$(g-2)_\mu$ and modified gravity

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The anomalous magnetic moment of leptons (electrons and muons) is of great interest for BSM (Beyond Standard Model) physics. It has a long history since Dirac and Schwinger:

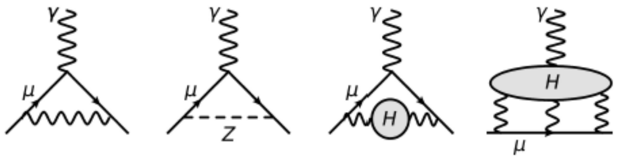
$$\vec{\mu} = g\frac{q}{2m}\vec{S} \qquad\qquad H = -\vec{\mu}.\vec{B}_{\mathrm{ext}}$$

The gyromagnetic factor g can be evaluated using Quantum Electrodynamics

$$g = 2, \quad \text{Dirac}$$

$$a \equiv g - 2 = \frac{\alpha}{2\pi}, \quad \text{Schwinger}$$

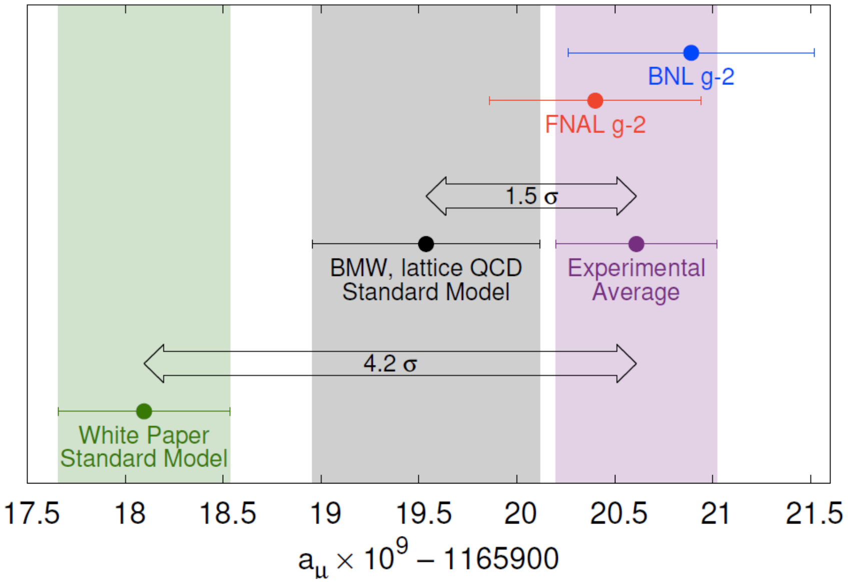
These days, very high order evaluations of the anomalous magnetic moment can be performed within the standard model:



For muons:

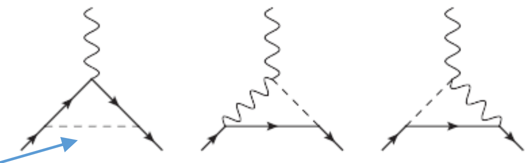
$$a_\mu = 116592000\ 10^{-11}$$

Recently a discrepancy between the standard model evaluation and the experimental results (Fermilab) has been reported:



$$a_\mu(\mathrm{exp}) - a_\mu(\mathrm{SM}) = (251 \pm 59)10^{-11}$$

The traditional explanation is to say that some new particle beyond the SM (SUSY? Etc…) could provide the necessary contribution:



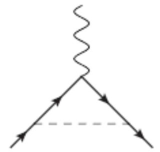
New particle

Could it be a light scalar?

