +\frac{\sqrt{2}v}{4}C^{psrt}_{\bar{d}LueH}$
$O^{S,prst}_{d\nu;RL}$	$(\overline{d_{Lp}}d_{Rr})(\overline{\nu^c_s}\nu_t)$	Not induced by $d = 7$ $\Delta L = 2$ SMEFT operators
$O^{S,prst}_{u\nu;LL}$	$(\overline{u_{Rp}}u_{Lr})(\overline{\nu^c_s}\nu_t)$	
$O^{T,prst}_{u\nu;LL}$	$(\overline{u_{Rp}}\sigma_{\mu\nu}u_{Lr})(\overline{\nu^c_s}\sigma^{\mu\nu}\nu_t)$	



Effective Approach to 0νββ

- Different mechanism beyond the standard scenario may contribute to 0νββ:



[Deppish et al. '18]

Atomic phase space

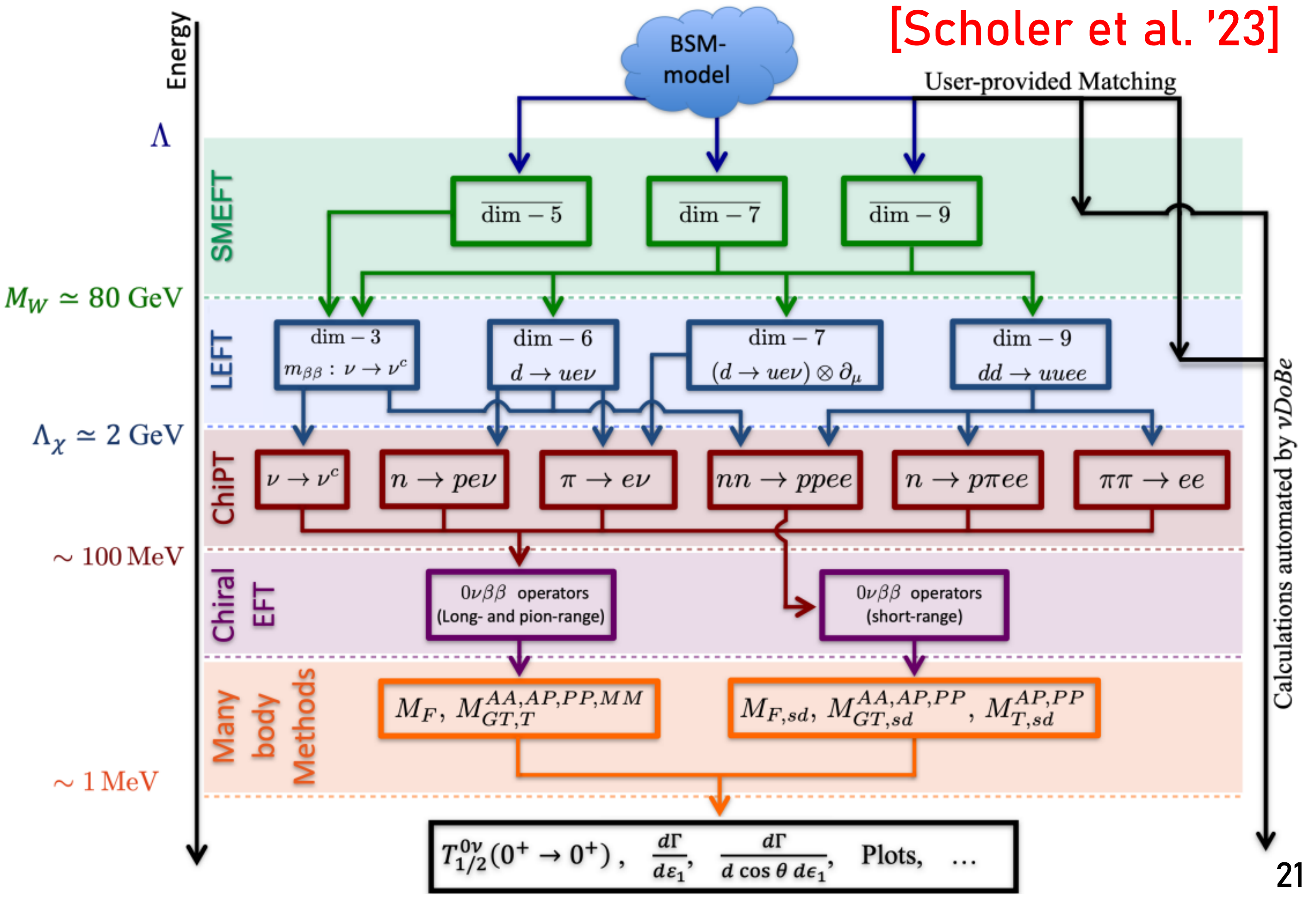
$$T_{1/2}^{-1} = |C|^2 G_{0\nu} |M_{0\nu}|^2$$

