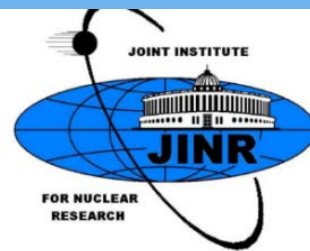


**EuCAPT Astroneutrino Theory Workshop 2021  
Prague, Czech Republic, September 20 – October 1, 2021**



*Massive Neutrinos and Neutrinoless Double Beta Decay*

Fedor Šimkovic



# OUTLINE

## I. *Introduction*

*(Majorana  $\nu$ 's)*

## II. *A generation of neutrino mass, $0\nu\beta\beta$ -decay mechanisms*

*(QCSS scenario, LR symmetric model)*

## III. *The $0\nu\beta\beta$ -decay NMEs – Current status*

*(deformation, SU(4) symmetry, ab initio... )*

## IV. *Quenching of $g_A$*

*(theory and experimental indications, novel approach for effective  $g_A$  )*

## V. *Outlook*

*Acknowledgements:* **P. Vogel** (Caltech), **S. Kovalenko** (Valparaiso U.), **M. Krivoruchenko** (ITEP Moscow), **A. Faessler** (Tuebingen), **D. Štefánik**, **R. Dvornický** (Comenius U.), **A. Babič**, **A. Smetana** (IEAP CTU Prague), **F.F. Deppisch** (Imperial College London), **L. Graf** (MPI Heidelberg ), ...



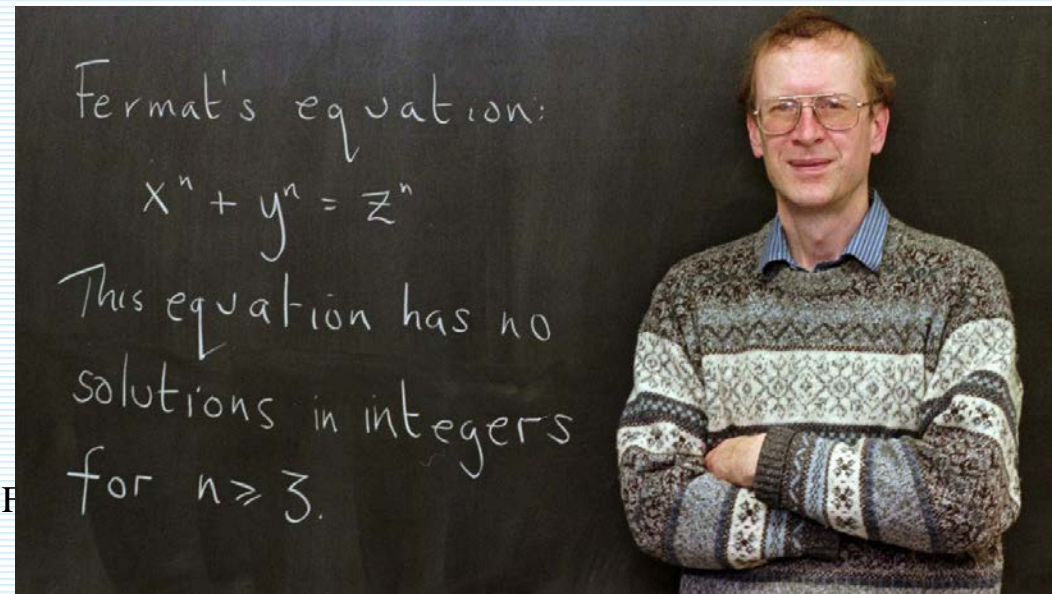
Around 1637, Fermat wrote in the margin of a book that the more general equation

