Neutrino oscillations Matter effects and Dark Matter

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Three lectures:

I. Oscillations

Generalized description, new aspects

II. Matter effects

Relevant for astrophysics and Cosmology

III. Oscillations and Dark matter

Neutrino mass and oscillations induced by interactions with very light DM

A kind of introduction to other topics of the workshop

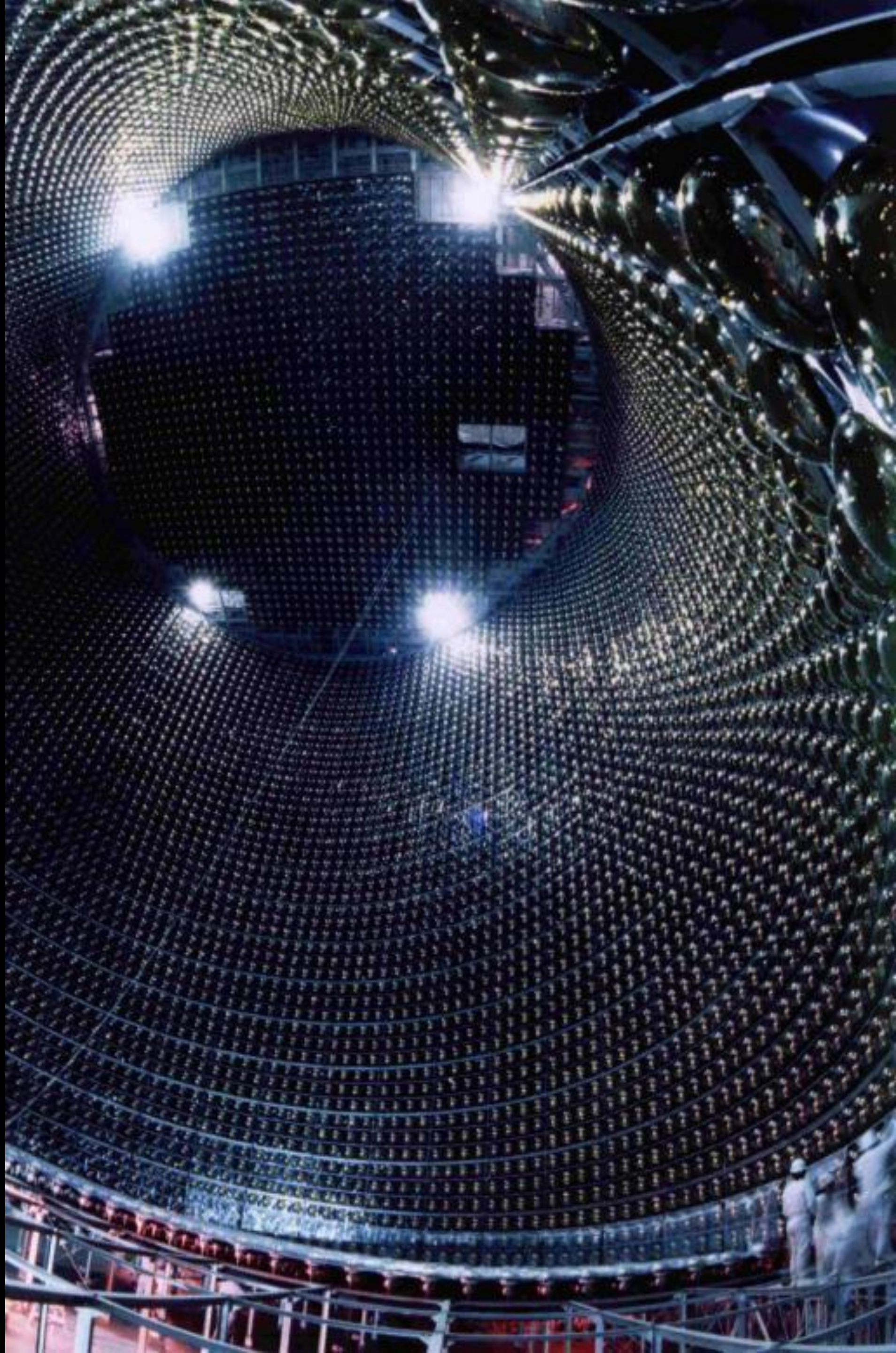
I. Oscillations

Oscillations in vacuum may have very non-trivial aspects:

1. Vacuum may have complicated structure

2. geometry of space is involved

Whatever affects vacuum, affects mass and oscillations



Content

1. Neutrino mass
2. Oscillation set-up and generalized description
3. Neutrinos wave packets and oscillations
4. Localization, Coherence, Entanglement

1. Neutrino mass

Oscillations probe the mass:
its existence and nature

Masses in the Standard model

Masses of quarks and leptons in the Standard Model are not fundamental constants or bare masses but dynamical quantities

They appear due to interactions with Higgs field

$$m = h \langle H \rangle$$

LHC - tests

Yukawa couplings may in turn depend on fields, and consequently x, t:

$$h = h(\phi\ (x,t))$$

Vacuum expectation value of the Higgs field (= value of the field in the low energy state)

Determined by minimum of Higgs potential

$$\langle H \rangle = \langle H \rangle\ (x,\ t,\ T\ldots)$$

Mass may depend on space-time coordinates, environment

Neutrinos – even more complicated case

The same for neutrinos? But

$h \sim 2\ 10^{-13}$ too small and with gap

New Higgs with small VEV?

S. Weinberg: $\frac{1}{\Lambda} L L H H \Rightarrow m_\nu = \frac{\langle H \rangle^2}{\Lambda}$

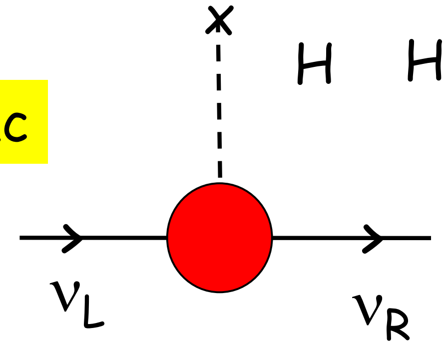
If not invent new scale, $\Lambda = M_{Pl} \Rightarrow m_\nu = 10^{-5}$ eV –too small

additional contribution, or new energy mass scale below M_{Pl} are nedeed

In seesaw: $\frac{h^2 \langle H \rangle^2}{M_R} \sim \frac{h^2 \langle H \rangle^2}{gV}$ may depend on two VEV's

Neutrino mass and EW Symmetry breaking

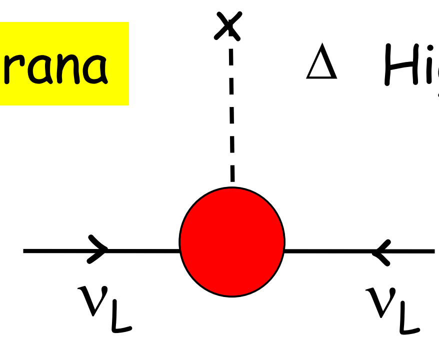
Dirac



Higgs doublet

$m_D = h \langle H \rangle$

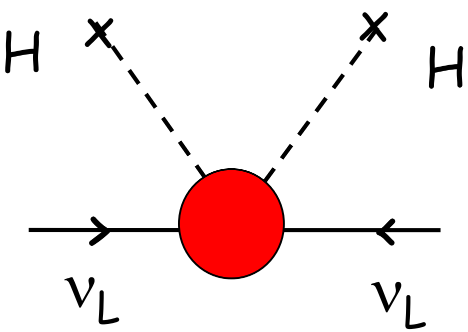
Majorana



Higgs triplet

$m_L = f \langle \Delta \rangle$

Elementary or composite higgs with $I_W = 1$



$m_L = f \langle H \rangle \langle H \rangle$

Mass and medium

VEV of Higgs field = can be treated as medium: classical scalar field

Mass as a result of interaction with medium

Mass as medium effect – as dynamical quantity

Medium: particles and fields, e.g. magnetic fields

Refraction on particles of medium → potential mean field approximation

System of scalar particles with high occupation number can act as classical scalar field

Non-trivial interplay of two media effects

VEV or EV?

2. Oscillation set up
Generalized description

Neutrino oscillations

Бруно Понтекорво

B. Pontecorvo, 1957

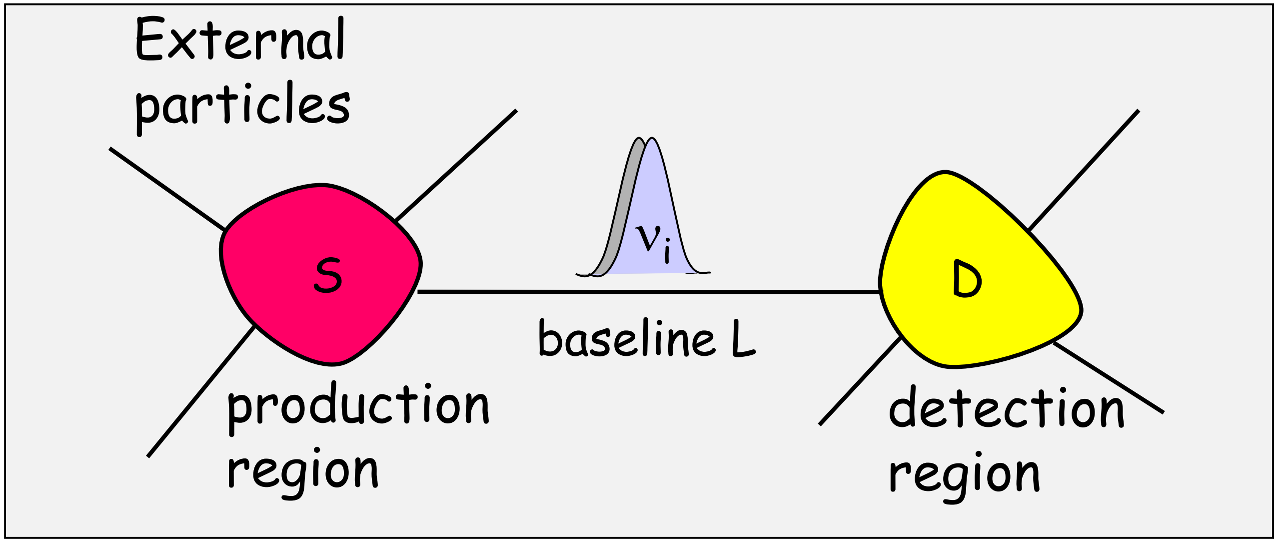
"Mesonium and antimesonium"
Zh. Eksp.Teor. Fiz. 33, 549 (1957)
[Sov. Phys. JETP 6, 429 (1957)]

periodic transformation of one neutrino species (flavor) into another

- Consequence of mixing: production of mixed states
- effect of propagation of mixed states
- interference effect
- implies coherence
- effect of the relative phase increase with time / distance

To avoid misconception – more general description is needed

Oscillation set-up. Localization



E. Akhmedov, A.S.

Oscillations as interference of amplitudes with intermediate ν_i

Finite space and time phenomenon

Two localized interaction regions determined by wave packets (wave functions) of external particles (e.g. nuclei)

Baseline L

Neutrinos: propagators for mass states or as real particles on mass shell

Oscillations: general picture

1. Identify the eigenstates of propagation in vacuum or medium, $\nu_{H\iota}$

In uniform medium $\nu_{H\iota}$ can be considered as asymptotic states (nothing happens with them in the course of propagation)

In the case of non-uniform medium one should consider the instantaneous eigenstates

Propagation of these states can be described by plane waves, $\exp(i\phi_i)$ or by propagators

2. Consider production of eigenstates $\nu_{H\iota}$ in a given process

Find the amplitudes of $\nu_{H\iota}$ production A_i^P

The produced neutrino state

$$\nu^P = N \Sigma_i A_i^P \nu_{H\iota}$$

Normalization factor $N^{-2} = \Sigma_i |A_i^P|^2$

Phase difference in the production region is neglected

Oscillations: general picture

3. Consider detection of the eigenstates $\nu_{H\iota}$ in a given process

Find the amplitude of $\nu_{H\iota}$ detection A_i^D

4. The amplitude of whole the process from initial observed state to final observed states $P \to D$

$$A^{P-D} = \Sigma_i \, A_i^D \, A_i^P \, e^{i\Delta\phi_i}$$

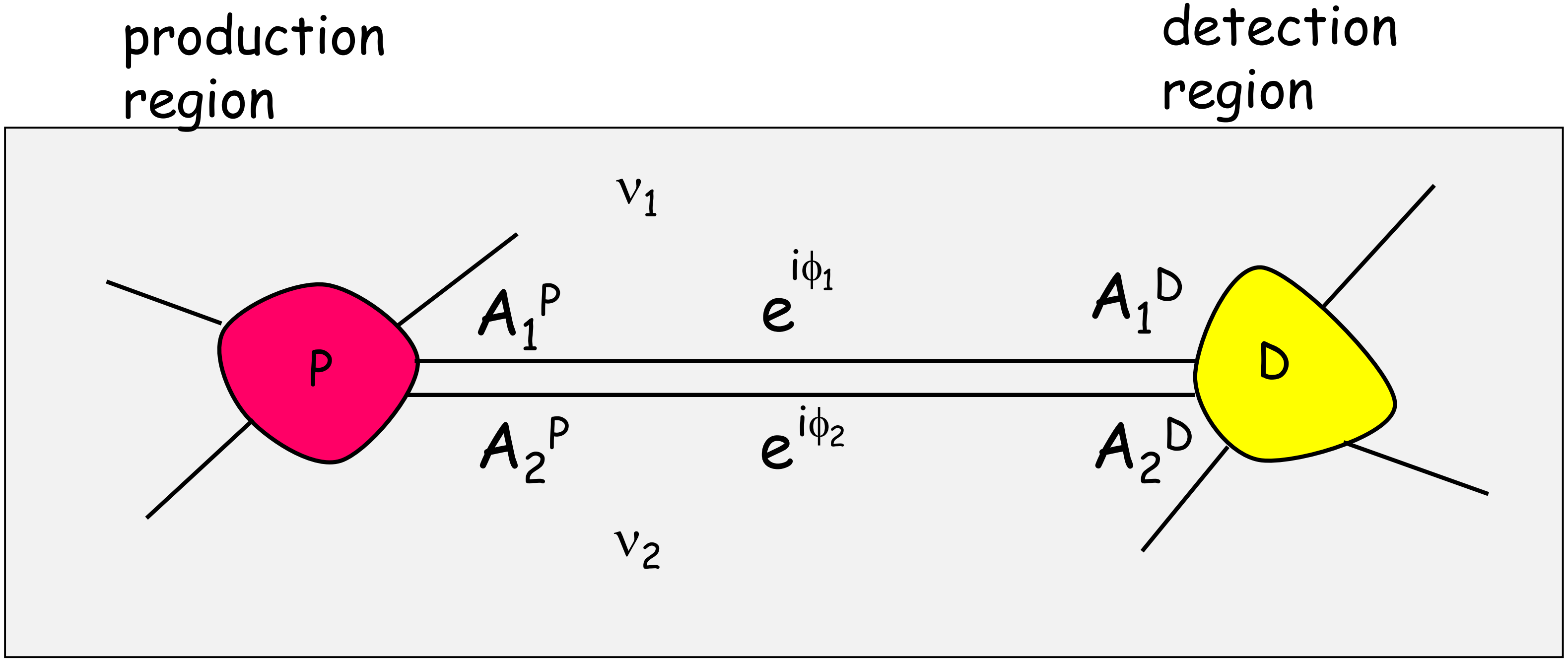
phase differences $\Delta\phi_i = \phi_i - \phi_1$

ν^P and ν^D are determined by external particles

Since neutrinos are not observed we can talk on transition between the state of initial external particles state to the state of final external particles