Typical Yukawa interaction

Motivated by dark energy or dark matter: $$\delta\mathcal{L} = \frac{\beta}{m_{\mathrm{Pl}}}\phi m_\psi \bar{\psi}\psi$$

If this coupling is Universal, then it has a gravitational effect on particles:



$$G_{\mathrm{eff}} = (1+2\beta^2)G_N$$

This comes from a mass varying effect which can be embedded in a **scalar-tensor theory**:

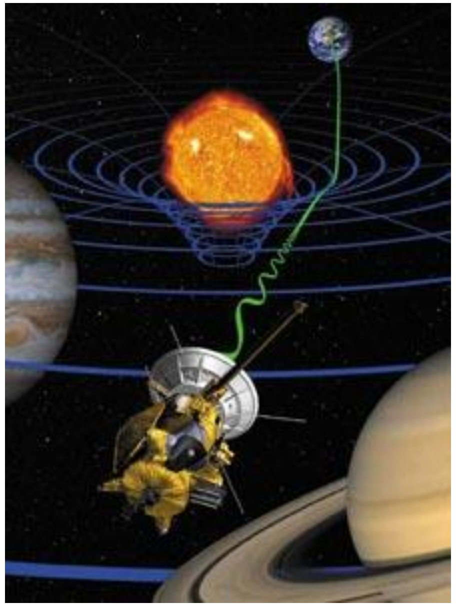
$$m_\psi(\phi) = A(\phi)m_\psi$$

$$g^J_{\mu\nu} = A^2(\phi)g_{\mu\nu}$$

$$S = \int d^4x\sqrt{-g}\left(\frac{R}{16\pi G_N} - \frac{(\partial\phi)^2}{2} - V(\phi)\right) + S_{\mathrm{matter}}(\psi, g^J_{\mu\nu})$$

Deviations from Newton's law are parametrised by:

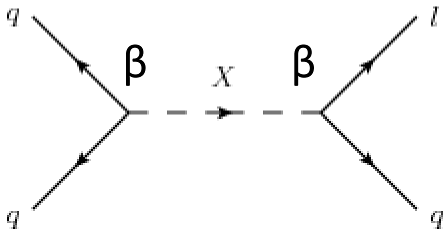
$$\Phi = -\frac{G_N M}{r}(1 + 2\beta^2 e^{-r/\lambda})$$



Bertotti et al. (2004)

For long range forces with large λ, the tightest constraint on the coupling β comes from the Cassini probe measuring the Shapiro effect (time delay):

$$\beta^2 \leq 4 \cdot 10^{-5}$$

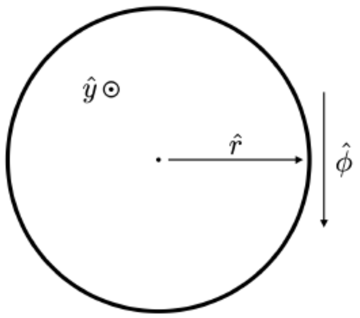


So tiny effects on (g-2)??

The particle physics vision: a scalar field exchanged between two fermions with coupling strength β

Is this all there is to it?

Not quite…

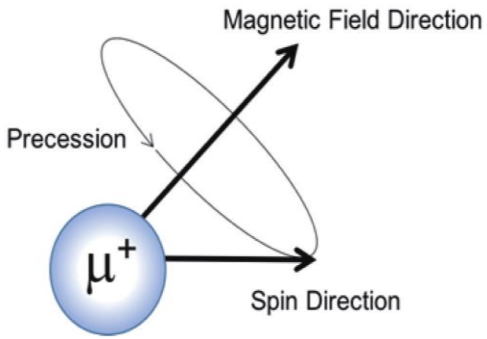


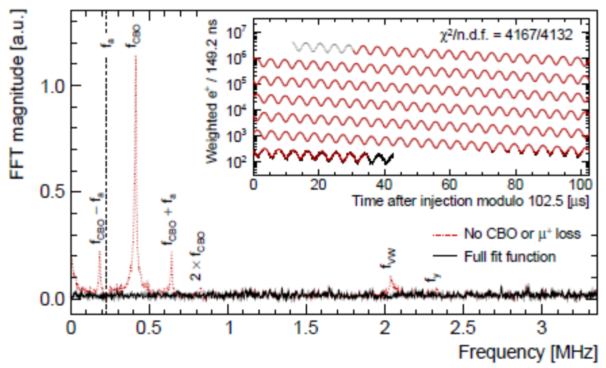
Antimuons going clockwise.

