Modified Gravity in a Nutshell

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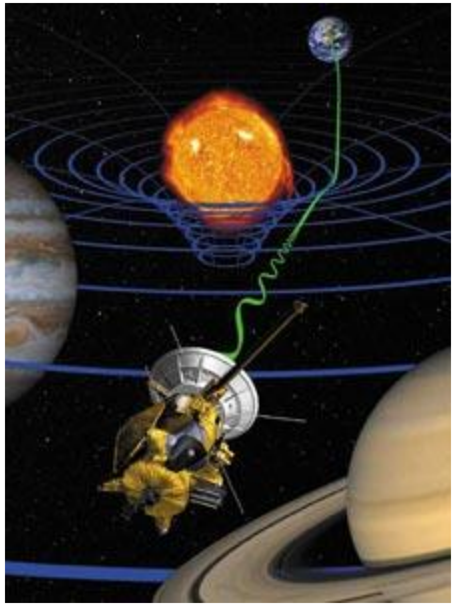
Deviations from Newton's law are parametrised by:

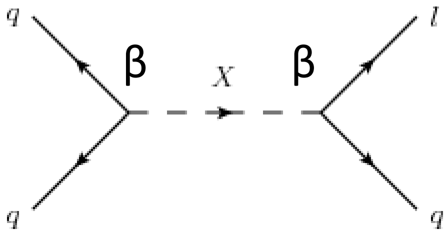
$$\Phi = -\frac{G_N M}{r}(1 + 2\beta^2 e^{-r/\lambda})$$

For long range forces with large λ, the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta^2 \leq 4 \cdot 10^{-5}$$



Bertotti et al. (2004)



The particle physics vision: a scalar field exchanged between two fermions with coupling strength β

Massive Gravity

Quite possible that the acceleration of the late Universe is due to a modification of General Relativity on large scales. The most natural way of introducing it is to give a mass to the graviton, i.e gravity would lose its influence on large distances:

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$$

Field associated to spin-2 graviton.

Propagation equation in GR:

$$\nabla^\rho \nabla_\rho (h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu} h) = -16\pi G_N T_{\mu\nu}$$

$$h = h^\mu_\mu, \quad T = T^\mu_\mu$$

$$\boxed{h_{\mu\nu} = \frac{16\pi G_N}{p^2}(T_{\mu\nu} - \frac{\eta_{\mu\nu}}{2}T)}$$

Massless field

Factor of ½ coming from 2 polarisations

Fierz-Pauli, in 1939, added a mass term to the graviton:

$$\mathcal{L}_{FP} = -\frac{m_G^2 m_{\mathrm{Pl}}^2}{8}(h_{\mu\nu}h^{\mu\nu} - (h^\mu_\mu)^2)$$

This modifies the propagation:

$$h_{\mu\nu} = \frac{16\pi G_N}{p^2 + m_G^2}(T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu})$$

Discontinuity vDVZ!

This discontinuity is due to the existence of a **SCALAR** polarisation:

$$5=2+2+1$$

Five polarisations

Scalar field!

The decomposition of the graviton into helicity-two and scalar is:

$$h_{\mu\nu} \sim \bar{h}_{\mu\nu} + \frac{\beta}{m_{\rm Pl}} \phi \eta_{\mu\nu}$$

The helicity-two part satisfies the usual equation:

$$\bar{h}_{\mu\nu} = \frac{16\pi G_N}{p^2 + m_G^2} (T_{\mu\nu} - \frac{T}{2} \eta_{\mu\nu})$$

Massive scalar coupled to matter

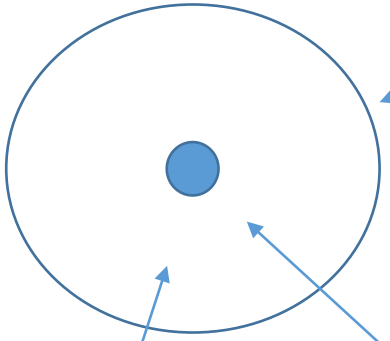
The scalar part satisfies:

$$\phi = \frac{\beta}{m_{\rm Pl}} \frac{1}{p^2 + m_G^2} T \longrightarrow \Box\phi - m_G^2 \phi = -\frac{\beta}{m_{\rm Pl}} T$$

$$\frac{1}{3} = \frac{1}{2} - \beta^2 \longrightarrow \beta = \frac{1}{\sqrt{6}}$$