The amplitudes A_i^D and A_i^P should take into account localization of external particles

Amplitudes of transition



$$A^{P\text{-}D} = \Sigma_i\, A_i^D\, A_i^P\, e^{i\phi_i}$$

... in a spirit of single Feynman diagram

Neutrinos: propagate as real particles on mass shell → as wave packets

The amplitudes should be integrated over intervals of momenta determined by localization of external particles (QM uncetainties)

Oscillations: general picture

The probability of transition

$$P^{P\text{-}D} = |\Sigma_i\, A_i^D\, A_i^P\, e^{i\Delta\phi_i}|^2$$

In 2 neutrino case

$$P^{P\text{-}D} = |A_1^D\, A_1^P|^2 + |A_2^D\, A_2^P|^2 + 2|A_1^D\, A_1^P\, A_2^D\, A_2^P|\cos\Phi$$

$\Phi = \Delta\phi + \xi$ $\qquad$ $\xi = \arg\,[A_1^D\, A_1^P\, A_2^D\, A_2^P\,]$

ξ can contribute to zero distance effect

$\Delta\phi = \Delta\phi\,(t,x)$ is function of distance and time
therefore the probability $P \rightarrow D$ varies with distance or time

What oscillates? Total probability of the whole process

In terms of neutrinos introduce a state ν^{ort} orthogonal to ν^P

$\langle \nu^P | \nu^{ort} \rangle = 0$, then ν^P - ν^{ort} oscillations proceed

Special cases: flavor oscillations

In vacuum, eigenstates of propagation are the states ν_i with definite masses m_i

If initial state is determined by electron and nuclei, then

$$A_1^P = \cos\theta\, F_1^P, \quad A_2^P = \sin\theta\, F_2^P$$

where $F_i^P = F_i^P(m_i, \{p\})$ are functions of kinematical variables p

If final state is determined by positron and nuclei:

$$A_1^D = \cos\theta F_1^D, \quad A_2^D = \sin\theta\, F_2^D$$

If kinematical differences related to neutrino masses can be neglected:

$$F_1^P = F_2^P = F^P, \quad F_1^D = F_2^D = F^D$$

$$P^{P-D} = |F^D F^P|^2 \left(\cos^4\theta + \sin^4\theta + \tfrac{1}{2} \sin^2 2\theta \cos\Delta\phi \right)$$

$|F^D F^P|^2$ disappears in normalization

standard oscillation formula

On the amplitudes of production and interaction

If $m_4 > Q$ (energy release, end point), it can not be produced in this process so

$$A_4^P = F_4^P = 0$$

No oscillations, zero distance effect

$$\boxed{P^{P\text{-}D} = |F_1^D F_1^P|^2 \cos^4\theta}$$

If $m_4 \sim Q$ big separation of WP in momentum space

Spin variables

State produced in the V-A interaction contains both helicities, although component with helicity h = +1 is suppressed by factor m/E.
For ultra-relativitic neutrinos component with “wrong” helicity h = +1 is usually neglected

This is also reflected in dispersion relation

$$E \sim p + m^2/2p \quad \Rightarrow \quad H \sim \frac{M^+ M}{2p}$$

M - mass matrix

Hamiltonian of evolution

Since mass flips chirality – the chirality is flipped by H twice → no change of spin variables

What is effect of wrong helicity?
What happens in the case of non-relativistic neutrinos

"Chiral" oscillations?

Minimal system: single Dirac neutrino with mass m.

In vacuum, the eigenstates of propagation are the states ν_h with definite helicity h = +/-

→ oscillations should be considered in terms of states ν_h

G. Raffelt, A.Y.S, to appear

The corresponding amplitudes of production and detection are

A_-^D, A_-^P, A_+^D, A_+^P

For fixed momentum p and single mass the propagation phases are equal : $\phi_- = \phi_+$ → $\Delta\phi_i = 0$

The probability: $P^{P-D} = |A_-^D A_-^P + A_+^D A_+^P|^2$

No phase difference, no dependence on time and distance → no oscillations, although it can be zero distance effect

"Chiral" oscillations

S. DeLeo, P. Rotelli, 96
about 30 papers …….
Xiao-Gang He, 2023
V. Bittencourt,
M. Blassone… 2024

Mass connects left and right components of fermions → transitions $\nu_L - \nu_R$ which have a character of oscillations with frequency 2E

Assumed that neutrino state produced in V-A interactions equals

$$\psi_L^h = \tfrac{1}{2}(1 - \gamma_5)\, u^h$$

u is usual bi-spinor

Explicitly: $$\psi_L^h(0) = \sqrt{\tfrac{1}{2}(1 - hp/E)} \begin{pmatrix} \xi^h \\ 0 \end{pmatrix}$$

ξ^h is spinor

Evolution of this initial state

$$\psi_L^h(t) = e^{-iHt}\, \psi_L^h(0)$$

$$H = \gamma_0 \gamma \mathbf{p} + \gamma_0 m$$

ψ_L^h is not the eigenstate of the Hamiltonian

"Chiral" oscillations

For neutrinos moving in z-direction an evolving state is

$$\psi_L^h(t) = \begin{pmatrix} [\cos Et + i \sin Et \,(p/E)\, \sigma_z]\, \xi^h \\ - i \sin Et \,(m/E)\, I\, \xi^h \end{pmatrix}$$

right component appears

Probability $P(L \to R) = (m/E)^2 \sin^2 Et$

- small depth for ultrarelativistic neutrinos
- length $2\pi/2E$ - smaller than de Brogle and Compton lengths

To express spinor $\psi_L^h(0)$ in terms of the eigenstates of propagation one need to use both u^h which corresponds to the positive energy and v^h which corresponds to negative energy:

$$\psi_L^h(t) = \cos\theta\, u^h e^{-iEt} + \sin\theta\, v^h e^{iEt}$$

$$\sin\theta = \sqrt{\tfrac{1}{2}(1 - hp/E)}$$

the phase difference: $\Delta\phi = 2E$

"Chiral" oscillations: what is wrong?

G. Raffelt, A.Y.S, to appear

Formally, one can construct any combination of 4 independent solutions of the Dirac equations. However, not all these combinations are produced in real processes.

In particular, the combination $\psi_L{}^h(t)$ of positive energy solution which corresponds to production of particles and negative energy solution corresponding to antiparticle is not produced

What is produced is the combination of states with the same energy and opposite helicities. But this combination does not oscillate in vacuum.

The chiral state $\psi_L{}^h$ is not produced and there is no chiral oscillations.

"Chiral" oscillations and relic neutrinos

Due to chiral oscillations of non-relativistic neutrinos, only half of neutrinos turns out to be active. Indeed, averaging oscillations with maximal mixing *give* $\frac{1}{2}\,\nu_L$ (which is active and) $\frac{1}{2}\,\nu_R$ (which is sterile).

However, the same result can be obtained without chiral oscillations:

At the ν decoupling epoch neutrinos are ultra-relativistic and so exist in the state with h =-1 ν_- (for anti ν: h = +1)

Helicity is conserved during further evolution till present epoch when neutrinos become non-relativistic $\nu_- \rightarrow \nu_-$

Non-relativistic ν_- is composed of ν_L and ν_R with equal fractions. So, only $\frac{1}{2}$ fraction is active.

What happens is change of mixing of ν_L and ν_R in a state with a given helicity: from very small to maximal. This is the adiabatic conversion $\nu_L \rightarrow \frac{1}{2}\,(\nu_L + \nu_R)$

“Chiral oscillations” in matter

Interactions with matter are different for components with helicity -1 (mostly active) and +1 (mostly sterile)

$E_- = p + V, \quad E_+ = p$ V is matter potential

That produces the energy split V and therefore the phase difference $\Delta\phi = Vt$

Matter does not affect mixing since due to angular momentum conservation , in the elastic forward scattering $\nu_- \rightarrow \nu_-$ and $\nu_+ \rightarrow \nu_+$

Thus in neutrino produced by chiral interactions the sterile component will oscillate with the period $2\pi/V$ and the depth $(m/2E)^2$

3. Neutrino wave packets and oscillations

Neutrino wave packets

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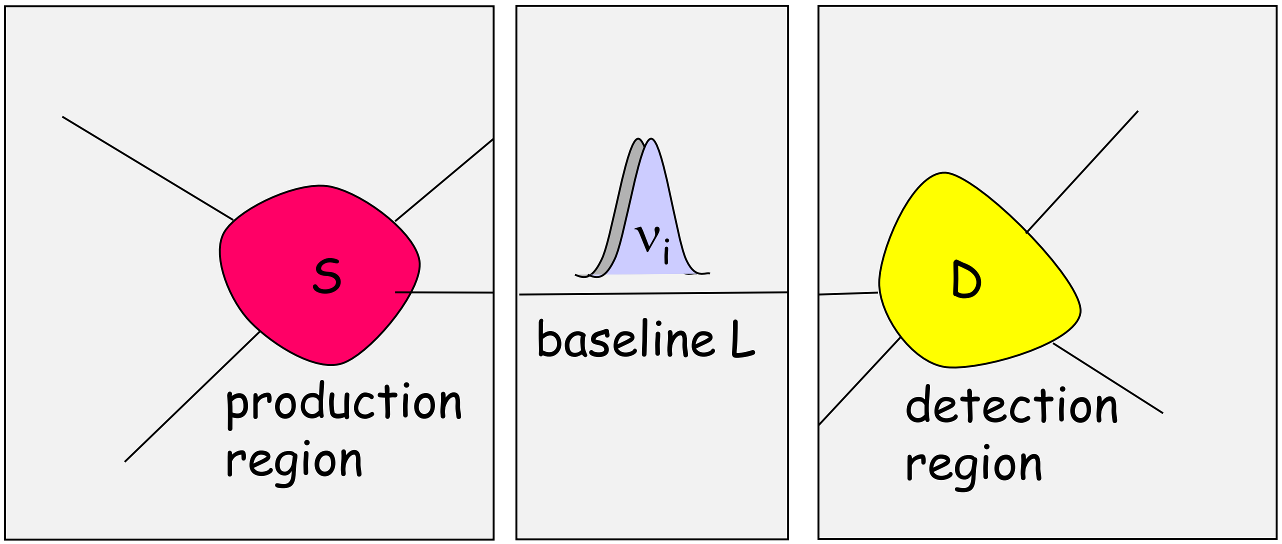
From interference of total amplitudes to interference of neutrino WP (internal WP picture)

Factorization, internal WP

Instead of whole picture with production and detection, one can consider neutrinos only, propagating as real particles. The WP of these neutrinos encode information about production and detection,that is on the WP of external accompanying particles

Factorizations



If oscillation effect in production/detection regions can be neglected

r_D , r_S << l_ν

Still the regions are large enough to compute mass states as asymptotic states

→ **factorization**

Production, propagation and detection can be considered as three independent processes

Neutrino wave packets

Suppose ν_α is produced in the source centered at $x \sim 0$, $t \sim 0$.
After formation of the wave packet (i.e., outside the production region):

$$|\nu_\alpha (x,t)\rangle = \Sigma_k U_{\alpha k}^* \Psi_k(x, t)|\nu_k\rangle$$

$$\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$$ - wave function of k-mass state

$E_k(p) = \sqrt{p^2 + m_k^2}$ - dispersion relation

$f_k(p - p_k)$ - the momentum distribution function peaked at

p_k - the mean momentum

Expanding $E_k(p)$ around mean momentum p_k :

describes spread of wave packets

$$E_k(p) = E_k(p_k) + (dE_k/dp)\Big|_{p_k}(p - p_k) + (dE_k^2/dp^2)\Big|_{p_k}(p - p_k)^2 + \ldots$$

$v_k = (dE_k/dp)\Big|_{p_k} = (p/E_k)\Big|_{p_k}$ - group velocity of ν_k

Shape and phase factors

Neglect spread of the wave packets: $E_k(p) = E_k(p_k) + v_k(p - p_k)$

Insert into $\Psi_k \sim \int dp\, f_k(p - p_k)\, e^{ipx - iE_k(p)t}$

$$\Psi_k \sim e^{ip_k x - iE_k(p_k)t}\, g_k(x - v_k t)$$

Phase factor

$$e^{i\phi_k}$$

$$\phi_k = p_k x - E_k t$$

depends on mean characteristics: p_k and corresponding energy: $E_k(p_k) = \sqrt{p_k^2 + m_k^2}$

Shape factor

$$g_k(x - v_k t) = \int dp\, f_k(p)\, e^{ip(x - v_k t)}$$

depends on x and t in combination $(x - v_k t)$ only and therefore it describes propagation of the wave packet with group velocity v_k without change of the shape

Mixing & mixed states

One needs to compute the produced state, i.e.

the shape factors, $g_k(x - v_k t)$

mean momenta p_k

taking into account
- fundamental interactions
- kinematics
- characteristics of parent and accompanying particles

Produced state is process-dependent

flavor states in Lagrangian differ from the produced states

Connection: $f_k(p - p_k) \sim A_k(p, m_k)$ - amplitudes discussed before

Propagation

Additional phase difference due to different masses

$$|\nu(x,t)\rangle = \cos\theta\, g_2(x - v_2 t)\, |\nu_2\rangle + \sin\theta\, g_3(x - v_3 t)\, e^{i\phi}\, |\nu_3\rangle$$

$\phi = \phi_3 - \phi_2$ - oscillation phase changes with (x,t)

for $\phi \neq 0$ components ν_τ in originally produced ν_μ will not cancel → appearance of ν_τ

Interference depends on phase difference

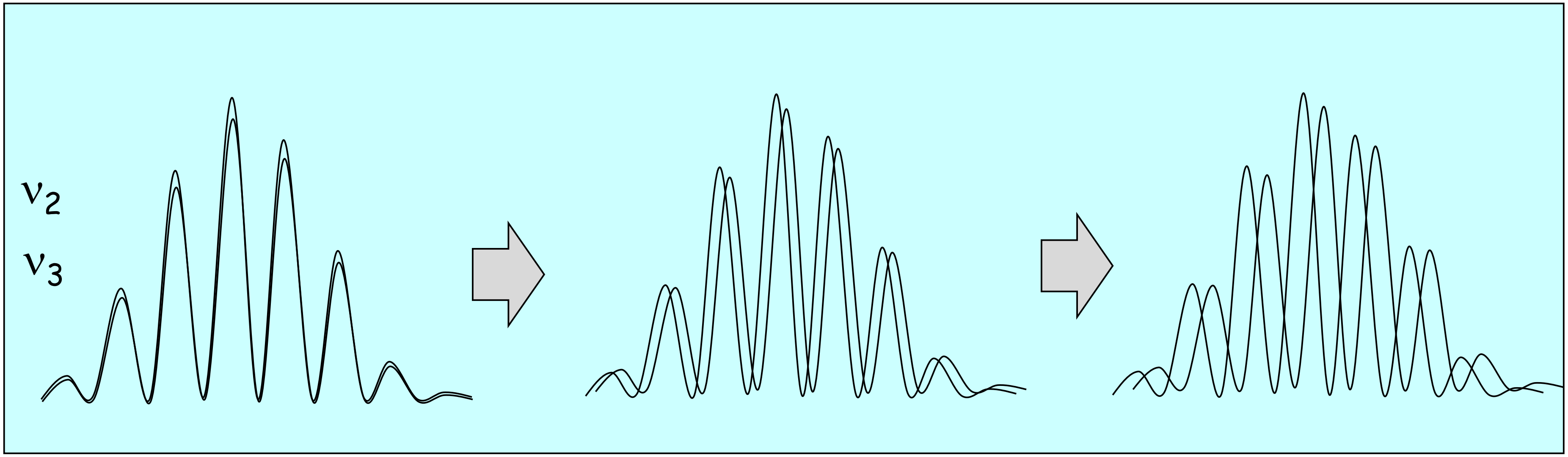
In terms of neutrino system propagation instead of interference of amplitudes of whole observable process