Wilson Coefficient

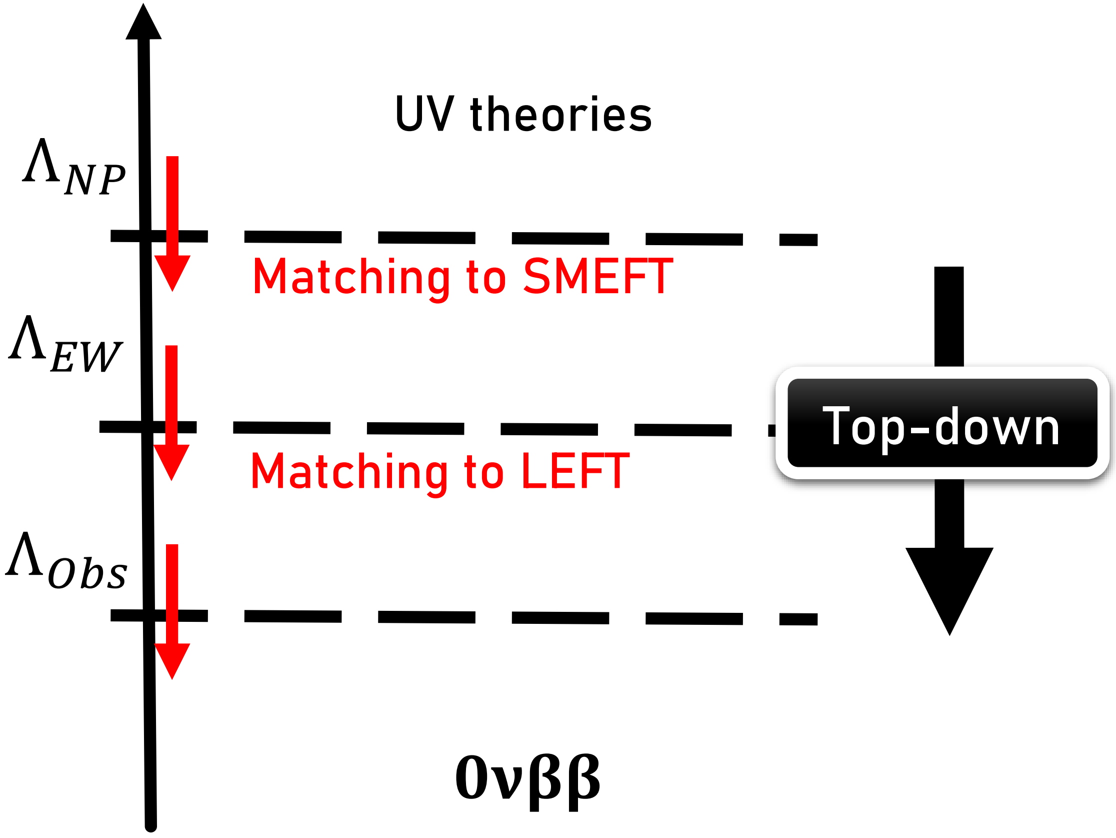
Nuclear matrix element

νDoBe: A Python Tool for 0νββ

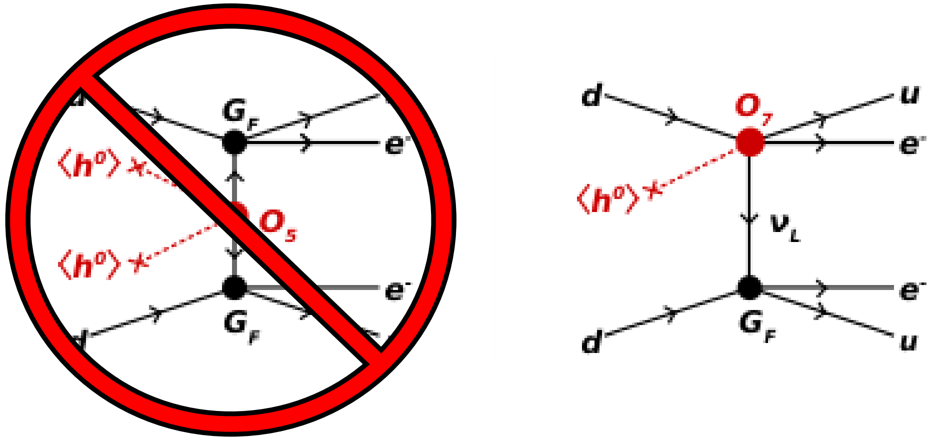
- User inputs:
  - Scale energy.
  - Operators.
  - Isotope.
  - Type of nuclear matrix elements
- Data inputs:
  - Nuclear matrix elements
  - Phase-space factors
  - Low-energy constants
- Output:
  - Half-life formula.
  - Limits
  - Plots: contour plots, $\mathbf{m_{\beta\beta}}$ vs $\mathbf{m_\nu}$, etc.