Cyclotron(Larmor) angular velocity

$$\vec{\omega}_c = -\frac{q}{m_\mu}\vec{B} \qquad q = e$$

$$\frac{d\vec{S}}{dt} = \vec{\omega}_S \wedge \vec{B}$$





When the muon rotate in the storage ring, their spin precesses too and their decay into positron is modulated by the spin precession in their rest frame:

$$N(t) \simeq e^{-t/t_{\mathrm{muon}}}(1 + A\cos(\omega_a t + \varphi))$$

$$\vec{\omega}_a = \vec{\omega}_S - \vec{\omega}_c$$

Key to new physics effects

Particle Dynamics

$$S = \int d\tau(-mA(\phi)\sqrt{-u^2} + qA^\mu u_\mu)$$

$$u^\mu = \frac{dx^\mu}{d\tau} \qquad m_E = A(\phi)m$$

Proper time

The light scalar field influences the dynamics as the mass becomes field-dependent.

The particle gyrates in the laboratory frame:

$$\frac{d\vec{v}}{dt} = \vec{\omega}_c \wedge \vec{v}$$

Scalar contribution

In the rest frame of the particle, the angular velocity vector is boosted to:

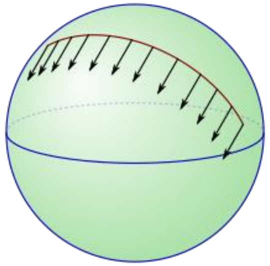
$$\vec{\omega}_c = -\frac{q}{m_E}(\vec{B} - \frac{\gamma}{\gamma+1}(\vec{v}.\vec{B})\vec{v}) + \frac{q\gamma}{m_E(\gamma^2-1)}\vec{v} \wedge \vec{E} - \boxed{\frac{\partial_\phi \ln A}{\gamma^2-1}\vec{v} \wedge \vec{\nabla}\phi}$$

Hypothesis: $\vec{E}.\vec{v} = 0, \quad \vec{\nabla}\phi.\vec{v} = 0$

Spin Precession I

In the absence of electromagnetic interactions, the spin is parallel transported in the Jordan geometry

$$\frac{dS^\mu}{d\tau_J} = 0$$



$$g^J_{\mu\nu} = \eta_{ij} e^i_\mu e^j_\mu$$

The rest frame is obtained by two operations: going to the locally Minkowskian frame and then boosting to the rest frame of the particle.

$$S^{\hat{i}} = \Lambda^{\hat{i}}_j e^j_\mu S^\mu \longrightarrow \frac{d\vec{S}}{d\tau_J} = \vec{\omega}_\phi \wedge \vec{S}$$

$$\vec{\omega}_\phi = (\partial_\phi A)\vec{u} \wedge \vec{\nabla}\phi \equiv \vec{0}$$

Lorentz boost

Vielbein

In the rest frame, no effect of the Jordan geometry

Spin Precession II

When electromagnetic effects are taken into account, the BMT equation (Bargman-Michel-Telegdi) in the rest frame:

$$\frac{dS^{\hat{i}}}{d\tau} = \underbrace{\frac{qg}{2m} F^{\hat{i}\hat{j}} S_{\hat{j}}}_{\text{Electromagnetic effect}} - \underbrace{\frac{d\Lambda^{\hat{i}}_{\mu}}{d\tau} \Lambda^{\mu}_{\hat{j}} S^{\hat{j}}}_{\text{Change of frame}}$$

This is also a precession equation:

$$\frac{d\vec{S}}{d\tau} = \vec{\omega}_S \wedge \vec{S}$$

Thomas precession

$$\vec{\omega}_S = -\frac{qg}{2m\gamma}\vec{B} + \frac{\gamma^2}{\gamma+1}\vec{a} \wedge \vec{v}$$