Oscillations

Difference of masses of ν_2 and ν_3 → Difference of phase velocities → Phase difference increases with distance

$$\phi = \frac{\Delta m^2 L}{2E}$$

$\Delta m^2 = m_3^2 - m_2^2$



distance

L

$\phi = 0$

ν_μ

shift of oscillatory patterns
within shape factors

Oscillation phase

$\phi = \phi_3 - \phi_2$ $\phi_i = - E_i t + p_i x$ $p_i = \sqrt{E_i^2 - m_i^2}$

E_i and p_i are the averaged characteristics of WP

$$\phi = \phi_3 - \phi_2 = \Delta E t - \Delta p x$$

For given ΔE

$$\Delta p = (dp/dE)\Delta E + (dp/dm^2)\Delta m^2 = 1/v_g \, \Delta E + (1/2p)\, \Delta m^2$$

group velocity

Insert into ϕ

$$\phi = (\Delta E/v_g)\, \underbrace{(v_g t - x)}_{< \sigma_x} + \underbrace{\frac{\Delta m^2}{2E} x}$$

standard oscillation phase

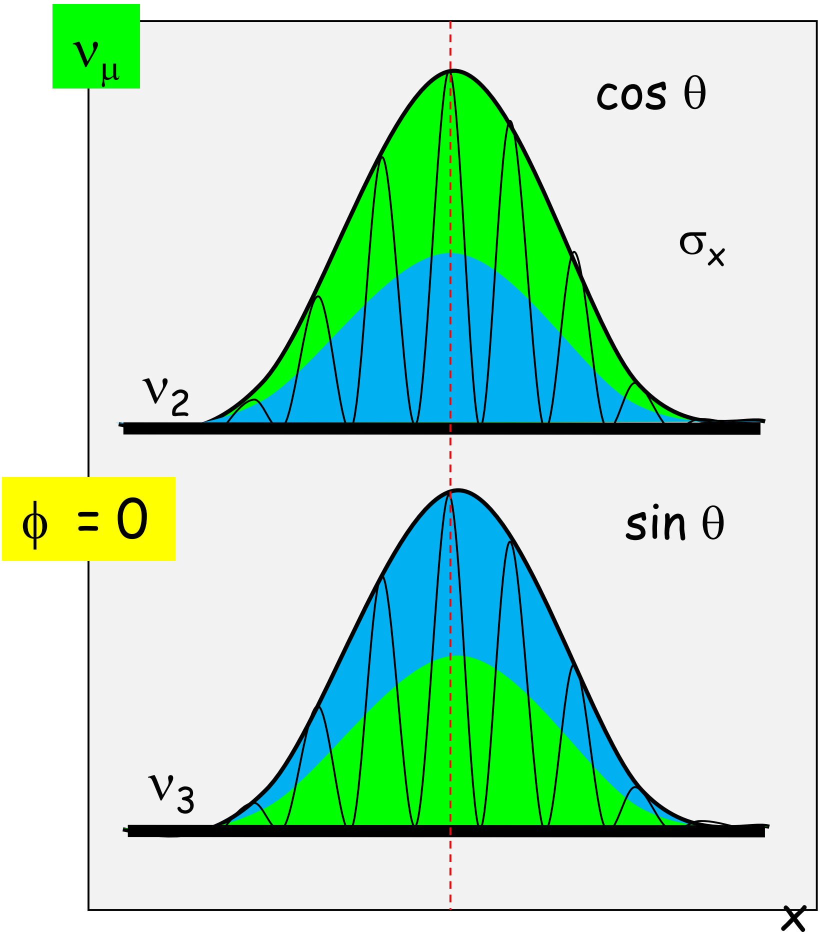
$\Delta E \sim \Delta m^2/2E$
(difference of the average energies)

usually- small

The first term: $< \sigma_x \Delta m^2/2E$ is oscillation phase over the size of WP

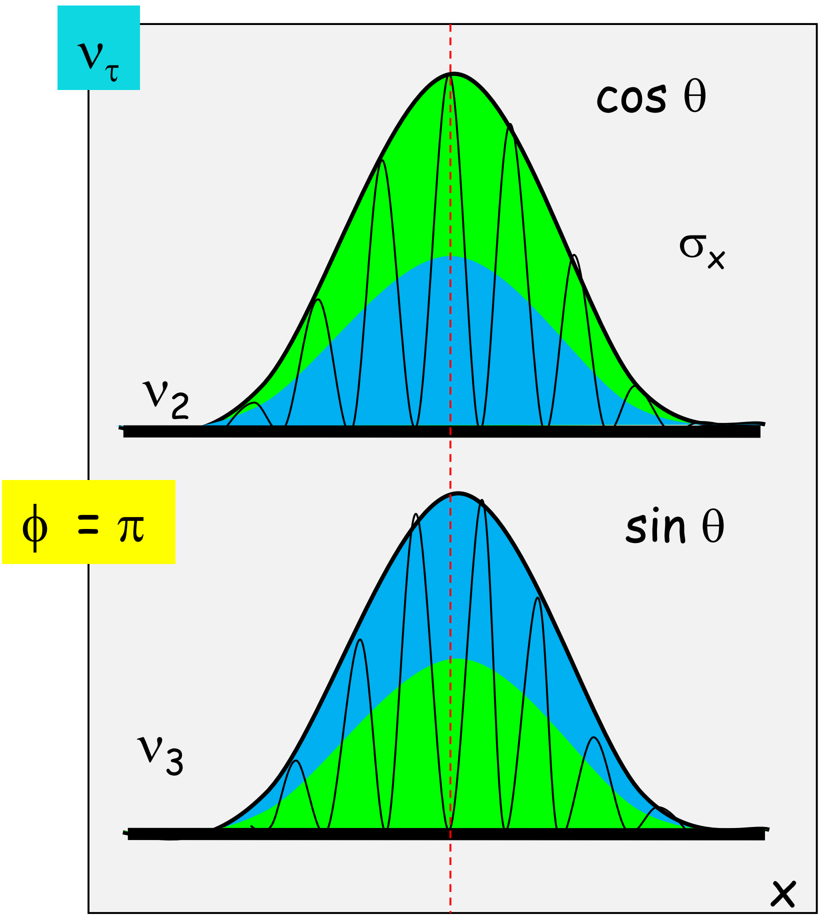
Phase difference along the wave packets is nearly the same

Oscillations



- Destructive interference of the tau parts
- Constructive interference of muon parts

Oscillations



- Destructive interference of the muon parts
- Constructive interference of tau parts

Detection

As important as production for computing observable effects and should be considered "symmetrically" with production

Detection effect can be included in the generalized shape factors

$g_k(x - v_k t) \rightarrow G_k(L - v_k t)$

$x \rightarrow L$ - distance between central points of the production and detection regions

Oscillation amplitude

ν_μ - ν_μ

generalized shape factors should be used

$$|\nu_\mu\rangle = \cos\theta\,|\nu_2\rangle + \sin\theta\,|\nu_3\rangle$$

$$|\nu(x,t)\rangle = \cos\theta\, g_2(x - v_2 t)|\nu_2\rangle + \sin\theta\, g_3(x - v_3 t)e^{i\phi}\,|\nu_3\rangle$$

ν_μ - survival amplitude

$$A(\nu_\mu) = \langle\nu_\mu|\nu(x,t)\rangle =$$

$$= \cos^2\theta\, g_2(x - v_2 t) + \sin^2\theta\, g_3(x - v_3 t)\, e^{i\phi}$$

Used that mass states are orthogonal and normalized: $\langle\nu_i|\nu_k\rangle = \delta_{ik}$

WF of detected state – does not depend on time

Oscillation probability

Probability to find ν_μ in the moment of time t

$$P(\nu_\mu, t) = \int dx \, |\langle \nu_\mu | \nu(x,t) \rangle|^2$$

$$= \cos^4\theta + \sin^4\theta + 2\sin^2\theta \cos^2\theta \cos\phi \int dx \, g_2(x - v_2 t) \, g_3(x - v_3 t)$$

since $\int dx |g|^2 = 1$

interference term

(similarly one can integrate over time for fixed x)

If $g_3 = g_2$ (separation is negligible)

$$P(\nu_\mu, x) = 1 - 2\sin^2\theta \cos^2\theta (1 - \cos\phi) = 1 - \sin^2 2\theta \sin^2 \tfrac{1}{2}\phi$$

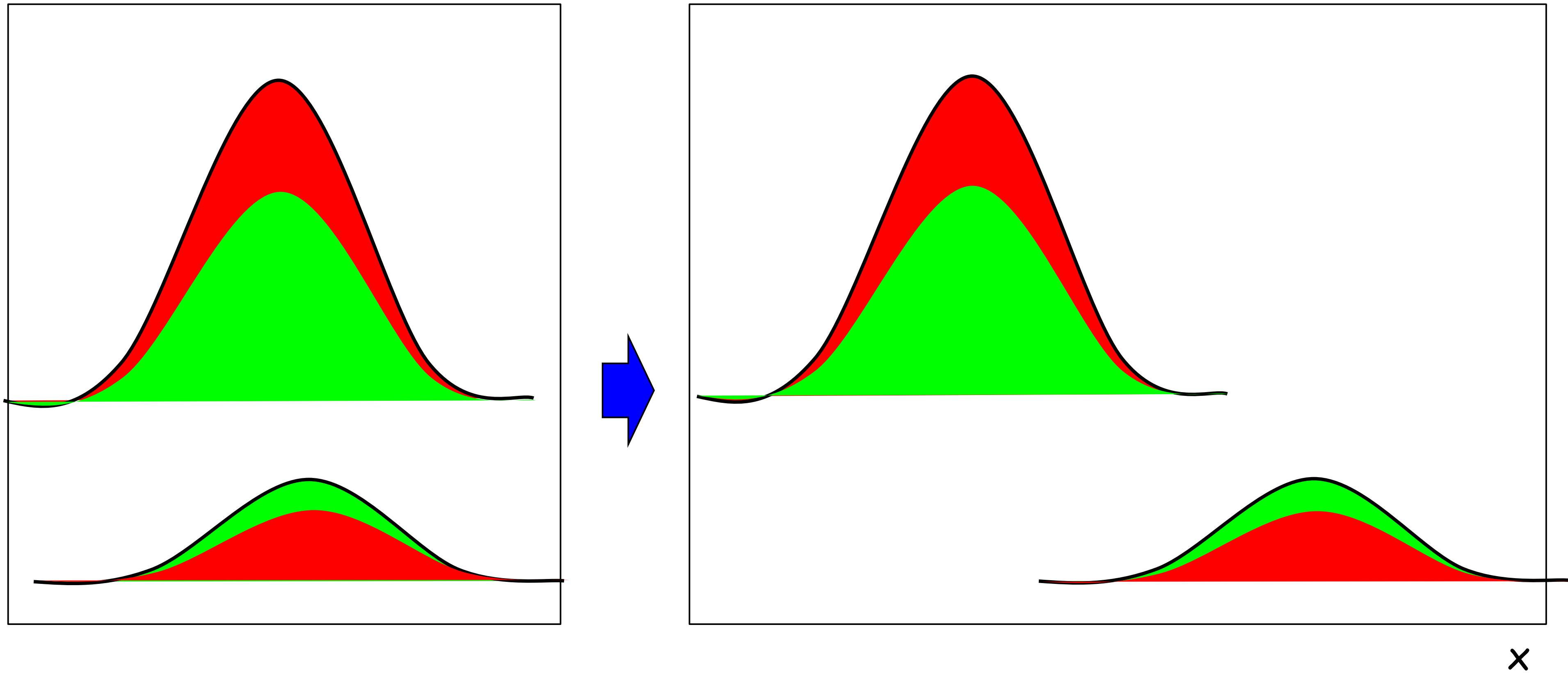
depth of oscillations

$$\phi = \frac{\Delta m^2 x}{2E} = \frac{2\pi x}{l_\nu}$$

$$l_\nu = \frac{4\pi E}{\Delta m^2}$$

- oscillation length

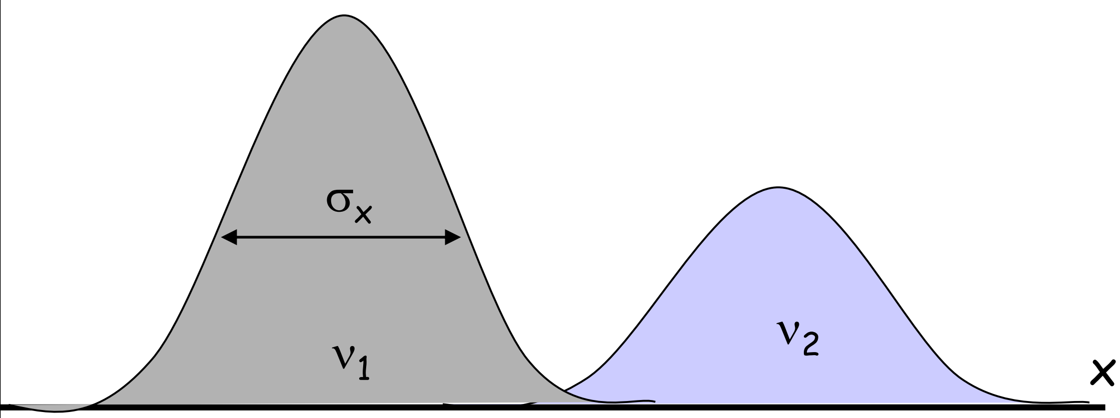
Loss of propagation coherence



Separation of the WP due to different group velocities

Propagation coherence

In the configuration space: separation of the wave packets due to difference of group velocities



$\Delta v_{gr} = \Delta m^2 / 2E^2$

Separation after propagation of distance L:

$\Delta v_{gr} L = \Delta m^2 L / 2E^2$

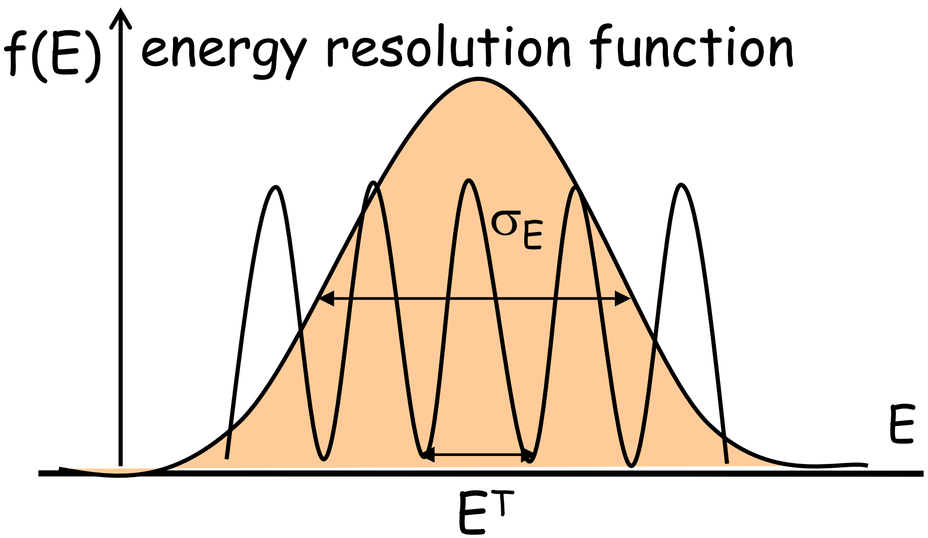
No overlap when: $\Delta v_{gr} L > \sigma_x$