Coupling of scalar graviton to matter

The coupling β exceeds the Cassini bound. Fortunately, non-linear terms in the action cure the discontinuity and save massive gravity:



$$r_V = \left(\frac{m}{m_{\rm Pl}^2 m_G^4}\right)^{1/5}$$

Vainshtein radius

$$h_{\mu\nu} = \frac{16\pi G_N}{p^2 + m_G^2}\left(T_{\mu\nu} - \frac{T}{3}\eta_{\mu\nu}\right) \longrightarrow h_{\mu\nu} = \frac{16\pi G_N}{p^2}\left(T_{\mu\nu} - \frac{\eta_{\mu\nu}}{2}T\right)$$

Inside the Vainshtein radius, usual gravity is restored and the scalar mode is **screened.**

$$\delta S \sim \int d^4x (\Delta\phi)^3$$

Dominates inside Vainshtein radius

Vainshtein Mechanism in a nutshell

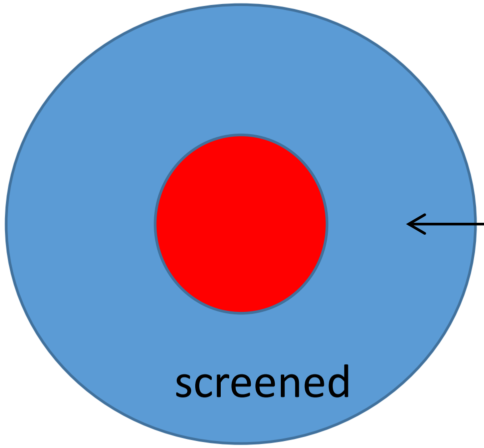
We can use a simple example with higher derivatives:

$$\mathcal{L}_\phi = -\frac{1}{2}(\partial\phi)^2 - \boxed{\frac{1}{2\Lambda^3}(\partial\phi)^2\Box\phi} + \frac{\beta\phi}{M_P}T \,.$$

$\Lambda^3 = m^2 m_{\rm Pl}$

m graviton mass

Non-linearity



$$\frac{F_\phi}{F_N} = 2\beta^2\left(\frac{r}{R_V}\right)^{3/2} \,.$$

$$R_V = \left(\frac{\beta M_c}{2\pi M_P}\right)^{1/3}\frac{1}{\Lambda} \,.$$

Well inside the Vainshtein radius, Newtonian gravity is restored. Well outside gravity is modified.

The Vainshtein radius is very large for stars, typically 0.1 kpc for the sun, and a mass for the graviton of the order of the Hubble rate .

There are four mechanisms to screen gravity, they can be easily understood using the perturbation Lagrangian of the scalar field fluctuation responsible for the fifth force:

Around a background figuration, for instance in the cosmos or the solar system, and in the presence of matter:

$$\mathcal{L} \supset -\frac{Z(\phi_0)}{2}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{M_P}\delta\phi\delta T\,,$$

$$\delta T = -\delta\rho$$

ϕ_0

Background value of the scalar field. For instance the value of the field in the cosmos or in the vacuum between planets.

- The ***chameleon mechanism*** makes the range of the scalar interaction become smaller in a dense environment by increasing the mass term m (Yukawa suppression).

- The ***Damour-Polyakov mechanism*** reduces the coupling strength β in a dense environment.

- ***The Vainshtein and K-mouflage mechanisms*** reduce the effectice coupling in a dense environment by increasing the field normalisation constant Z.

Derivative screening:

$$\mathcal{L} \supset \boxed{-\frac{Z(\phi_0)}{2}}(\partial\delta\phi)^2 - \frac{m^2(\phi_0)}{2}\delta\phi^2 + \frac{\beta(\phi_0)}{M_P}\delta\phi\delta T \ ,$$

The K-mouflage and Vainshtein mechanisms reduce the coupling by increasing Z

By writing the static equations of motion of the scalar fluctuation:

$$\Delta\delta\phi = \frac{\beta(\phi_0)}{m_{\rm Pl} Z(\phi_0)}\delta\rho$$

The Effective Newtonian potential:

$$\Phi = (1 + \frac{2\beta^2(\phi_0)}{Z(\phi_0)})\Phi_N$$

$$\Phi = \Phi_N + \frac{\beta}{m_{\rm Pl}}\delta\phi$$

For theories with second order equations of motion, the factor Z can be expanded in powers of the derivatives involving only the first two derivatives:

$$Z(\phi) = 1 + a(\phi)L^2 \frac{\Box\phi}{m_{\rm Pl}} + b(\phi)\frac{(\partial\phi)^2}{\mathcal{M}^4} + \dots$$