Acceleration including the scalar effects

Anomalous spin precession

The anomalous spin precession depends on the scalar field only via the Thomas precession.

$$\vec{\omega}_a = -\frac{q}{m}[(\vec{B} - \frac{\gamma}{\gamma+1}(\vec{v}.\vec{B})\vec{v})a_\mu + (a_\mu - \frac{1}{\gamma^2-1})\vec{v}\wedge\vec{E}] + \frac{\gamma}{\gamma^2-1}(\partial_\phi \ln A)\vec{v}\wedge\vec{\nabla}\phi$$

Focus on ideal case with electric field and trajectories in the plane perpendicular to the magnetic field. The magnetic and scalar contributions have the same sign:

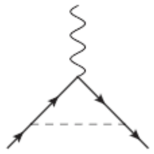
$$a_\mu(\exp) = \frac{m_\mu}{eB}(\omega_a - \frac{\gamma}{\gamma^2-1}(\partial_\phi \ln A)v|\vec{\nabla}\phi|)$$

Measured

Scalar effect

The quantum effects modify the theoretical expectation:

$$a_\mu(\mathrm{th}) = a_\mu(\mathrm{SM}) + 3(\partial_\phi \ln A)^2 (\frac{m_\mu}{4\pi})^2$$



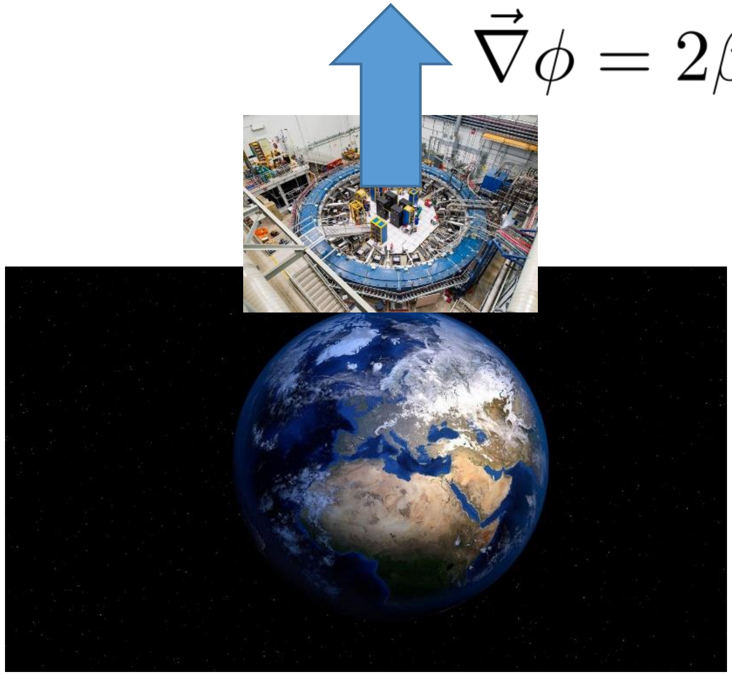
The resulting effects of the scalar field on the deviation from the Standard Model are

$$\boxed{\delta a_\mu = \frac{m_\mu}{eB}\frac{1}{\gamma v}(\partial_\phi \ln A)|\vec{\nabla}\phi| + 3(\partial_\phi \ln A)^2 (\frac{m_\mu}{4\pi})^2}$$

Classical

quantum

Yukawa interactions



$$\vec{\nabla}\phi = 2\beta m_{\mathrm{Pl}} g \vec{e}_r$$

Applying the Cassini bound:

$$\frac{\delta\omega_a}{\omega_a} \leq 10^{-20}$$

$$\delta a_\mu(\mathrm{th}) \leq 10^{-43}$$

Negligible!

Need strong coupling in the laboratory!