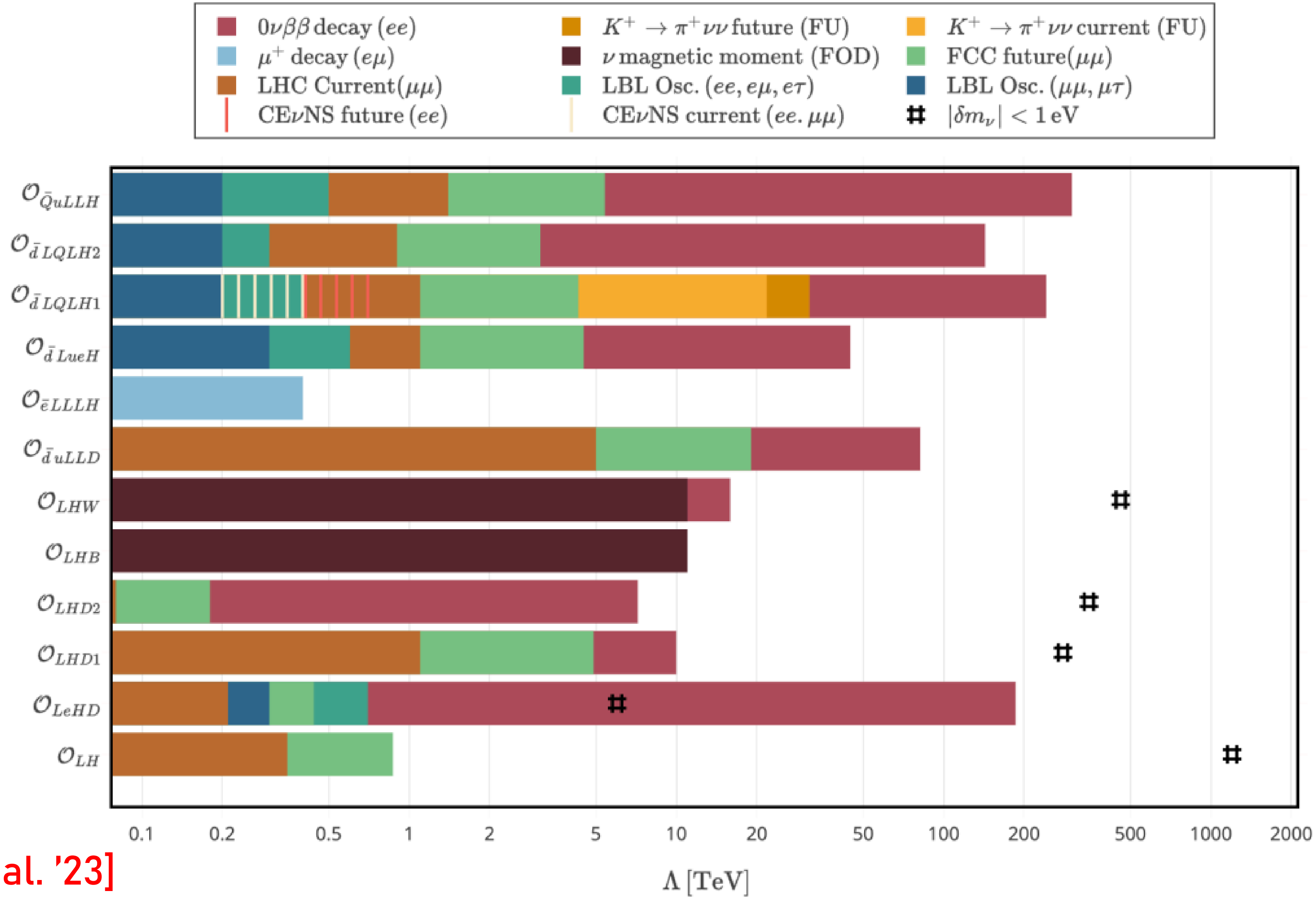
[Gráf, Hati, AM, Scholer in preparation]



- Could we distinguish between LNV due to Weinberg operator or dimension 7 operator?



- But also can be tested by a variety of other observables!



[Fridell et al. '23]

Conclusions

- EFTs are very versatile and useful for the hunting of new physics.
- The functional method is another valid way of efficiently computing EFTs.
- Computational tools are being developed to ease the burden of calculations.
- EFT approach to study LNV processes could help us to distinguish between new physics scenarios.
- $\mathbf{0\nu\beta\beta}$ together with other observables could potentially distinguish between the different dimension 7 LNV operators of SMEFT.

Backup slides

Method of regions

$$I = I_{hard} + I_{soft}$$

Hard expansion ($\boldsymbol{p} \sim \boldsymbol{M}$)

$\frac{1}{p^2 - M^2}$ remains the same

$$\frac{1}{p^2 - m^2} \approx \frac{1}{p^2}\left(1 + \frac{m^2}{p^2} + \mathcal{O}(p^{-4})\right)$$

Soft expansion ($\boldsymbol{p} \sim \boldsymbol{m} \ll \boldsymbol{M}$)

$$\frac{1}{p^2 - M^2} \approx \frac{1}{M^2}\left(1 - \frac{p^2}{M^2} + \mathcal{O}(M^{-4})\right)$$