The difference between the Vainshtein and K-mouflage screenings come from the type of dominant term in Z

Natural scales to act on cosmological scales

Vainshtein

K-mouflage

$$\mathcal{M}^4 \sim 3H_0^2 m_{\rm Pl}^2, \quad L \sim H_0^{-1}$$

Vainshtein:

$$\frac{\delta\phi}{m_{\mathrm{Pl}}} = \frac{2\beta}{Z}\Phi_N \longrightarrow \Delta\Phi_N \gg \frac{\Delta\delta\phi}{2\beta m_{\mathrm{Pl}}}$$

Large Z

Screening when Z>>1: $\Delta\delta\phi \gg \frac{m_{\mathrm{Pl}}}{L^2}$

Newtonian gravity retrieved when the curvature is large enough:

$$\Delta\Phi_N \geq \frac{1}{2\beta L^2}$$

Poisson equation

$$\Delta\Phi_N = \frac{3}{2}\Omega_{m_0} H_0^2 \delta$$

On small scales (solar system, galaxies) screening only occurs within the Vainshtein radius:

Mass of object

$$R_V = \left(\frac{3\beta M}{4\pi m_{\mathrm{Pl}}^2 H_0^2}\right)^{1/3} \sim \left(\frac{\rho}{\rho_\Lambda}\right)^{1/3} R \gg R$$

$$\frac{\rho_\odot}{\rho_\Lambda} \sim 10^{30}$$

Screening radius much larger than the solar system

On large cosmological scales, this tells us that overdensities such as galaxy clusters are screened :

$$\delta \geq \frac{1}{3\Omega_{m0}\beta}$$

K-mouflage

Newtonian gravity retrieved when the gravitational acceleration is large enough:

$$|\nabla\Phi_N| \geq \frac{\mathcal{M}^2}{2\beta m_{\rm Pl}}$$

On small scales (solar system, galaxies) screening only occurs within the K-mouflage radius:

$$R_K = (\frac{\beta m}{4\pi m_{\rm Pl}\mathcal{M}^2})^{1/2}$$

Dwarf galaxies are not screened.

On large cosmological scales, this tells us that overdensities such as galaxy clusters are not screened :

$$\frac{k}{H_0} \leq \beta\delta$$

Only very large scales are screened

Non-Derivative Screening

This type of screening can appear in "simple" (compared to the general case of Horndeski) scalar-tensor theories:

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{1}{2}(\partial\phi)^2 - V(\phi) + \mathcal{L}_m(\psi_m, A^2(\phi)g_{\mu\nu})\right)$$

Coupling function, only conformal coupling to matter

In these theories the coupling β can become field dependent:

$$\beta = m_{\rm Pl}\frac{\partial \ln A}{\partial\phi}$$

$$A(\phi) = e^{\beta\phi/m_{\rm Pl}}$$

General case

Constant Yukawa coupling

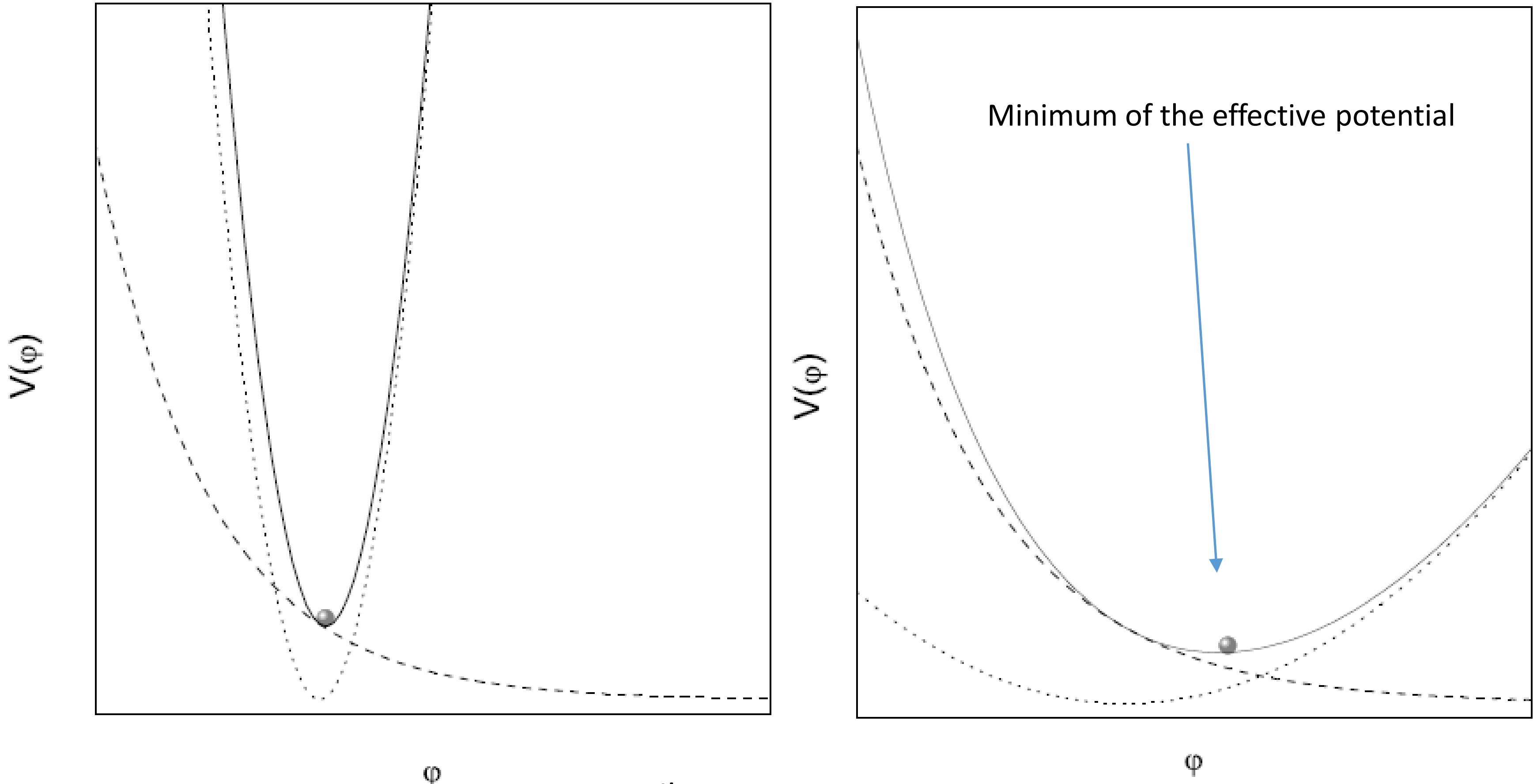
Damour-Polyakov screening

$$\beta(\phi) \sim \frac{A_2}{m_{\rm Pl}}(\phi - \phi_\star)$$

In dense environment

$$\phi = \phi_\star$$

where β=0



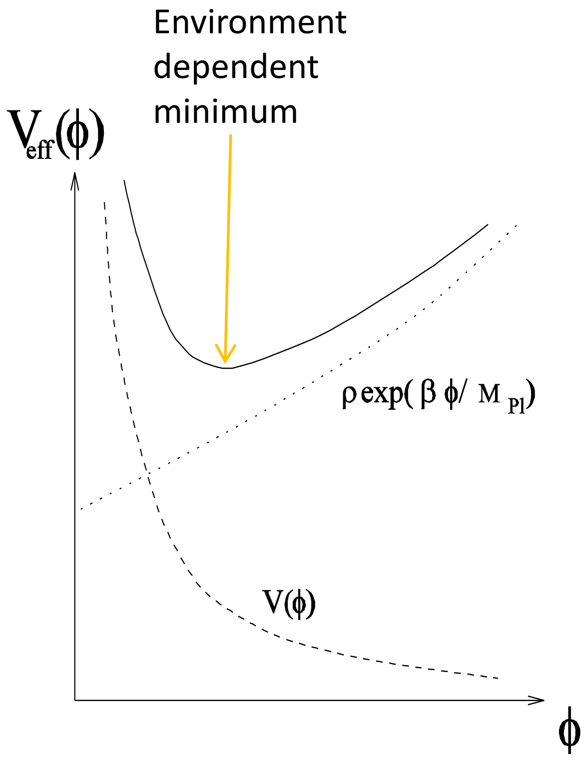
Dilaton

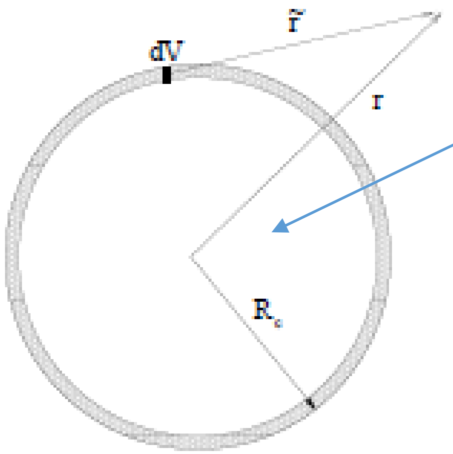
$$V(\phi) = V_0 e^{-\phi/m_{\rm Pl}}, \quad A(\phi) = 1 + \frac{A_2}{2m_{\rm Pl}^2}(\phi - \phi_\star)^2$$

Chameleon Screening

When coupled to matter, scalar fields have a ***matter dependent effective potential***

$$V_{eff}(\phi) = V(\phi) + \rho_m(A(\phi) - 1)$$





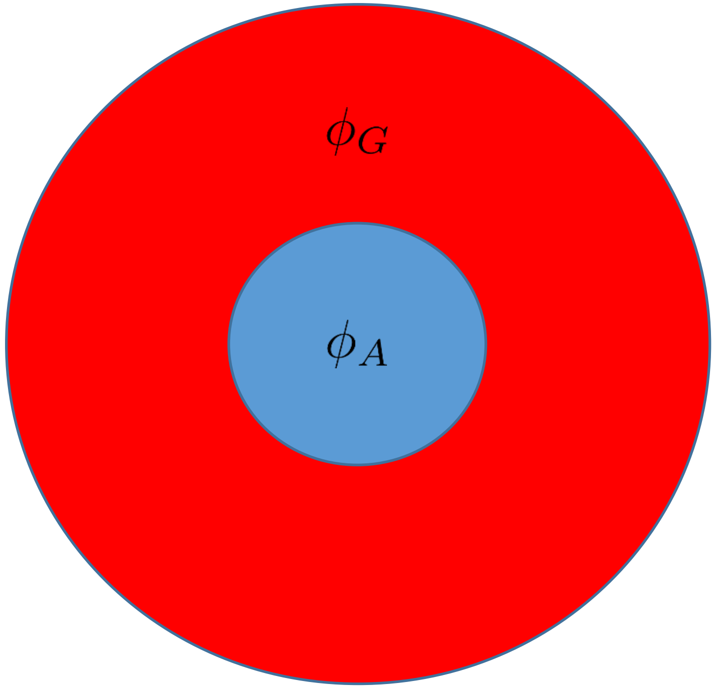
Large mass inside object

$$m_{\rm in} R \gg 1$$

The field generated from deep inside is Yukawa suppressed. Only a *thin shell* radiates outside the body. Hence suppressed scalar contribution to the fifth force.