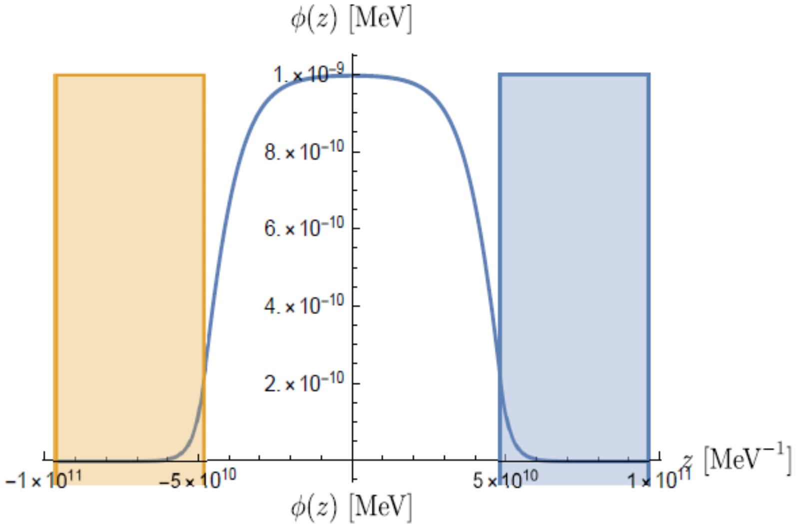
The symmetron is screened in the solar system and coupled strongly in the laboratory:

$$V(\phi) = -\frac{\mu^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4 \qquad A(\phi) = 1 + \frac{\phi^2}{2M^2} + \dots \Longrightarrow \beta(\phi) = \frac{m_{\rm Pl}\phi}{M^2}$$

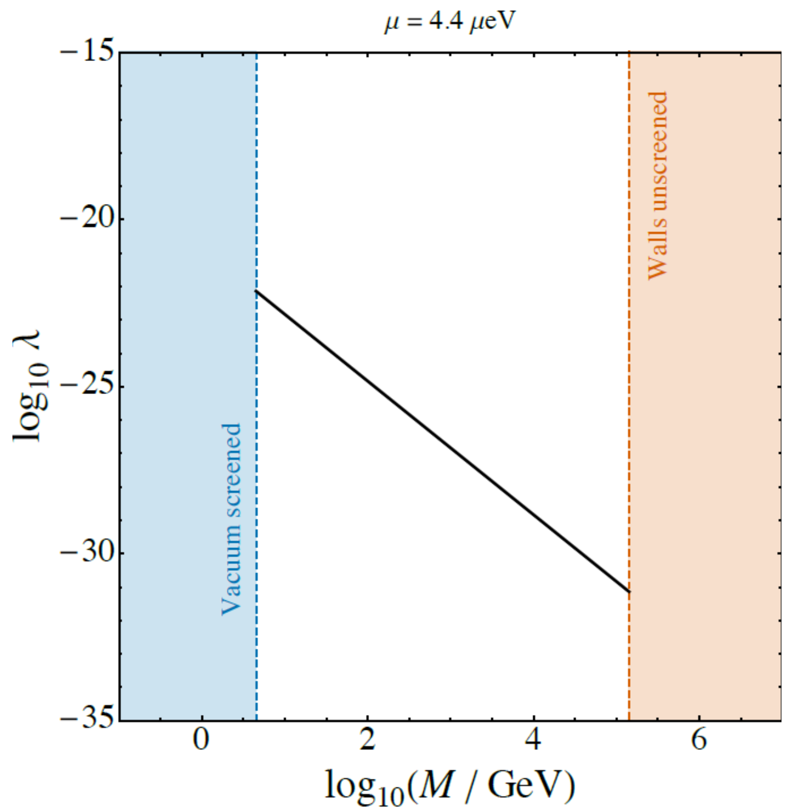
$$V_{\rm eff}(\phi) = V(\phi) + (A(\phi) - 1)\rho$$