Coherence length –distance at which overlap disappears

$L_{coh} = \sigma_x E^2 / \Delta m^2$

Coherence in the energy space

In the energy space: separation of WP is equivalent to averaging of oscillations over energy



Oscillatory period in the E space is determined by the condition $\Delta\phi\ (E^T) = 2\pi$

$$\phi = \Delta m^2\ L/2E$$

$$E^T = 4\pi\ E^2/(\Delta m^2\ L)$$

Averaging (= loss of coherence) if energy resolution σ_E is

$$E^T < \sigma_E$$

this leads to the same coherence length if $\sigma_E \sim 2\pi\ /\sigma_x$

$$L_{coh} = \sigma_x\ E^2/\Delta m^2$$

If $\sigma_E < E^T$ (good energy resolution) , the restoration of coherence occurs even if the wave packets separated

Observing propagation decoherence

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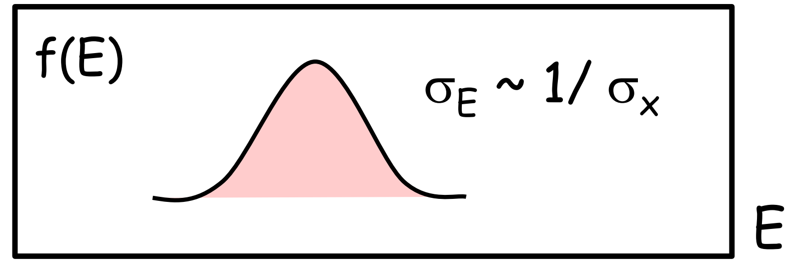
equivalent to integration over the energy uncertainty



Suppression of interference → damping of oscillations

Survival probability :

$$P_{ee} = \overline{P}_{ee} + \tfrac{1}{2} D(E, L) \sin^2 2\theta \cos \phi$$

Damping factor for Gaussian WP (proportional to overlap of WP)

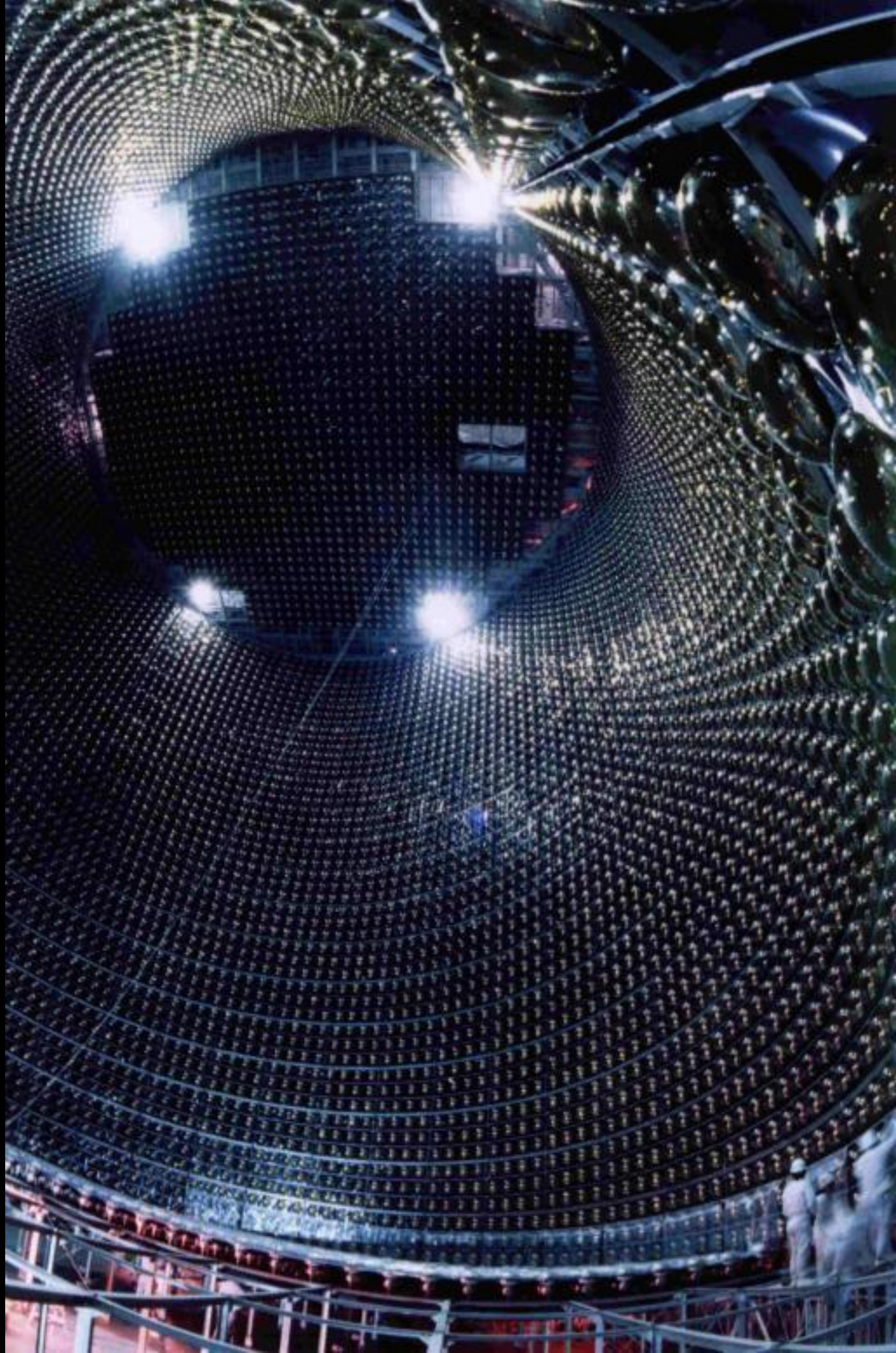
$$D(E, L) = \exp\left[- \tfrac{1}{2}(L/L_{coh})^2\right]$$

Coherence length

$$L_{coh} = \sigma_x \frac{E^2}{\Delta m^2}$$

Information is not lost and can be restored at detection

4. Localization
Coherence
Entanglement



Space-time localization diagram

E.Kh. Akhmedov,
D. Hernandez, A.Y.S.
1201.4128 [hep-ph]

Produced and propagated neutrino state

$$|\nu^P\rangle = \psi_1^P|\nu_1\rangle + \psi_2^P|\nu_2\rangle$$

$\psi_i^P = \psi_i^P(x - v_i t)$, produced WP, width σ_t^P , v_i - group velocities, $i = 1,2$

$$\psi_i^P \sim e^{i\phi_i} g_i(x - v_i t)$$

Detected state: $|\nu^D\rangle = \psi_1^D|\nu_1\rangle + \psi_2^D|\nu_2\rangle$

$\psi_i^D = \psi_i^D(x - x_D, t - t_D)$ - the detection WP with width σ_t^D

Amplitude = projection of propagated state onto detection state:

$$A(L, t_D) = \langle\nu^D|\nu^P\rangle = \Sigma_i \int dt\, \psi_i^{D*}(t - t_D)\, \psi_i^P(L - v_i t)$$ L- baseline

where for simplicity $\psi_i^D(x - x_D, t - t_D) = \delta(x - L)\, \psi_i^D(t - t_D)$,
so that integration over **x** is removed

$(x_D = L)$

Space-time localization diagram

E.Kh. Akhmedov and A.Y.S. JHEP 11 (2022) 082, 2208.03736 [hep-ph]

Oscillation probability

$$P(L) = \int dt_D |A(L, t_D)|^2 =$$

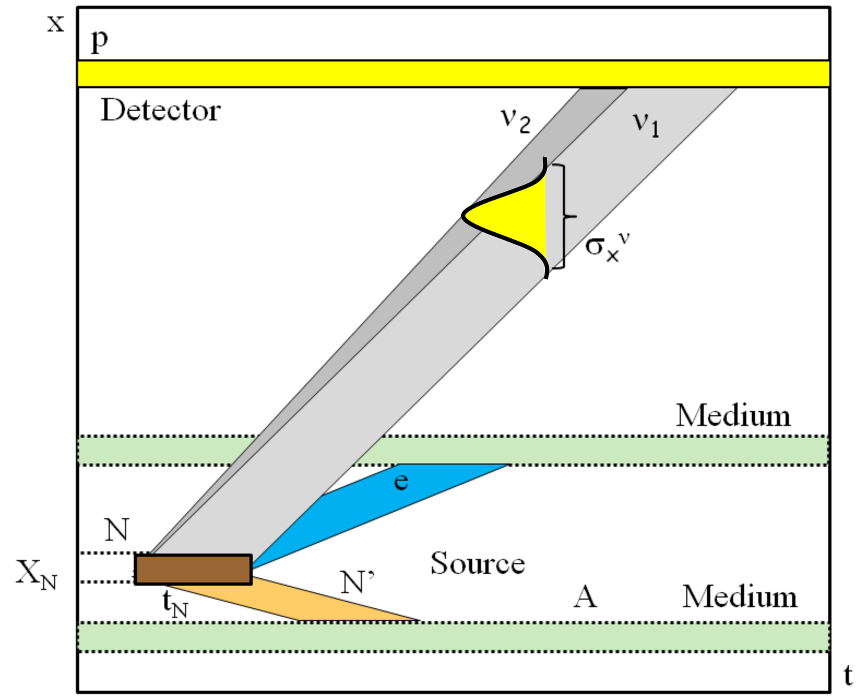
$$\int dt_D \left[|A_1(L, t_D)|^2 + |A_2(L, t_D)|^2\right]$$

$$+ 2\mathrm{Re}\int dt_D\, A_1(L, t_D)^* A_2(L, t_D)$$

interference

$A_i(L, t_D)$ – essentially the generalized WP

Further integration over interval of baseline L due to finite sizes of the source and detector



The slopes of bands are determined by group velocities

Detection

two extreme cases

A Y Smirnov, 2212.10242 [hep-ph]

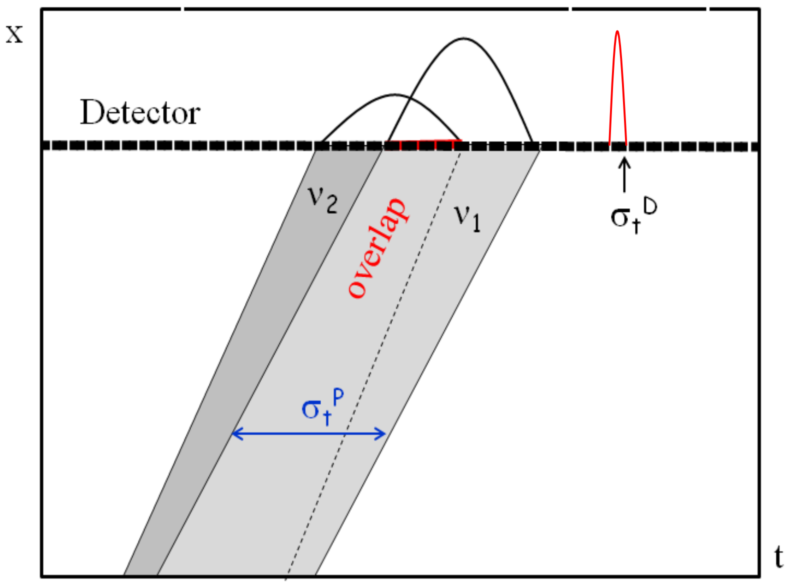
$\sigma_t^D \ll \sigma_t^P$

short detection coherence time

$\psi_i^D (t - t_D) \sim \delta (t - t_D)$

$A_i(L, t_D) \sim \psi_i^P(L - v_i t_D)$

Interference is determined by overlap of produced WP



$\sigma_t^D \gg \sigma_t^P$

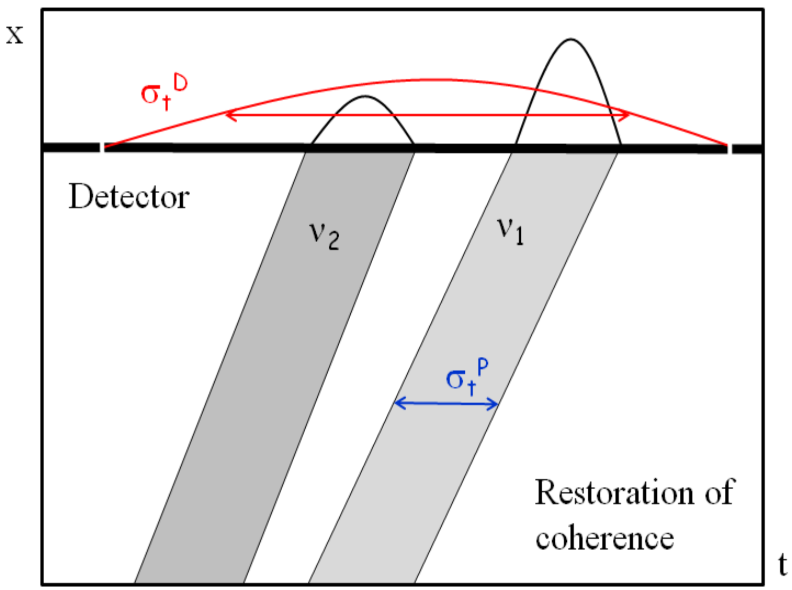
long detection coherence time

$\psi_i^P (t - t_D) \sim \delta (L - v_i t)$

$A_i(L, t_D) \sim \psi_i^D(L/v - t_D)$

restoration of coherence if

$\sigma_t^D \gg t_{sep}$



$\sigma_t^D \ll \sigma_t^P$

short detection coherence time

$\psi_i^D (t - t_D) \sim \delta (t - t_D)$

$A_i(L, t_D) \sim \psi_i^P(L - v_i t_D)$

Interference is determined by overlap of produced WP

$$2\mathrm{Re}\int dt_D \psi_1^P(L - v_1 t_D)^* \psi_2^P(L - v_2 t_D)$$

$\sigma_t^D \gg \sigma_t^P$

long detection coherence time

$\psi_i^P (t - t_D) \sim \delta (L - v_i t)$

$A_i(L, t_D) \sim \psi_i^D(L/v - t_D)$

restoration of coherence if

$\sigma_t^D \gg t_{sep}$

$$2\mathrm{Re}\int dt_D \psi_1^D(L/v_1 - t_D)^* \psi_2^D(L/v_2 - t_D)$$

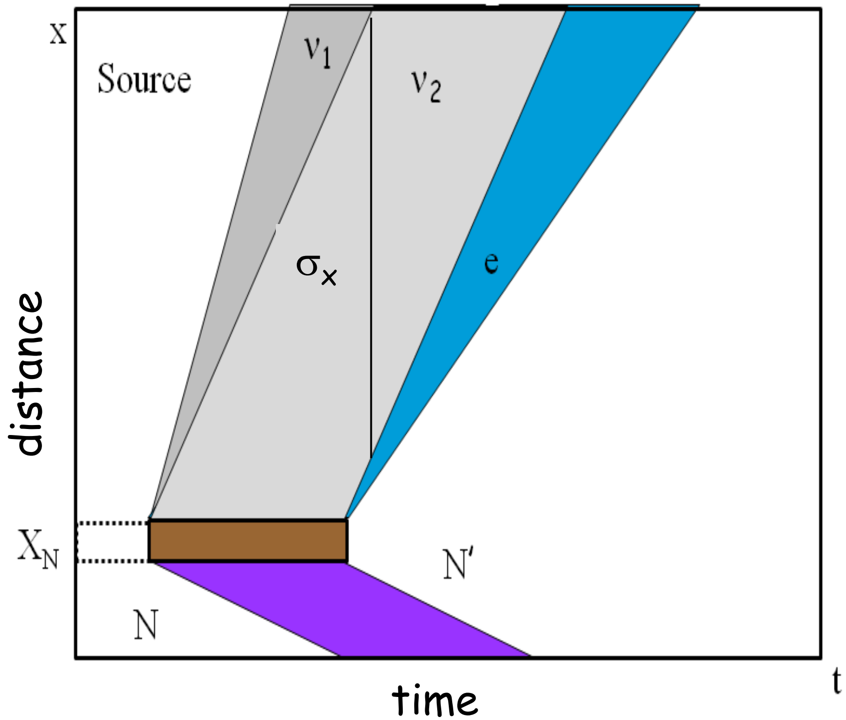
$$t_{sep} = L \frac{\Delta m^2}{2E^2}$$

Production

A Y Smirnov, 2212.10242 [hep-ph]

WP's are determined by localization region of the production process: overlap of localization regions of all particles involved but neutrinos.