$\frac{1}{p^2 - m^2}$ remains the same

Toy model result

$$\mathcal{L}_{EFT}^{1-loop} = \mathcal{L}_{EFT}^{n=1} + \mathcal{L}_{EFT}^{n=2} + \mathcal{L}_{EFT}^{n=3} =$$

$$-\frac{1}{4}\frac{k^2}{M^2}\frac{1}{16\pi^2}\phi\partial^2\phi - \frac{1}{96}\frac{1}{16\pi^2}\frac{\lambda_{\phi\Phi}^2}{M^2}\phi^2\partial^2\phi^2 - \frac{1}{96}\frac{1}{16\pi^2}\frac{1}{M^2}\lambda_{\phi\Phi}^2\phi^6$$

$$+\frac{1}{16}\frac{1}{16\pi^2}\frac{1}{M^2}\lambda_\Phi k^2\left(\frac{1}{\epsilon}+1+\log\frac{\tilde{\mu}^2}{M^2}\right)\phi^4 + \frac{1}{16}\frac{1}{16\pi^2}\lambda_{\phi\Phi}^2\left(\frac{1}{\epsilon}+\log\frac{\tilde{\mu}^2}{M^2}\right)\phi^4$$

$$-\frac{1}{2}\frac{1}{16\pi^2}\frac{1}{M^2}k^2\lambda_{\phi\Phi}\left(\frac{1}{\epsilon}+\frac{3}{2}+\log\frac{\tilde{\mu}^2}{M^2}\right)\phi^4 + \frac{1}{4}\frac{1}{M^2}k^2\lambda_\phi\left(\frac{1}{\epsilon}+1+\log\frac{\tilde{\mu}^2}{M^2}\right)\phi^4$$

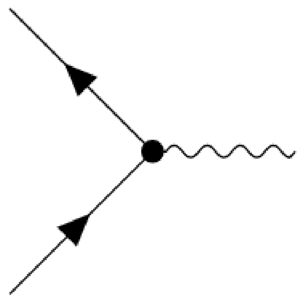
$$+\frac{1}{4}\frac{1}{16\pi^2}\lambda_{\phi\Phi}M^2\left(\frac{1}{\epsilon}+1+\log\frac{\tilde{\mu}^2}{M^2}\right)\phi^2 + \frac{1}{2}\frac{1}{16\pi^2}k^2\left(1+\frac{m^2}{M^2}\right)\left(\frac{1}{\epsilon}+1+\log\frac{\tilde{\mu}^2}{M^2}\right)\phi^2$$

Two-Higgs-Doublet Model

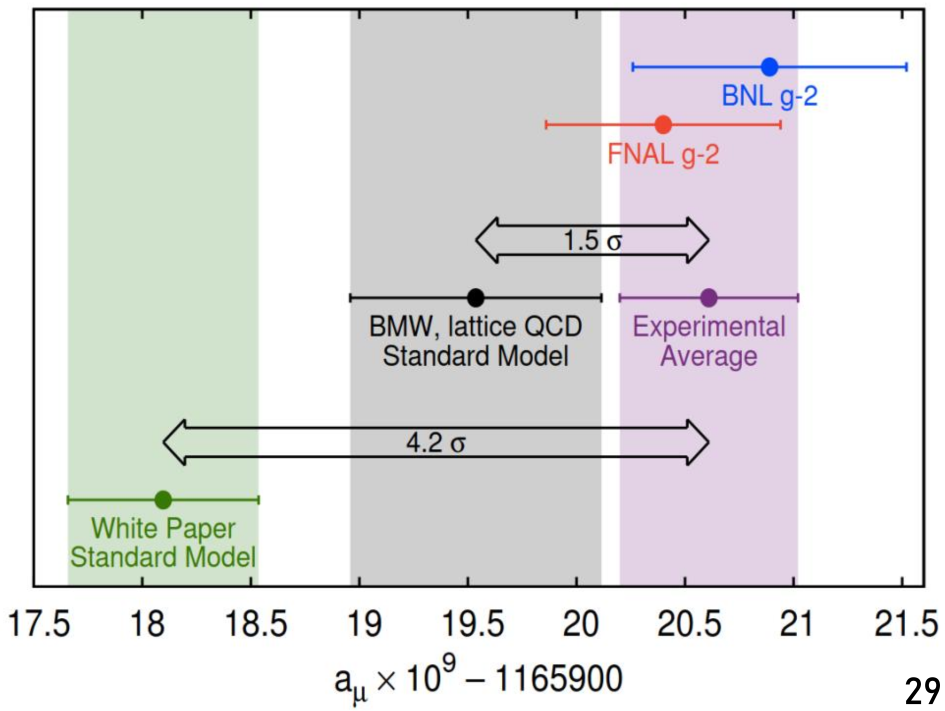
- SM + heavy doublet $SU(2)_L$ [Cornet '21, Celis '14]
- It couples to fermions as