Chameleons:

The screening criterion for an object BLUE embedded in a larger region RED expresses the fact that the Newtonian potential of an object must be larger than the variation of the field:



Scalar charge: $$Q_A = \frac{|\phi_G - \phi_A|}{2m_{\rm Pl}\Phi_A}$$

$$Q_A \leq \beta_G$$

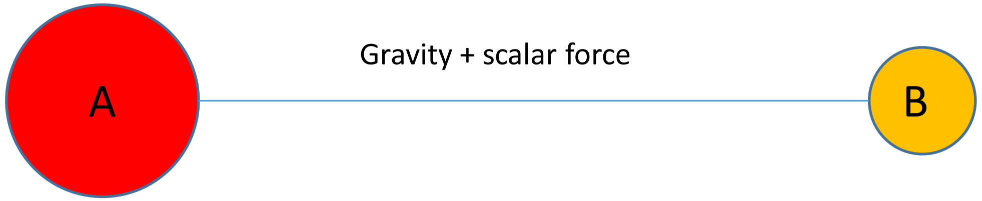
Screening criterion

Φ_A Newton's potential at the surface

Due to the scalar interaction, within the Compton wavelength of the scalar field, the inertial and gravitational masses differ for screened objects:

$$G_{A,B} = G_N(1 + 2Q_A Q_B)$$

Interaction rate depending on the objects



Value of the field far away

$$Q_A = \frac{\phi_\infty}{2m_{\mathrm{Pl}}\Phi_A}$$

Newtonian potential at the surface of the body.

$$Q_A \leq \beta_\infty$$

Screening criterion for compact objects

Massive bodies with differ scalar charges fall differently. Hence a violation of the strong equivalence principle.