E.g. in the β decay, $N \rightarrow N' + e^- + \bar{\nu}$



If N' and e^- are not detected or their interactions can be neglected

localization of process is given by localization of atom N

The latter is determined by time between two collisions of N, t_N

Then the size of ν WP

$$\sigma_x \sim v_\nu t_N \sim X_N\, c/v_N$$

enhancement factor wrt σ_N

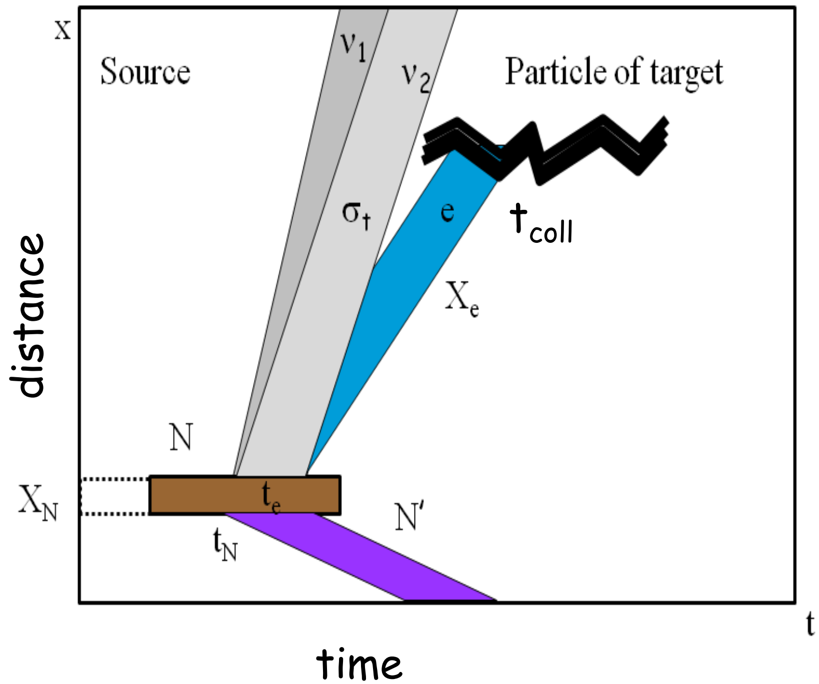
Entanglement and correlations

If N′ or/and e^- are detected or interact, this may narrow their WP's and therefore the neutrino WP. Consequently, enhance decoherence

If e^- is detected during time interval $t_e < t_N$, the size of ν WP will be determined by t_e

If e^- interacts with particles of medium which have very short time between collisions t_{coll}, then $\sigma_x \sim c t_{coll}$

Similar to the EPR paradox



Consider ν emission and interactions of e^- as unique process; contributions to its amplitude from different interactions regions of e appear with random phases ξ_k - incoherent $A_{tot} = A_k e^{i\xi_k}$

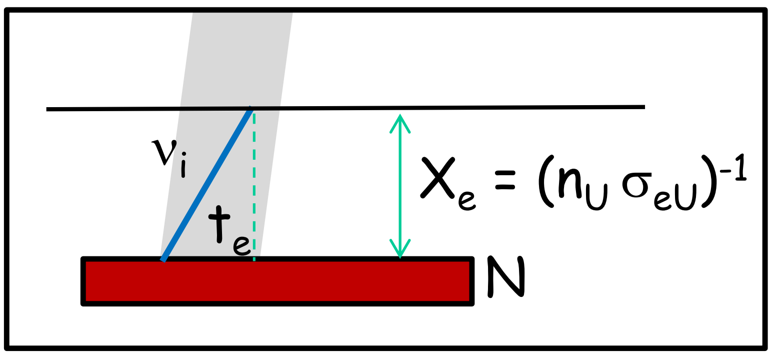
Effect of accompanying particles

Duration of ν production process is given by the shortest mean free time among particles involved

Electrons have the shortest

$\sigma_t = t_e = X_e/v_e$

X_e is determined by ionization of uranium, σ_{eU}

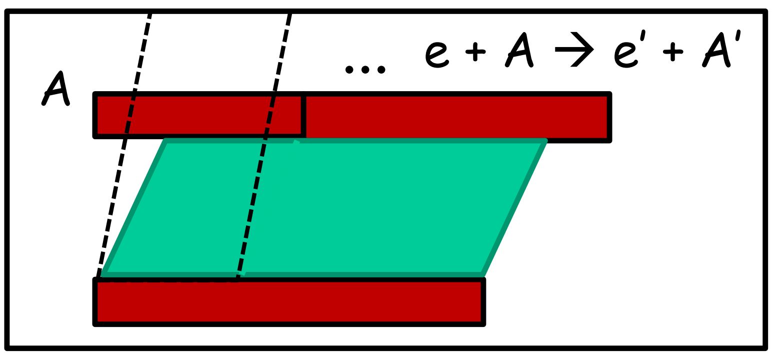


$\sigma_x = 2 \times 10^{-5}$ cm

"short cut" estimation: can be considered as the upper bound

Consideration of x-t localization of interactions of accompanying particles.

Chain of k processes of secondary interactions till equilibration (thermalization)

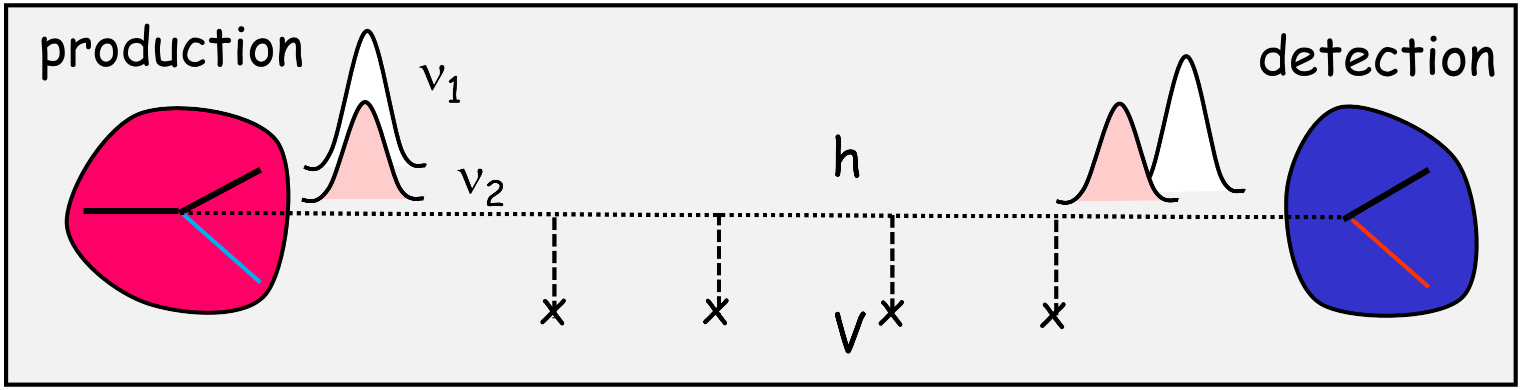


$\sigma_t \sim t_N / 2^k$

$\sigma_x = (5 - 10) \times 10^{-5}$ cm

Oscillations in vacuum

Wave packets of the eigenstates of propagation ν_i



Entanglement with accompanying particles

Vacuum : VEV V(x,t), interactions of ν with VEV
h V → m, θ, h = h(< τ >)

Interference: coherence at production propagation, detection

ν oscillations:

effect of propagation in space - time → Modification of geometry of x-t, metrics, GR, NO in the GW background

Quantum mechanical effect (superposition, interference) → Tests of QM, modification of QM, evolution equation..

Back-up

Vacuum and properties of oscillations

G.Dvali , L Funcke,
1602.03191 [hep-ph]

Neutrino vacuum condensate due to gravity. Order parameter

$$\langle \Phi_{\alpha\beta} \rangle = \langle \nu_\alpha^T C \nu_\beta \rangle \sim \Lambda_G = \text{meV} - 0.1 \text{ eV}$$

Cosmological phase transition at $T \sim \Lambda_G$

Neutrinos get masses $m_{\alpha\beta} \sim \langle \Phi_{\alpha\beta} \rangle$

Flavor is fixed by weak (CC) interactions and charged leptons with definite mass generated by usual Higgs field

$$m \sim U(\theta)^T \langle \Phi \rangle U(\theta)$$

$\langle \Phi \rangle = \text{diag}(\Phi_{11}, \Phi_{22}, \Phi_{33})$, mixing matrix

$T < \Lambda_G$ Relic neutrinos form bound states $\phi = (\nu_\alpha^T \nu_\beta)$ decay and annihilate into ϕ (neutrinoless Universe)

Symmetry of system SU(3)xU(1) spontaneously broken by neutrino condensate - ϕ are goldstone bosons

ϕ get small masses due explicit symmetry breaking by WI via loops

Mixing and topological defects

G.Dvali , L Funcke,
T Vachaspati
2112.02107 [hep-ph]

Symmetry breaking: $SU(3) \rightarrow Z_2 \times Z_2 \rightarrow I$ ⇒ string-wall network

global strings domain walls

Length scale of strings ~ inter-string separation

$$\xi = 10^{14}\ \mathrm{m}\ (\lambda/a_G) \left(\frac{\Lambda_G}{1\ \mathrm{meV}}\right)^{7/2}$$

(self-coupling of string field Φ/scale factor of phase transition)

Travelling around string winds VEV $\langle\Phi\rangle$ by the SU(3) transformation:

$$\langle \Phi (\theta_S) \rangle = \omega(\theta_W)^T \langle\Phi\rangle\, \omega(\theta_W)$$

$\omega(\theta_W)$ path - O(3) transformation with angles $\theta_W = (\theta_W^{12}, \theta_W^{13}, \theta_W^{23})$.
After the path ω lepton mixing changes as $U = U(\theta)\, \omega(\theta_W)$
over length ξ, $\theta_W = O(1)$

Solar system moves through the frozen string-DW background with v = 230 km/sec. For 6 years (operation of Daya Bay) $d = vt = 4 \times 10^{13}$ m - comparable with expected ξ

**

Standard Model + “m_ν”

Oscillations
Adiabatic conversion

Physics BSM responsible for m_ν can be introduced in such a way that feedback on SM is negligible

Dirac mass

$$y \bar{L} \nu_R H$$

Majorana mass

$$\frac{1}{\Lambda} LL\, HH$$

D5 Weinberg operator

Effective mass

$$m_\nu (E, n, ..)$$

generated by interactions with medium, e.g. DM

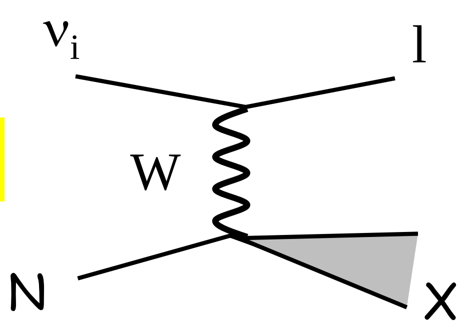
Propagation in terms of mass eigenstates

**

Without flavor states

(Eigenstates of propagation)

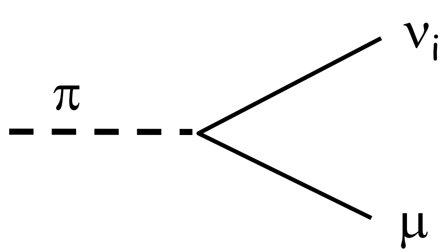
Scattering



$$\underbrace{\frac{g}{2\sqrt{2}}\, U^{PMNS}_{li}}_{\text{interaction constant}} \; \bar{l}\, \gamma^\mu (1 - \gamma_5)\, \nu_i\, W_\mu + h.c.$$

Eigenstates of the Hamiltonian in vacuum

$\pi \rightarrow \mu\ \nu_i$



Lagrangian of interactions

wave functions of accompanying particles

compute the wave functions of neutrino mass eigenstates = wave packets

Set-up. Remarks

**

Flux not a single neutrino

Quantum mechanics: formulate problem in terms of observables
to avoid misconception

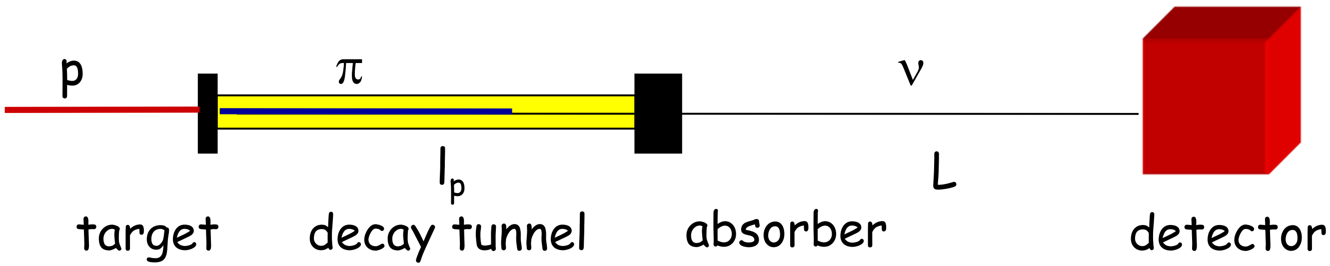
External –internal wave packets pictures

Factorization

Usually many complications disappear in normalization

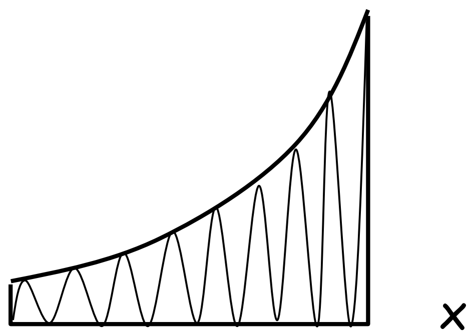
Example of neutrino wave packets

**



D. Hernandez, AS
E. Kh Akhmedov,
D. Hernandez, AS
arXiv:1110.5453

ν wave packet



The length of ν WP emitted
in the forward direction:

$$\sigma = l_p \frac{v - v_\pi}{v_\pi}$$

Doppler effect

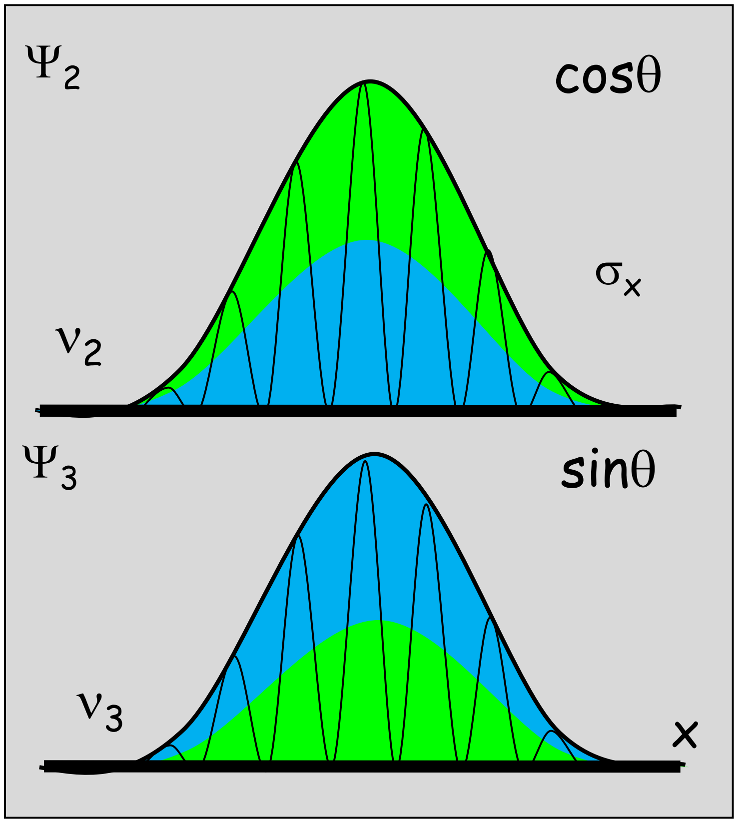
Interplay of interaction (at production and detection) and propagation