$$a^n + b^n = c^n$$

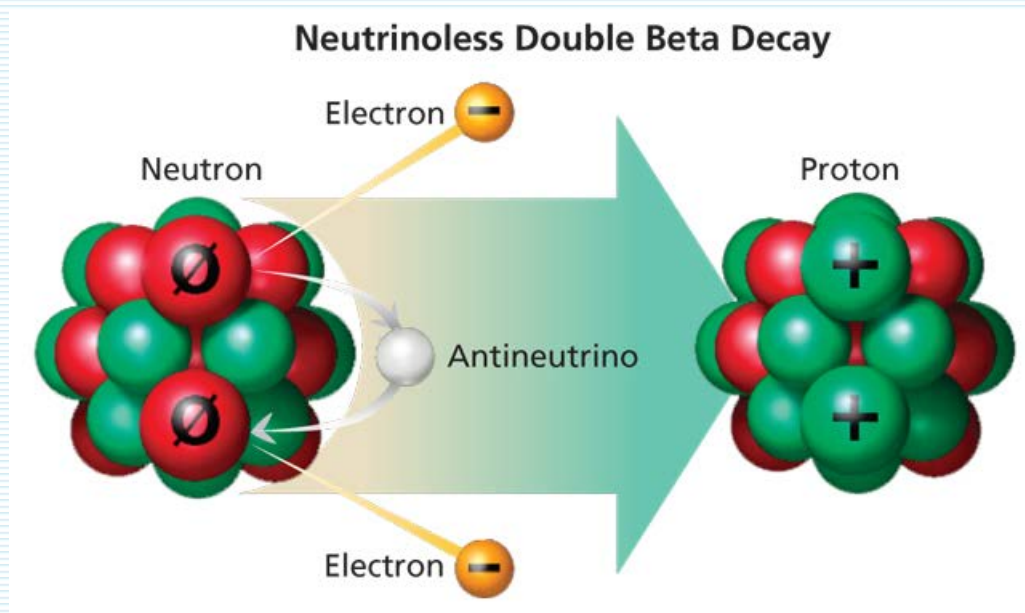
had no solutions in positive integers if  $n$  is an integer greater than 2.

After **358** years

The corrected proof was published by **Andrew Wiles** in 1995.



# $\bar{\nu}$ ARE NEUTRINOS THEIR OWN ANTI PARTICLES?



# Majorana fermion



[https://en.wikipedia.org/wiki/File:Ettore\\_Majorana.jpg](https://en.wikipedia.org/wiki/File:Ettore_Majorana.jpg)



CNNP 2018, Catania, October 15-21, 2018

9/29/2021

Fedor S

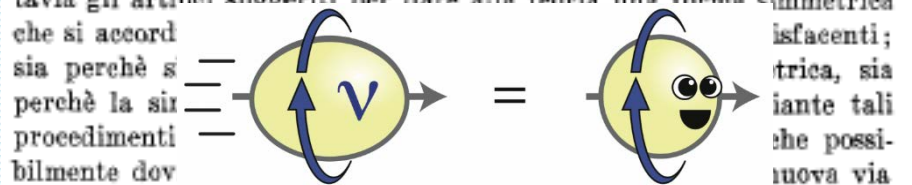
## TEORIA SIMMETRICA DELL'ELETTRONE E DEL POSITRONE

Nota di ETTORE MAJORANA

### Symmetric Theory of Electron and Positron Nuovo Cim. 14 (1937) 171

**Sunto.** - Si dimostra la possibilità di pervenire a una piena simmetrizzazione formale della teoria quantistica dell'elettrone e del positrone facendo uso di un nuovo processo di quantizzazione. Il significato delle equazioni di DIRAC ne risulta alquanto modificato e non vi è più luogo a parlare di stati di energia negativa; nè a presumere per ogni altro tipo di particelle, particolarmente neutre, l'esistenza di « antiparticelle » corrispondenti ai « vuoti » di energia negativa.

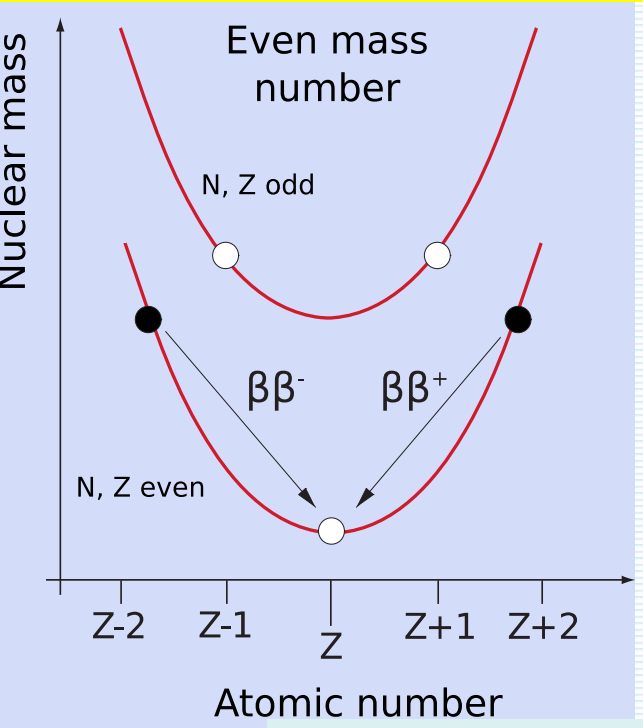
L'interpretazione dei cosiddetti « stati di energia negativa » proposta da DIRAC <sup>(1)</sup> conduce, come è ben noto, a una descrizione sostanzialmente simmetrica degli elettroni e dei positroni. La sostanziale simmetria del formalismo consiste precisamente in questo, che fin dove è possibile applicare la teoria girando le difficoltà di convergenza, essa fornisce realmente risultati del tutto simmetrici. Tuttavia gli artifici suggeriti per dare alla teoria una forma simmetrica che si accorda sia perchè sia perchè la sir procedimenti bilmente dov che conduce più direttamente alla meta.