$$\mathcal{L}_Y \supset -\left(\bar{l}_{Lp}\left[y_e^{pr} H + y_\Phi^{pr} \Phi\right] e_{Rr} + \text{h.c.}\right)$$

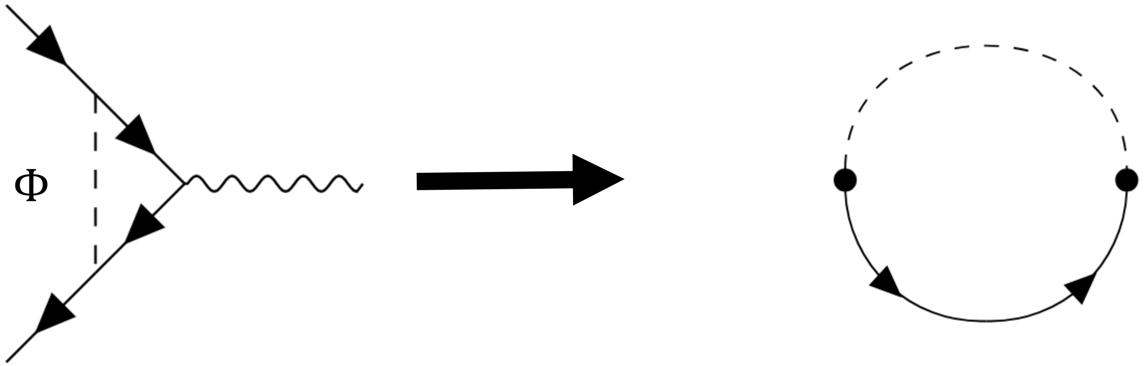
- Goal: see effects on $(\boldsymbol{g}-\mathbf{2})_{\boldsymbol{\mu}}$



[Pilato '21]