Description of neutrino state in terms of WP

**

ν_μ

$\phi = 0$



$$\Psi_k = g_k(x - v_k t)\, e^{i\phi_k}$$

Shape factor

v_k - group velocity

Phase factor

$\phi_k = p_k x - E_k t$

k = 2,3

	ϕ
$\nu_\mu = \cos\theta\, \nu_2 + \sin\theta\, \nu_3$	0
$\nu_\tau = -\sin\theta\, \nu_2 + \cos\theta\, \nu_3$	π

$e^{i\pi}$

differ by
- mass content
- phase difference

Inverting

$$\nu_2 = \cos\theta\, \nu_\mu - \sin\theta\, \nu_\tau$$

$$\nu_3 = \cos\theta\, \nu_\tau + \sin\theta\, \nu_\mu$$

Interference of the same flavor parts of the WP which depends on phase

Equivalence of considerations in p- and x- spaces **

Averaging over energy – loss of coherence due to separation of WP

L. Stodolsky

in stationary state approximation

or if there is no time tagging

→

Wave packets are unnecessary for computation of observable effects

The only what is needed is to make correct integration over

Production energy spectrum

Energy resolution of a detector

But this is nothing but WP in the energy space

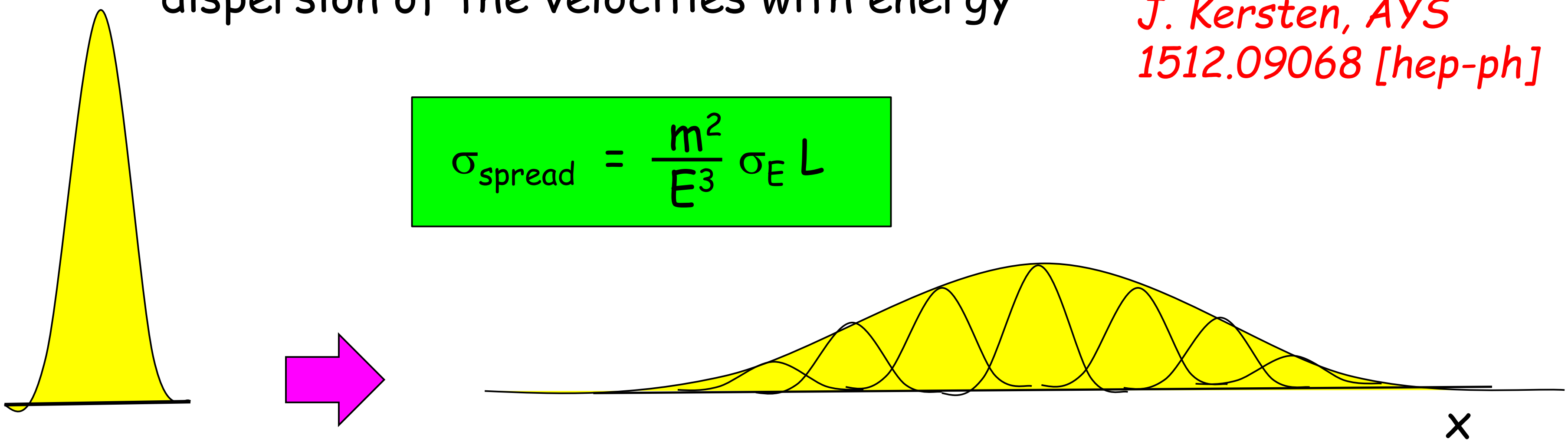
Spread of wave packets

**

Due to presence of waves with different energies in a packet dispersion of the velocities with energy

J. Kersten, AYS 1512.09068 [hep-ph]

$$\sigma_{spread} = \frac{m^2}{E^3}\sigma_E L$$



Beryllium neutrinos

$\sigma_x = 2\pi/\Gamma_{Be} = 6\ 10^{-8}$ cm → $\sigma_{spread} \sim 4\ 10^{-6}$ cm

$\Delta x_{sep}^S \sim 2\ 10^{-3}$ cm → $\Delta x_{sep}^S \gg \sigma_{spread}$

Oscillations of mass states in the Earth

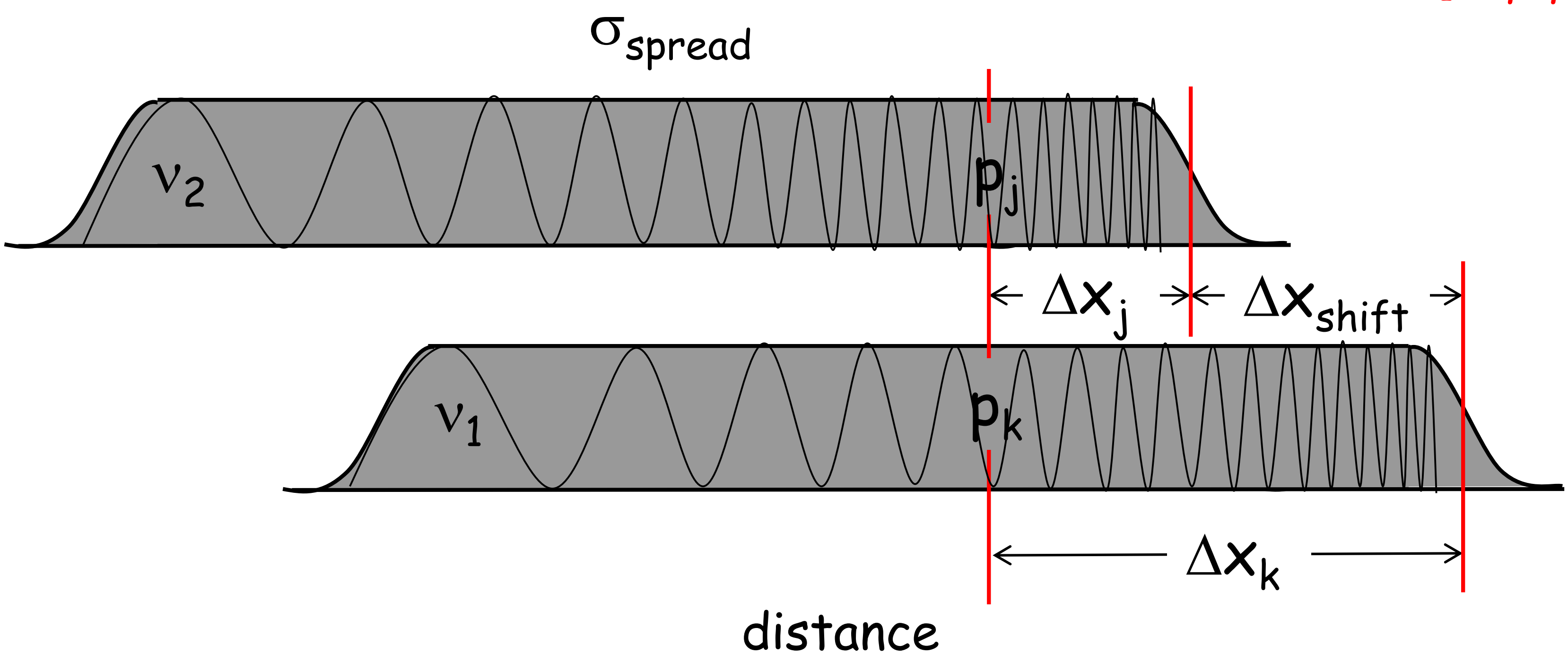
$\Delta x_{sep}^E \sim 5\ 10^{-8}$ cm → $\sigma_x \sim \Delta x_{sep}^E \ll \sigma_{spread}$

Loss of coherence: YES NO?

Spread of the wave packets and coherence**

Spread does not improve coherence

J. Kersten, AYS
1512.09068 [hep-ph]

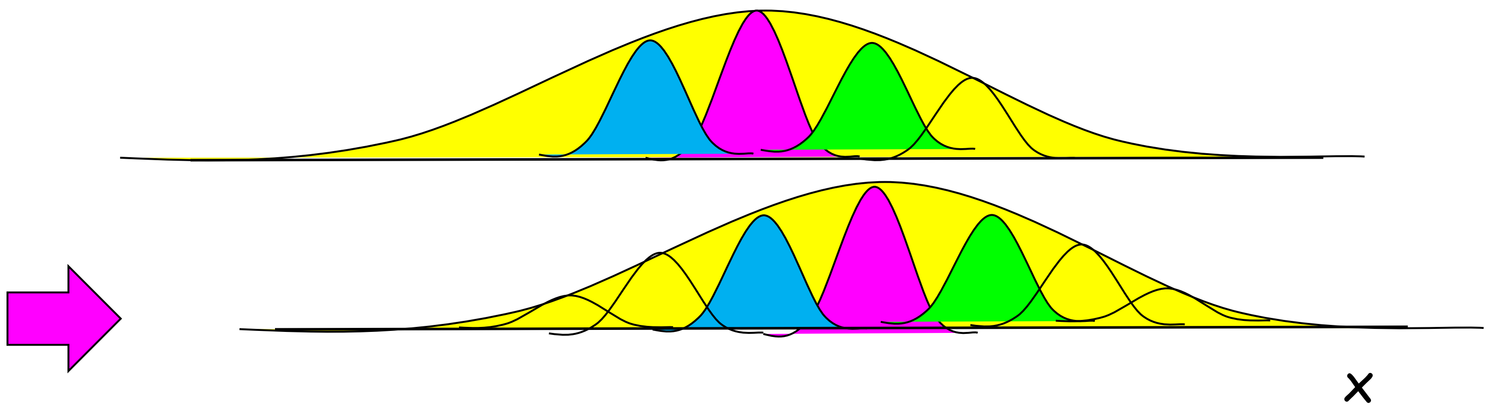


Averaged (effective) momenta in overlapping parts are different → averaging

Spread of wave packets

**

Coherent parts (same color)
- stop to overlap



Loss of coherence between overlapping parts of the WP

WP becomes "classical": describing that the highest energy neutrinos arrive first

No effect on coherence if considered in the p-space

Propagation of wave packets

**

What happens?

Phase difference change	Due to different masses (dispersion relations) → phase velocities	**Oscillations**
Separation of wave packets	Due to different group velocities	**Loss of coherence**
Spread of individual wave packets	Due to presence of waves with different momenta and energy in the packet	**Loss of coherence within WP**

Evolution equation

If loss of coherence and other complications related to WP picture are irrelevant – `` point-like" picture

$$i \frac{d\,\Psi}{d\,t} = H\,\Psi$$

$$\Psi = \begin{pmatrix} \psi_e \\ \psi_\mu \\ \psi_\tau \end{pmatrix}$$

What is H?

For ultra relativistic ν

$$H = E \sim p + \frac{m^2}{2E}$$

omit p, substitute $m^2 \rightarrow M\,M^+$

M is the 3x3 mass matrix

$$H = \frac{M\,M^+}{2E}$$

$$M\,M^+ = U\,M_{diag}^{\ 2}\,U^+$$

$$M_{diag}^{\ 2} = \text{diag}\,(m_1^2,\ m_2^2,\ m_3^2)$$

U - mixing matrix

Hamiltonian for 2ν

In the flavor basis $\nu_f = (\nu_e , \nu_\tau)^T$

$$H_{tot} = \frac{M M^+}{2E}$$

$$M M^+ = U M_{diag}^2 U^+ \qquad U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

$$M_{diag}^2 = \mathrm{diag}\,(m_1^2,\ m_2^2)$$

$$H_{tot} = \frac{\Delta m^2}{4E} \begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix}$$

Neutrino polarization vector in the flavor space

$$\psi = \begin{pmatrix} \nu_e \\ \nu_\tau \end{pmatrix} \Rightarrow$$

Def. polarization vector:

$$\mathbf{P} = \psi^+ \sigma/2\, \psi \quad (*)$$

Pauli matrices

$$\mathbf{P} = \begin{pmatrix} \mathrm{Re}\, \nu_e^+ \nu_\tau \\ \mathrm{Im}\, \nu_e^+ \nu_\tau \\ \nu_e^+ \nu_e - 1/2 \end{pmatrix}$$

elements of density matrix

Evolution equation:

$$i \frac{d\Psi}{dt} = H\Psi \Rightarrow i \frac{d\Psi}{dt} = -(\mathbf{B}\,\sigma/2)\,\Psi$$

where $\mathbf{B} = \frac{2\pi}{l_\nu}(\sin 2\theta, 0, \cos 2\theta)$

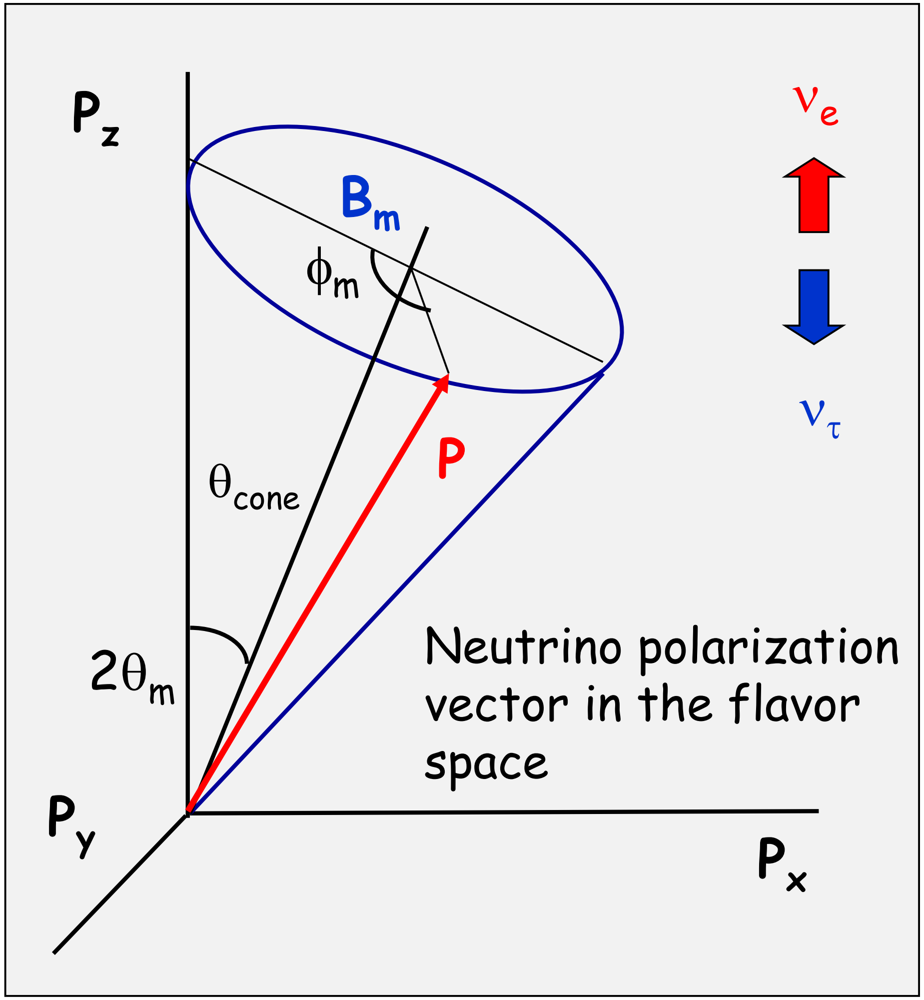
Differentiating **P** from (*) and using equation of motion for Ψ

$$\frac{d\mathbf{P}}{dt} = -[\mathbf{B} \times \mathbf{P}]$$

Coincides with equation for the electron spin precession in the magnetic field

Graphic representation

of oscillations with the electron spin precession in the magnetic field



$|\mathbf{P}| = \frac{1}{2}$ polarization vector

$$\mathbf{B} = \frac{2\pi}{l_\nu}(\sin 2\theta,\ 0,\ \cos 2\theta)$$

$$P_{ee} = \nu_e^+ \nu_e = P_Z + 1/2$$

$\phi = 2\pi t / l_\nu$ oscillation phase

In general, degrees of freedom in matter:

$\theta_m (n, E)$ - mixing angle
$\phi_m (n, E)$ - phase
$\theta_{cone} (dn/dx)$ - cone angle

Subscript m means in medium
for vacuum -

Based on

[1] Neutrino production coherence and oscillation experiments.
E. Akhmedov, D. Hernandez, A. Smirnov, JHEP 1204 (2012) 052,
arXiv:1201.4128 [hep-ph]

[2] Neutrino oscillations: Entanglement, energy-momentum conservation and QFT.
E.Kh. Akhmedov, A.Yu. Smirnov, Found. Phys. 41 (2011) 1279-1306
arXiv:1008.2077 [hep-ph]

[3] Paradoxes of neutrino oscillations.
E. Kh. Akhmedov, A. Yu. Smirnov Phys. Atom. Nucl. 72 (2009) 1363-1381
arXiv:0905.1903 [hep-ph]

[4] Active to sterile neutrino oscillations: Coherence and MINOS results.
D. Hernandez, A.Yu. Smirnov, Phys.Lett. B706 (2012) 360-366
arXiv:1105.5946 [hep-ph]

[5] Neutrino oscillations: Quantum mechanics vs. quantum field theory.
E. Kh. Akhmedov, J. Kopp, JHEP 1004 (2010) 008
arXiv:1001.4815 [hep-ph]

In principle:

Starting from the first principles

Lagrangian

$$\frac{g}{2\sqrt{2}}\, \bar{l}\, \gamma^{\mu}(1-\gamma_5)\nu_l\, W^{+}_{\mu}$$

$$-\tfrac{1}{2}\, m_L\, \nu_L{}^{T} C \nu_L$$

$$-\bar{l}_L\, m_l\, l_R + h.c.$$

QFT

QM

Amplitudes, probabilities of processes

Observables, number of events, etc..

Actually not very simple

Quantum mechanics at macroscopic distances

What is the problem?

Set-up

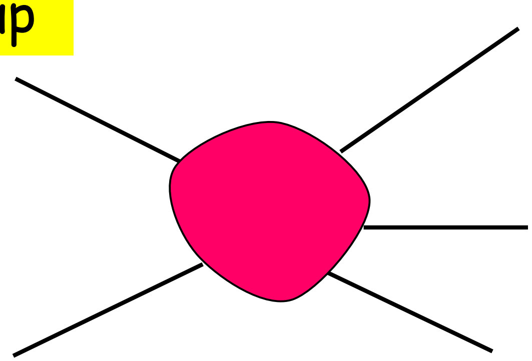
Formalism should be adjusted to specific physics situation

Initial conditions

Recall, the usual set-up

asymptotic states described by plane waves

- enormous simplification



single interaction region

Approximations

Approximations, if one does not want to consider whole history of the Universe to compute e.g. signal in Daya Bay

Truncating the process

Localizations

of some initial and final particles

... for maximal mixing

**

$\theta = \pi/4$

$$\nu_\mu = (\nu_2 + \nu_3) / 2^{1/2}$$
$$\nu_\tau = (- \nu_2 + \nu_3) / 2^{1/2}$$

The difference in phase only, mass composition is the same!
Interaction of neutrino state depends on the phase difference between mass eigenstates

$$\nu(\phi) = (e^{i\phi} \nu_2 + \nu_3)/2^{1/2}$$

$$\nu(\phi) = \begin{cases} \nu_\mu & \phi = 0 \\ \nu_\tau & \phi = \pi \\ \nu_\mu, \nu_\tau & 0 < \phi < \pi \end{cases}$$

Flavor composition (interaction properties) depends on ϕ

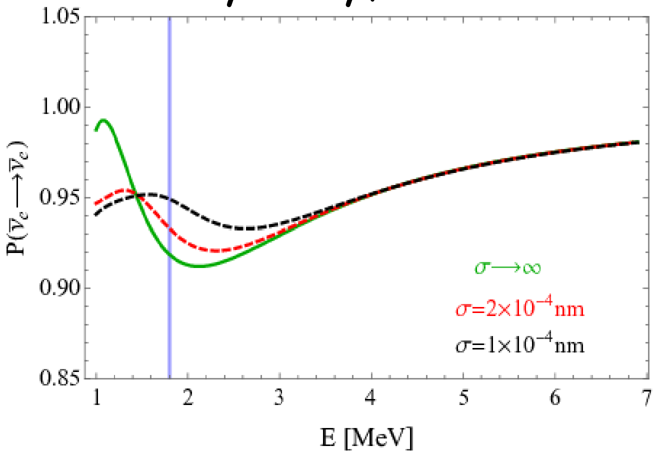
Decoherence of reactor neutrinos

A de Gouvea, V De Romeri, C.A. Termes, 2104.05806 [hep-ph]

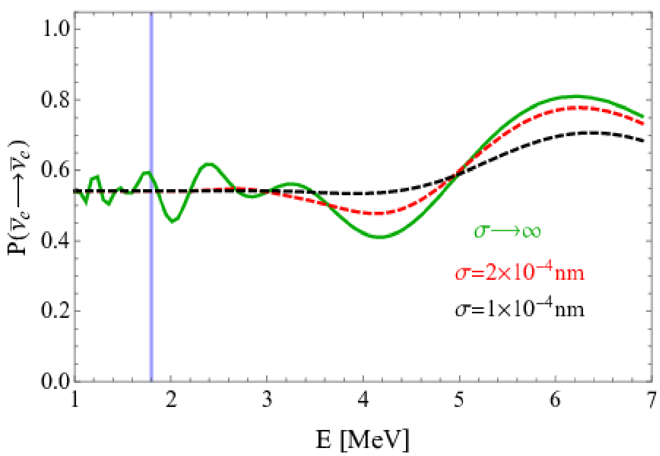
Bound on size of the WP

Expected damping effect

Daya Bay, RENO



KamLAND



Absence of decoherence (damping) effect means

$$L \ll L_{coh} \Rightarrow \sigma_x > L \frac{\Delta m^2}{2E^2}$$

Analysis of data: $\sigma_x > 2.1 \times 10^{-11}$ cm (90% C.L.)

The bound corresponds to the energy resolution of detectors δ_E

$$\sigma_x \sim 1/\delta_E$$

Other studies

Daya Bay: decoherence due to finite momentum spread σ_p

$\sigma_p /p < 0.23$ (95% C.L.)

for p = 3 MeV: $\sigma_x \sim 1/\sigma_E = 2.8 \times 10^{-11}$ cm

F.P. An, et al, 1608.01661 [hep-ex]

JUNO in future may set the limit

$\sigma_p /p < 10^{-2}$ (95% C.L.) $\rightarrow \sigma_x > 2.3 \times 10^{-10}$ cm

J. Wang et al. 2112.14450 [hep-ex]

Decoherence in oscillations active – eV scale sterile

Damping effects in various experiments computed for $\sigma_x = 2.1 \times 10^{-11}$ cm (as found in A de Gouvea et al).

C.A.Arguelles et al, 2201.05108 [hep-ph]

Claims:

- decoherence allows to reconcile BEST result with reactor bounds;
- results of analysis should be presented in two forms: with and without decoherence

Propagation decoherence and energy resolution

integration over the energy resolution of setup
– another sources of damping

E.Kh. Akhmedov and A.Y.S.
2208.03736[hep-ph]

$R(E_r, E)$ energy resolution in experimental set-up (width δ_E):
- spectrum of produced neutrinos (line), or
- energy resolution of a detector

$f(E, \bar{E})$ – WP of produced neutrino in energy representation acts on oscillations, as R does, and can be attached to $R(E_r, E)$

Effective resolution function

$$R_{eff}(E_r, E) = \int d\bar{E}\, R(E_r, \bar{E})\, |f(E, \bar{E})|^2$$

For Gaussian f and R, R_{eff} is also Gaussian with width

$$\delta_E^2 + \sigma_E^2$$

The problem: to disentangle the two contributions

WP's of reactor neutrinos

Source: β-decays of fragments N of nuclear fission

$$N \rightarrow N' + e^- + \bar{\nu}$$

N quickly thermalise $\rightarrow$ in equilibrium with medium in the moment of decay $\rightarrow$ the average velocity:

$$v_N \sim [3T/\, m_N]^{-1/2}$$

If N' and e^- are not detected or their interactions can be neglected, localization of ν production process is given by localization of N.

$$\sigma_x \sim v_\nu t_N \sim X_N\, c/v_N$$

t_N - time between two collisions of N with other atoms

$$t_N \sim [\sigma_{AA}\, n_U\, v_N]^{-1}$$

σ_{AA} geometric cross-section $\sigma_{AA} \sim \pi(2r_{vdW})^2$ Van der Waals radius

n_U - number density of Uranium

$$\sigma_x = 2.8 \times 10^{-3}\ \text{cm}$$

Implications

1. $\sigma_x / \sigma_x^{exp} = 10^5 - 10^6$ $\sigma_x \gg \sigma_x^{exp}$

2. Corresponding energy uncertainty $\sigma_E \sim 1$ eV
   while energy resolution $\delta_E \sim 10^5$ eV

   To be sensitive to WP separation energy resolution function should be known with better that 10^{-5} accuracy

3. For Cr source: $\sigma_x = 1.4 \times 10^{-4}$ cm

4. Large Δm^2 does not help since oscillatory pattern shows up at $L \sim l_\nu$

   but $L_{coh} \sim l_\nu \sim 1/\Delta m^2$ $\rightarrow$ Δm^2 cancels in damping factor

5. If some additional damping is found, it is due to some new physics and not due to WP separation

6. Experiments with $L \sim L_{coh}$? Lower energies? Widening lines?

Neutrino oscillations

**

Simple but not as simple as in many textbooks (pointlike or plane wave)

Conceptually wrong but gives correct final results for physically relevant situations

Wave packet picture (or QFT) should be used to avoid misconception.
Complications related to WP disappear in final result in physically relevant cases

…disappear in normalization

Physics summary

Appearance probability

$$P(\nu_\mu) = \sin^2 2\theta \, \sin^2 \frac{\pi x}{l_\nu}$$

All complications are "absorbed" in normalization
or reduced to partial averaging of oscillations
or lead to negligible corrections of order $m/E \ll 1$

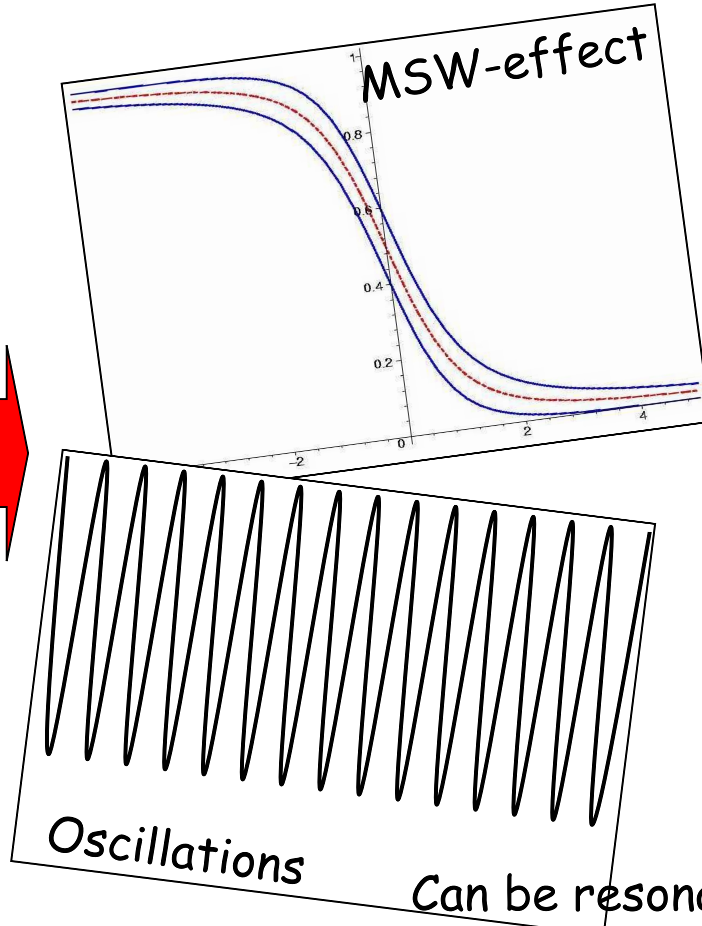
Oscillations - effect of the phase difference increase between mass eigenstates

Admixtures of the mass eigenstates ν_i in a given neutrino state do not change during propagation

Flavors (flavor composition) of the eigenstates are fixed by the vacuum mixing angle

Measurements

MINOS
T2K
NOvA
Solar neutrinos
KamLAND
Double Chooz
Daya Bay
RENO
NEOS
Atmospheric neutrinos
SK
DeepCore
Antares



Can be resonantly enhanced in matter

Δm^2

θ

Plane wave and point-like descriptions

- in many presentations leading to confusion

Infinite space, but neutrino oscillations are finite space phenomenon

Additional constraints required

Mass states have different velocities, separate in space interference is not possible

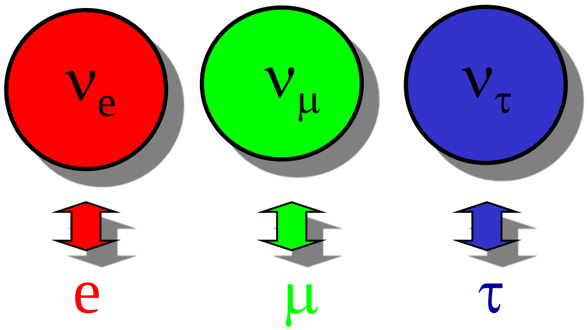
Wave packets treatment intermediate between the two above cases

Leo Stodolsky: In most of the cases from practical point of view consideration in terms of spatial WP is not necessary

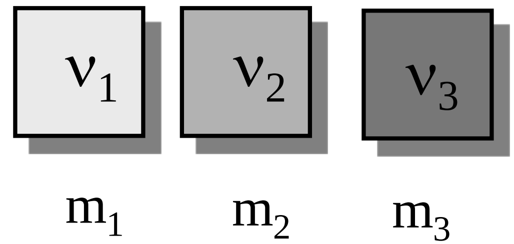
But WP consideration allows to avoid various misconceptions