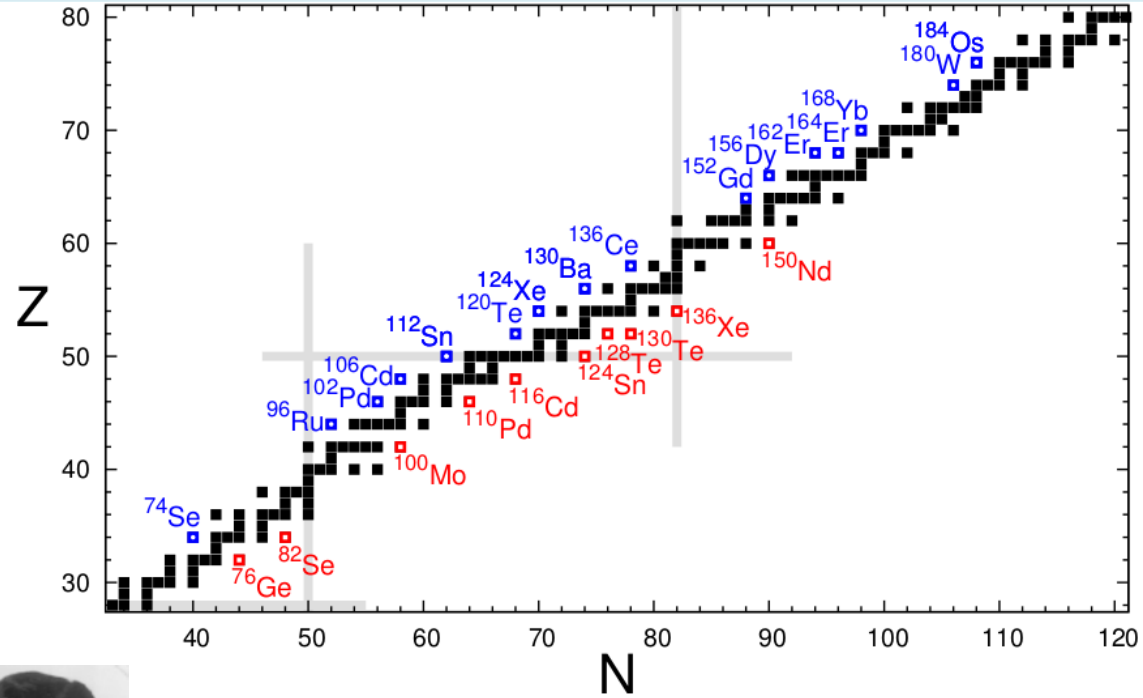
Per quanto riguarda gli elettroni e i positroni, da essa si può veramente attendere soltanto un progresso formale; ma ci sembra importante, per le possibili estensioni analogiche, che venga a cadere la nozione stessa di stato di energia negativa. Vedremo infatti che è perfettamente possibile costruire, nella maniera più naturale, una teoria delle particelle neutre elementari senza stati negativi.

<sup>(1)</sup> P. A. M. DIRAC, « Proc. Camb. Phil. Soc. », **30**, 150, 1924. V. anche W. HEISENBERG, « ZS. f. Phys. », **90**, 209, 1934.