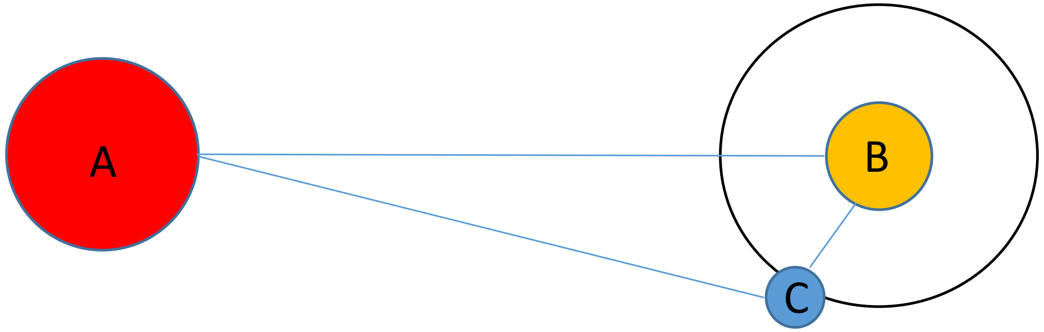
Due to the scalar interaction, within the Compton wavelength of the scalar field, the inertial and gravitational masses differ for screened objects:

VIOLATION OF THE STRONG EQUIVALENCE PRINCIPLE

$$\Phi_{\oplus} \sim 10^{-9}$$

$$\Phi_{\odot} \sim 10^{-6}$$

$$\Phi_{\rm moon} \sim 10^{-11}$$



The tightest constraint on planetary motion is obtained by the Lunar Laser Ranging experiment measuring the distance between the Earth and the Moon. The bound is on the Eotvos parameter A=Sun B= Earth C=Moon

$$\eta_{\rm moon-earth} \leq 10^{-13}$$

$$\eta_{BC} \equiv \left|\frac{a_C - a_B}{a_C + a_B}\right| = Q_A |Q_C - Q_B|$$

For this system, this is related to the scalar charge of the Earth:

$$\eta_{\rm moon-earth} \approx 10^{-1} Q_{\oplus}^2 \longrightarrow Q_{\oplus} \leq 10^{-6}$$

Newton's Law:

Newton's law is the non-relativistic version of the geodesic equation. This is a consequence of matter conservation:

$$\nabla_\mu T_m^{\mu\nu} \to \dot{u}^\mu = 0$$

$$\dot{f} \equiv u^\mu \nabla_\mu f$$

In a scalar-tensor theory, matter is not conserved so modification of the geodesic equation:

$$\nabla_\mu T_m^{\mu\nu} = -\frac{\beta}{m_{\rm Pl}} T_m \partial^\nu \phi$$

The energy momentum tensor of matter is:

$$T_{\mu\nu}^m = \rho u_\mu u_\nu$$

$$u^\mu = \frac{dx^\mu}{d\tau}$$

Proper time

Proof: total matter is conserved

$$\nabla_\mu (T_m^{\mu\nu} + T_\phi^{\mu\nu}) = 0$$

Use Klein-Gordon: $\Box \phi = \frac{\partial V_{\rm eff}}{\partial \phi}$

and the energy-momentum tensor

$$T_{\mu\nu}^\phi = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} \left(\frac{(\partial\phi)^2}{2} + V(\phi)\right)$$

$$\dot{\rho} u^\mu + 3h\rho u^\mu + \rho \dot{u}^\mu = -\frac{\beta_\phi}{m_{\rm Pl}} \rho \dot{\phi} u^\mu$$

Contract with u^μ

$u^2 = -1$

$$\dot{\rho} + 3h\rho = \frac{\beta}{m_{\rm Pl}} \rho \dot{\phi}$$

$\rho = A\rho_m \qquad 3h = \nabla_\mu u^\mu$

$$\boxed{\dot{\rho}_m + 3h\rho_m = 0}$$

Conservation of matter

$$\boxed{\dot{u}^\mu + \frac{\beta}{m_{\rm Pl}} \dot{\phi} u^\mu = -\frac{\beta}{m_{\rm Pl}} \partial^\mu \phi}$$