Flavors and mixing

Flavor neutrino states:



Mass eigenstates



Mixing

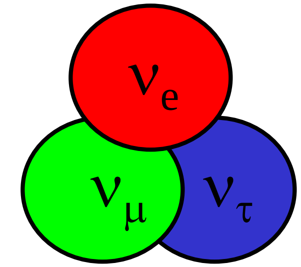
Flavor states – Weak interaction states

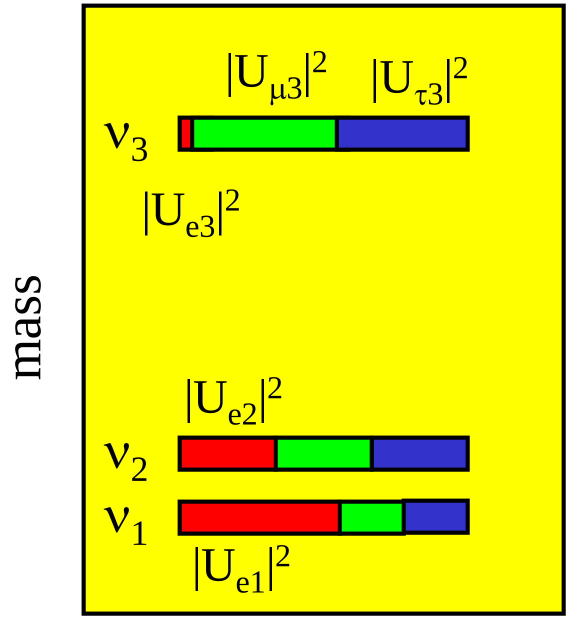
Flavor states $\neq$ Mass eigenstates

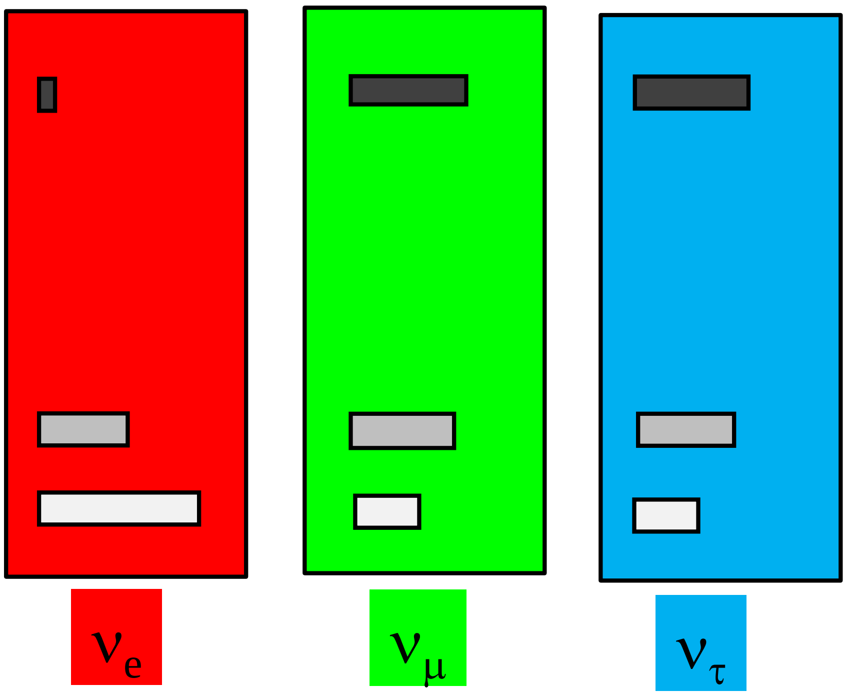
Combinations of mass states described by mixing matrix U_{PMNS}

$$\nu_f = U_{PMNS}\,\nu_{mass}$$

Mixing: dual role







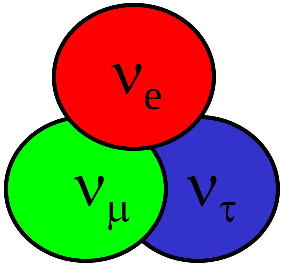
Flavor content of the mass states

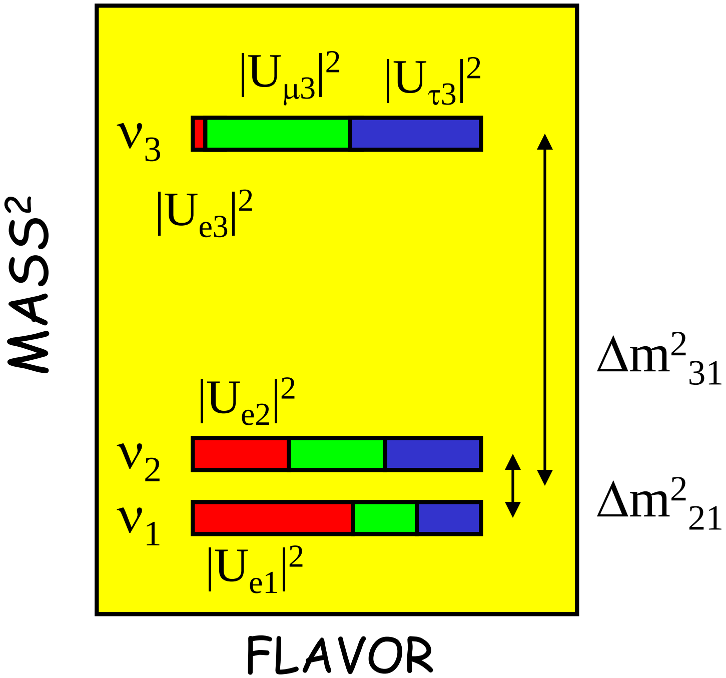
$$\nu_{mass} = U_{PMNS}^{+}\,\nu_f$$

Mass content of the flavor states

$$\nu_f = U_{PMNS}\,\nu_{mass}$$

Mixing angles





Normal mass hierarchy

$$\Delta m^2_{31} = m^2_3 - m^2_1$$

$$\Delta m^2_{21} = m^2_2 - m^2_1$$

Mixing parameters

$$\tan^2\theta_{12} = |U_{e2}|^2 \ / \ |U_{e1}|^2$$

$$\sin^2\theta_{13} = |U_{e3}|^2$$

$$\tan^2\theta_{23} = |U_{\mu3}|^2 \ / \ |U_{\tau3}|^2$$

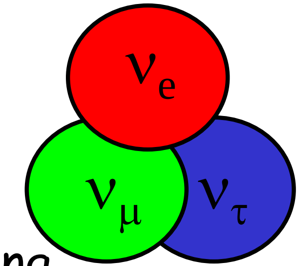
Mixing matrix:

$$\nu_f = U_{PMNS}\, \nu_{mass}$$

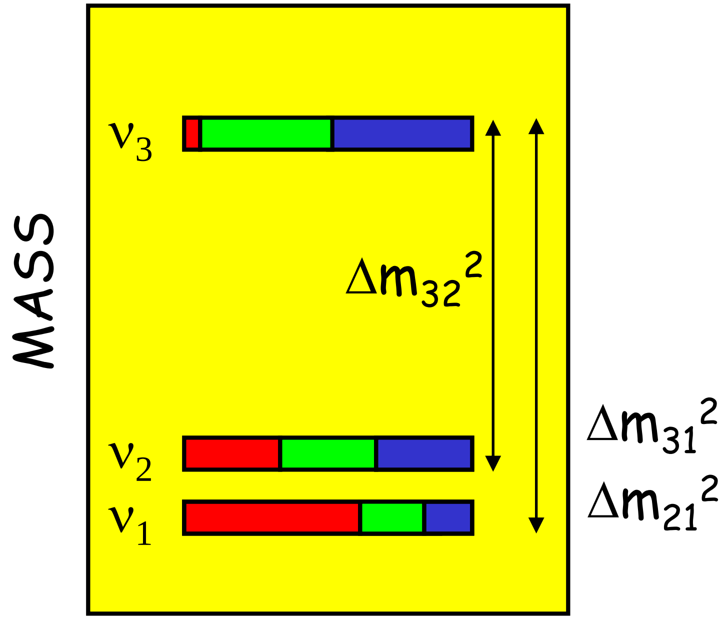
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{PMNS} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

$$U_{PMNS} = U_{23} I_\delta U_{13} I_{-\delta} U_{12}$$

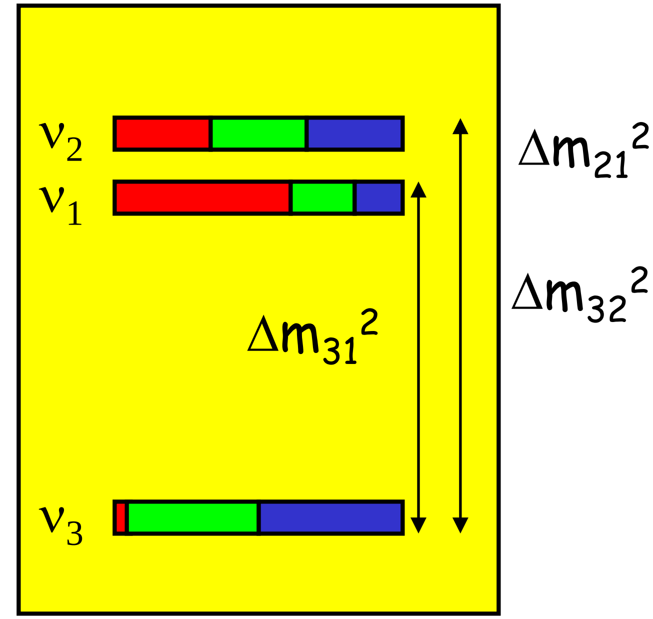
Neutrino mass ordering



Normal ordering



Inverted ordering



Preferable by
global fit of
oscillation data

Vacuum mixing

$$\nu_f = U_{PMNS}\, \nu_m$$

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Standard parametrization

$U_{PMNS} = U_{23} I_{\delta} U_{13} I_{-\delta} U_{12}$ I_M

$I_{\delta} = \mathrm{diag}\,(1, \; 1, \; e^{i\delta})$
Dirac phase matrix

$$
\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & -s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}
\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}
\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} I_M
$$

$I_M = \mathrm{diag}\,(1, e^{i\alpha_{21}/2}, e^{i\alpha_{31}/2})$ - matrix of Majorana phases

Not unique parametrization
Convenient for phenomenology, especially for oscillations in matter
Insightful for theory?

Mixing and mass matrices

Origin of mixing: off-diagonal neutrino mass matrix in basis of flavor states $M_l \neq M_\nu$

Diagonalization:

Mixing matrix

Mass spectrum (Elements of diagonalized Matrix)

$M_l = U_{lL} m_l^{diag} U_{lR}^+$

$m_l^{diag} = (m_e, m_\mu, m_\tau)$

$M_\nu = U_{\nu L} m_\nu^{diag} U_{\nu L}^T$

$m_\nu^{diag} = (m_1, m_2, m_3)$

for Majorana neutrinos

CC in terms of mass eigenstates: $\bar{L} \gamma^\mu (1 - \gamma_5) U_{PMNS} \nu_{mass}$

$\Rightarrow U_{PMNS} = U_{lL}^+ U_{\nu L}$

Flavor basis: $M_l = m_l^{diag}$ $U_{PMNS} = U_{\nu L}$

Masses of fermions in SM

$$m = h \langle H \rangle$$

h is constant yukawa coupling
Confirmed by LHC with single Higgs boson

Why not the same for neutrinos?

$h \sim 2\ 10^{-13}$

Still OK

In the first approximation in SM $m_\nu = 0$.
Neutrino mass as correction to (perturbation of) the SM ?

Weinberg approach

EFT and the neutrino mass

S. Weinberg

Only observed SM particles (multiplets) are involved.
New physics should be parametrized by high dimensional (non-renormalizable) operators in the Hamiltonian.
Analogy with Fermi theory of WI at low energies

Next to the SM operators (D4) is the D5 operator

$$\boxed{\frac{1}{\Lambda} LLHH}$$

Large scale of new physics

generates the Majorana neutrino mass

$$\frac{\langle H \rangle^2}{\Lambda}$$

Other new physics appears in order Λ^2 → strongly suppressed

That's all? Will we learn more? Violation of universality, unitarity

Right handed neutrinos are the key!

Their existence and properties play fundamental role
in understanding of masses and mixing of neutrinos .

Formally, they do not exist in the SM and their appearance is considered as physics BSM

Reasons:

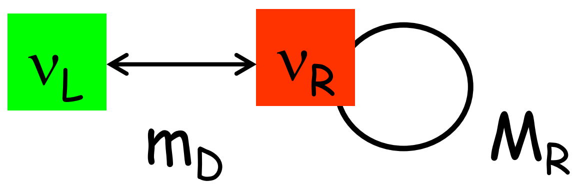
- they do not have electroweak gauge interactions in the SM
- automatically explain zero mass of neutrino - dominant idea in early days

- Aesthetics: why not, if other SM fermions have?
- Justified in plausible gauge extensions of SM: B – L and L - R symmetric models

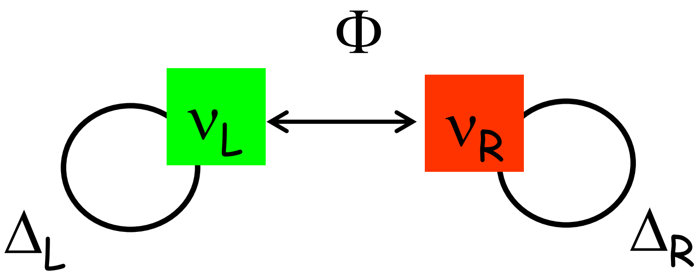
$$U(1)_{B-L} \rightarrow SU(2)_L \times SU(2)_R \times U(1)$$

- Gravitational anomaly cancellations

See-saw and L-R



Naturally embedded into $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$



$\langle \Delta_R \rangle \rightarrow M_R$, m_{W_R}

Seesaw scale is determined by the scale of L- R symmetry violation

$\langle \Delta_L \rangle \neq 0$ → type II

Type I + II

Introduction

The neutrino flavor transformations play the key role in developments of neutrino physics with applications to the Earth based neutrino experiments, to solar, atmospheric and supernova neutrinos, to cosmic neutrinos of high energies.

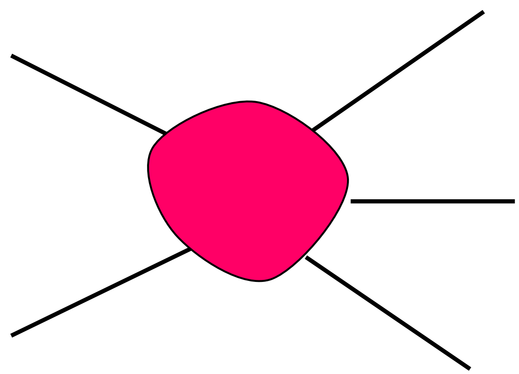
Flavor transformations in vacuum and various media include oscillations, adiabatic conversion in matter parametric effects, transformations in multi-layer media, collective transformations.

Physics derivations of the results are given.
Applications and possible manifestations of physics beyond the standard model will be outlined.

My “short cut” way to get results and understand things

Oscillations vs. scattering

Scattering set-up



- Single interaction region
- Asymptotic states described by plane waves
- Integration over infinite space-time

Enormous simplification:

Integration gives delta functions which reflect exact enrgy- momentum conservation, etc

Neutrino propagation

Equation of motion

Mass

Medium

Eventually everything will be
around neutrino mass
its nature and origins
and about possibilities to establish
this nature and origins

Oscillations: general picture

Vacuum and medium

…disappear in normalization

Mass and oscillations

Whatever affect vacuum,
affects mas and oscillations

…disappear in normalization