Matching magnetic dipole

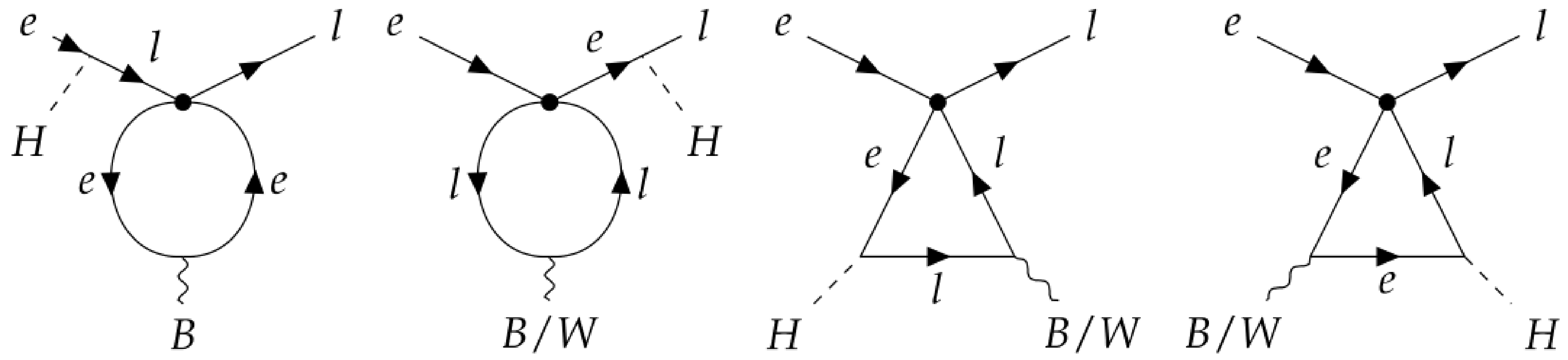


$$\left\{ \begin{array}{l} \Delta_l X_{\bar{l}\Phi} \Delta_\Phi X_{\bar{\Phi} l} \\ \Delta_e X_{\bar{e}\Phi} \Delta_\Phi X_{\bar{\Phi} e} \end{array} \right.$$

$$\mathcal{L}_{dipole}^{tree\ level} = \frac{\bar{y}_\Phi^{rs} y_\Phi^{tp}}{M^2} \frac{1}{2} \left(\bar{l}_{Lr} \gamma^\mu l_{Ls}\right)\left(\bar{e}_{Rt} \gamma_\mu e_{Rp}\right)$$

$$\mathcal{L}_{dipole}^{1-loop} = \frac{\mathrm{i}}{48} \frac{1}{16\pi^2} \frac{1}{M_\Phi^2} g_Y \bar{y}_\Phi^{st} \left(5 y_e^{sp} y_\Phi^{rt} + 2 y_e^{rt} y_\Phi^{sp} + 18 y_e^{ts} y_\Phi^{pr}\right) H B^{\mu\nu} \left(\bar{l}_L^r \gamma_\nu \gamma_\mu e_R^p\right)$$
$$+ \frac{\mathrm{i}}{24} \frac{1}{16\pi^2} \frac{1}{M_\Phi^2} \left(g_L y_L^{sp} \bar{y}_\Phi^{st} y_\Phi^{rt} - 3 g_L y_e^{ts} y_\Phi^{rp} \bar{y}_\Phi^{st}\right) H W^{\mu\nu\mathrm{I}} \left(\bar{l}_L^r \gamma_\nu \gamma_\mu e_R^p\right) \tau^I + \mathrm{h.c.}$$