Newton's law

Variation of the particle's mass

Force due to the scalar

The Two Newtonian potentials

In the Einstein frame:

$$ds^2 = a^2(-(1+2\Phi_N)d\eta^2 + (1-2\Phi_N)dx^2)$$

In the Jordan frame:

$$ds^2 = A^2 a^2(-(1+2\Phi_N)d\eta^2 + (1-2\Phi_N)dx^2)$$

Separate the background dynamics

$$ds^2 = a_J^2(-(1+2\Phi)d\eta^2 + (1-2\Psi)dx^2)$$

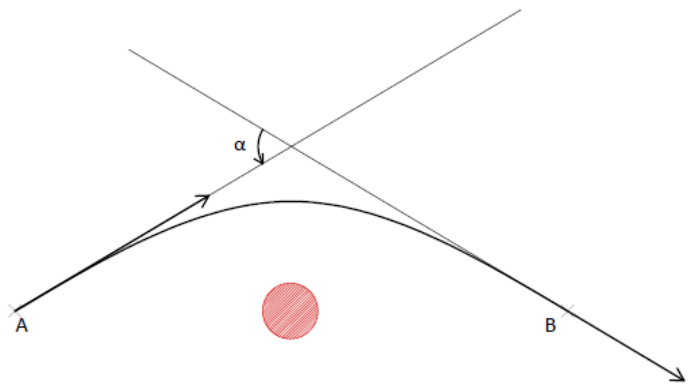
So far background value of A taken to be unity, more general here.

$$A = \bar{A} \times \frac{A}{\bar{A}}$$

$$a_J = \bar{A} a$$

$$\boxed{\Phi = \Phi_N + \ln\frac{A}{\bar{A}}, \quad \Psi = \Phi_N - \ln\frac{A}{\bar{A}}}$$

Background no spatial dependence



$$\Phi_L = \frac{\Phi + \Psi}{2}$$

The two Newtonian potentials have an effect on lensing:

Integral along the path of the gradient of lensing potential projected perpendicularly to the path

$$\vec{\alpha} = \frac{2}{c^2} \int_A^B ds \vec{\nabla}_\perp \Phi_L$$

Gravity modified because:

$$\Phi \neq \Psi, \quad \Phi \neq \Phi_N$$

Putting it all together, we find the growth equation:

Time and scale dependent Newton constant

$$\delta'' + (\mathcal{H} + \frac{A'}{A})\delta' - \frac{3}{2}\Omega_m \mathcal{H}^2 (1+\epsilon)\delta = 0$$

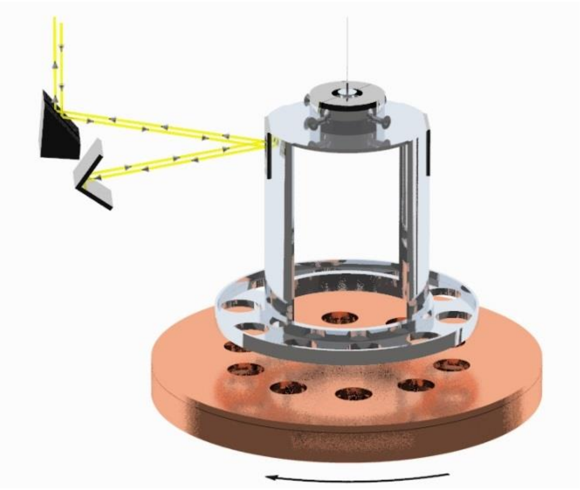
depending on:

$$\epsilon(a,k) = \frac{2\beta^2(a)}{1 + \frac{m^2(a)a^2}{k^2}}$$

Screened ε=0

Unscreened $\epsilon = 2\beta^2$

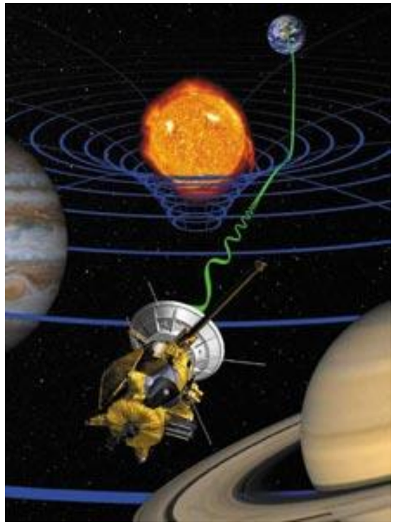
Gravity has been wonderfully tested on many length scales:



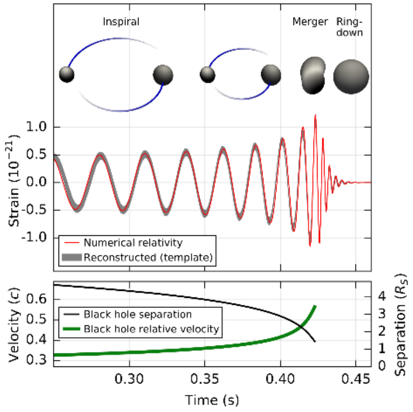
Laboratory experiments (Eotwash) tests of fifth forces and equivalence principle
0.1 mm

$$\beta^2 \leq 10^{-5}$$

Cassini probe test of fifth forces
1 a.u., 150 million km.

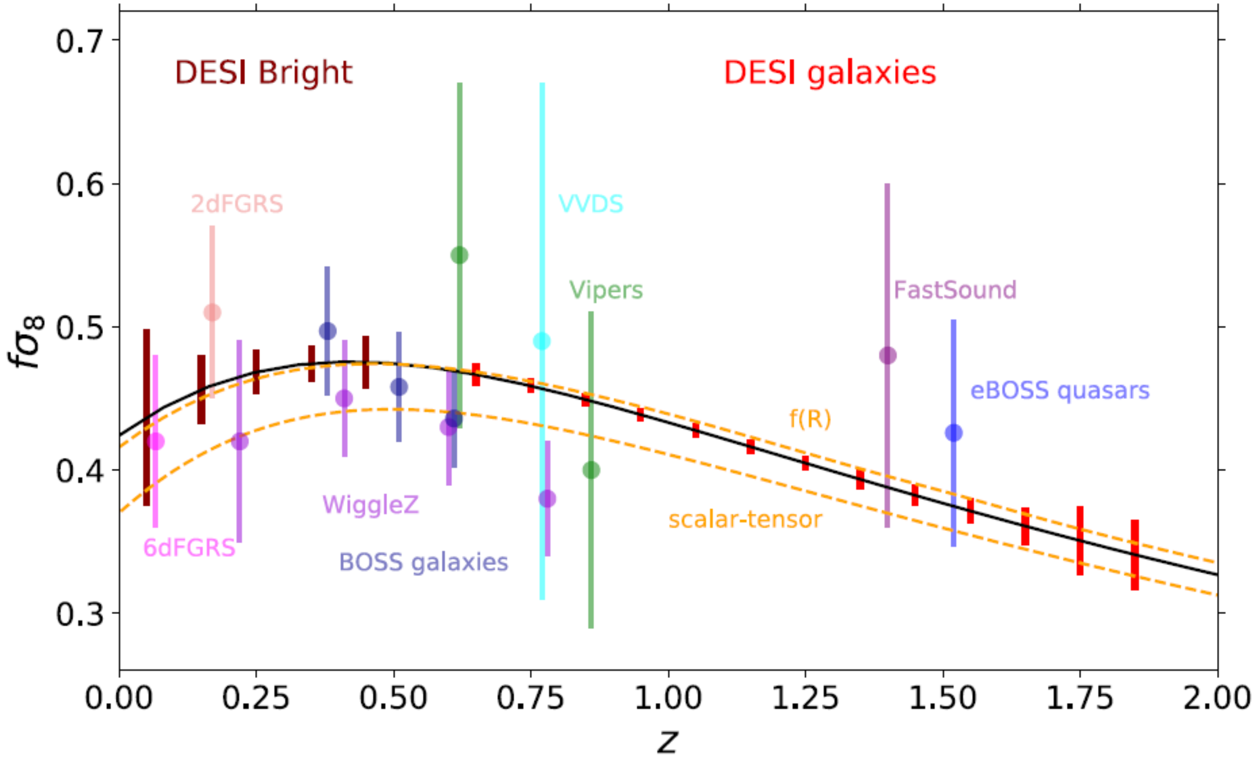


Lunar ranging tests of strong equivalence principle and time variation of Newton's constant,
400 000 km



Gravitational wave emissions from black hole and neutron star mergers
50 Mpc

In the near future (Euclid, LSST, DESI …) gravity will be tested on cosmological scales by comparing lensing and growth of structure:



$$f = \frac{d \ln \delta}{d \ln a}$$