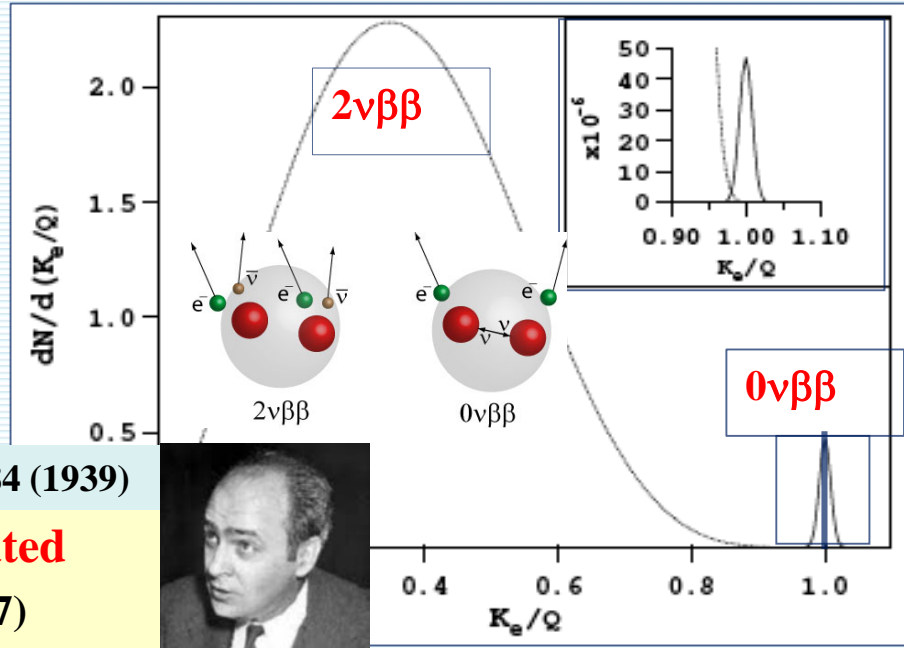
# Nuclear double- $\beta$ decay (even-even nuclei, pairing int.)



Phys. Rev. 48, 512 (1935)



**Two-neutrino double- $\beta$  decay – LN conserved**  
 $(A,Z) \rightarrow (A,Z+2) + e^- + e^- + \nu_e + \nu_e$   
 Goepert-Mayer – 1935. 1<sup>st</sup> observation in 1987



Nuovo Cim. 14, 322 (1937)

Phys. Rev. 56, 1184 (1939)

**Neutrinoless double- $\beta$  decay – LN violated**  
 $(A,Z) \rightarrow (A,Z+2) + e^- + e^-$  (Furry 1937)

Not observed yet. Requires massive Majorana  $\nu$ 's

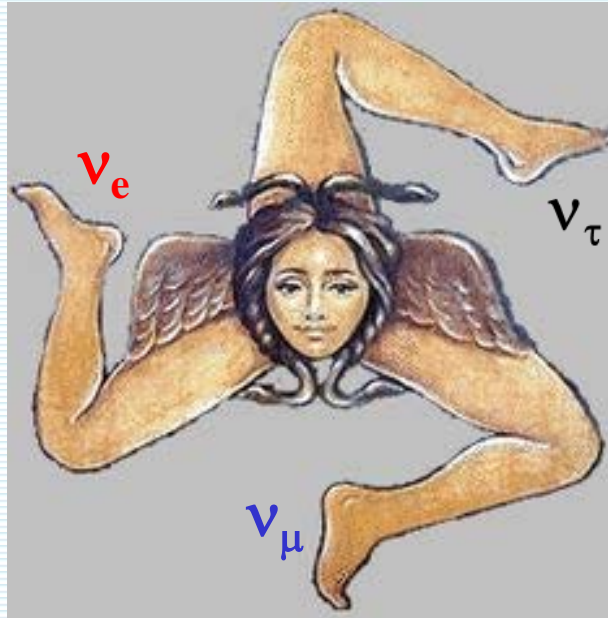


After 90/64 years  
we know

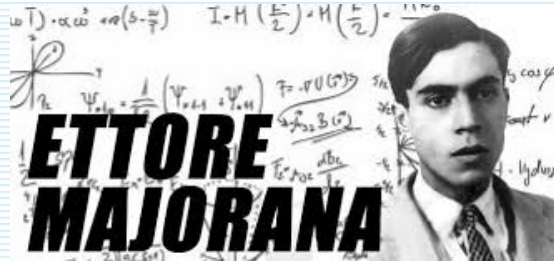
## Fundamental $\nu$ properties

No answer yet

- 3 families of light (V-A) neutrinos:  
 $\nu_e, \nu_\mu, \nu_\tau$
- $\nu$  are massive:  
we know mass squared differences
- relation between flavor states and mass states (neutrino mixing)