$$\rho \geq M^2\mu^2 \qquad \langle\phi\rangle = 0 \to \beta \equiv 0$$

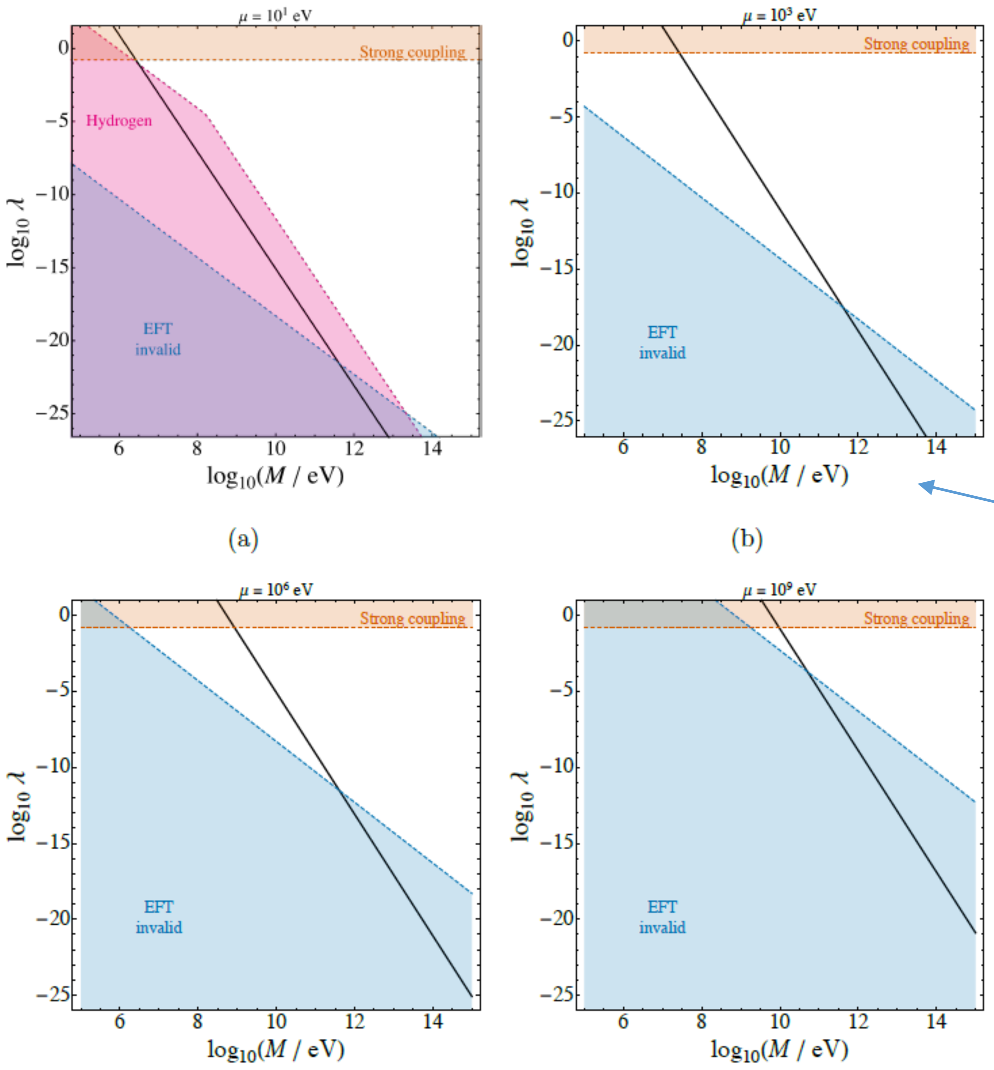
DENSE OBJECTS DECOUPLE



Symmetron profile in a ring



The classical contribution dominates (g-2) but excluded by quantum bouncing neutrons.



Strong-coupling scale around the EW scale

$$1\ \mathrm{keV} \leq \mu \leq 1\ \mathrm{GeV}$$

Conclusions

- There is more than meets the eyes: quantum effects can be complemented by interesting spin effects. Here for (g-2).
- Is there a link with Dark Energy? Here NO, but in other models YES
- The same classical techniques can be used to study the effects of scalar fields on the GW emitted from spinning binaries.