Evanescent contributions



$$[R_{le}]^{prst} = (\bar{l}_L e_R)(\bar{e}_L l_R) \overset{d=4}{=} -\frac{1}{2}(\bar{l}_{Lp}\gamma_\mu l_{Lt})(\bar{e}_{Rs}\gamma^\mu e_{Rr}) := -\frac{1}{2}[Q_{le}]^{ptsr}$$

$$\Delta[C_{eB}]^{pr} = \frac{3 g_Y y_e^{ts}}{128\pi^2}[c_{le}]^{prst} \qquad \Delta[C_{eW}]^{pr} = -\frac{g_L y_e^{ts}}{128\pi^2}[c_{le}]^{prst}$$

Effects on $(g-2)_\mu$

$$\Delta a_\mu^{M_\Phi} = \mathrm{Re}\left[2.9\times10^{-3}\,C_{eB\mu\mu} - 1.6\times10^{-3}\,C_{eW\mu\mu} - 7.9\times10^{-8}\,c_{le}^{\mu\tau\tau\mu} - 4.6\times10^{-9}\,c_{le}^{\mu\mu\mu\mu}\right]$$

[Aebischer et al. '23]

$$\downarrow$$

$$\Delta a_\mu^{M_\Phi} = \frac{\Lambda^2}{M_\Phi^2}\{(1.12873(7)\times10^{-8} - 4.6\times10^{-9})\left|y_\Phi^{\mu\mu}\right|^2 + 4.6275(3)\times10^{-11}y_\Phi^{\mu\mu}y_\Phi^{ee}$$
$$+(1.60909(16)\times10^{-7} - 7.9\times10^{-8})y_\Phi^{\mu\mu}\,y_\Phi^{\tau\tau}\}$$