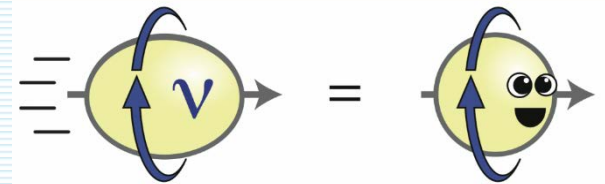


- Are  $\nu$  Dirac or Majorana?
- Is there a CP violation in  $\nu$  sector?
- Are neutrinos stable?
- What is the magnetic moment of  $\nu$ ?
- **Sterile neutrinos?**
- Statistical properties of  $\nu$ ? Fermionic or partly bosonic?



Currently main issue

*Nature, Mass hierarchy, CP-properties, sterile  $\nu$*



The observation of neutrino oscillations has opened a new excited era in neutrino physics and represents a big step forward in our knowledge of neutrino properties

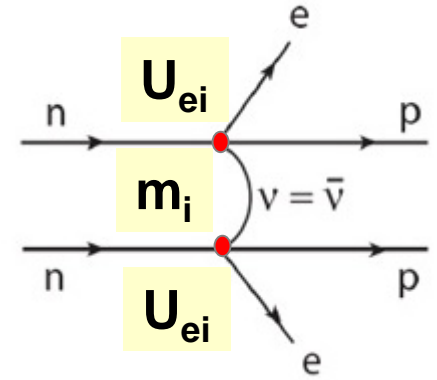
# $0\nu\beta\beta$ -decay (traditional picture)

$$(A,Z) \rightarrow (A,Z+2) + e^- + e^-$$

$$(T_{1/2}^{0\nu})^{-1} = \left| \frac{m_{\beta\beta}}{m_e} \right|^2 g_A^4 |M_\nu^{0\nu}|^2 G^{0\nu}$$

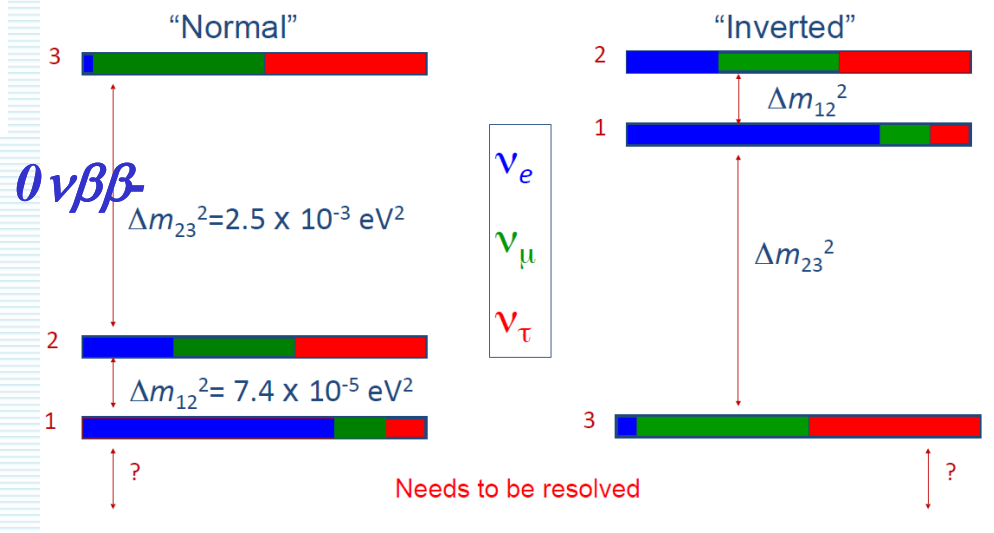
Phase factor well understood

NME must be evaluated using tools of nuclear theory



$$m_{\beta\beta} = \left| c_{13}^2 c_{12}^2 e^{i\alpha_1} m_1 + c_{13}^2 s_{12}^2 e^{i\alpha_2} m_2 + s_{13}^2 m_3 \right|$$

**Effective Majorana mass can be evaluated. It depends on  $m_1, m_2, m_3, \theta_{12}, \theta_{13}, \alpha_1, \alpha_2$**   
 (3 unknown parameters:  $m_1/m_3, \alpha_1, \alpha_2$ )

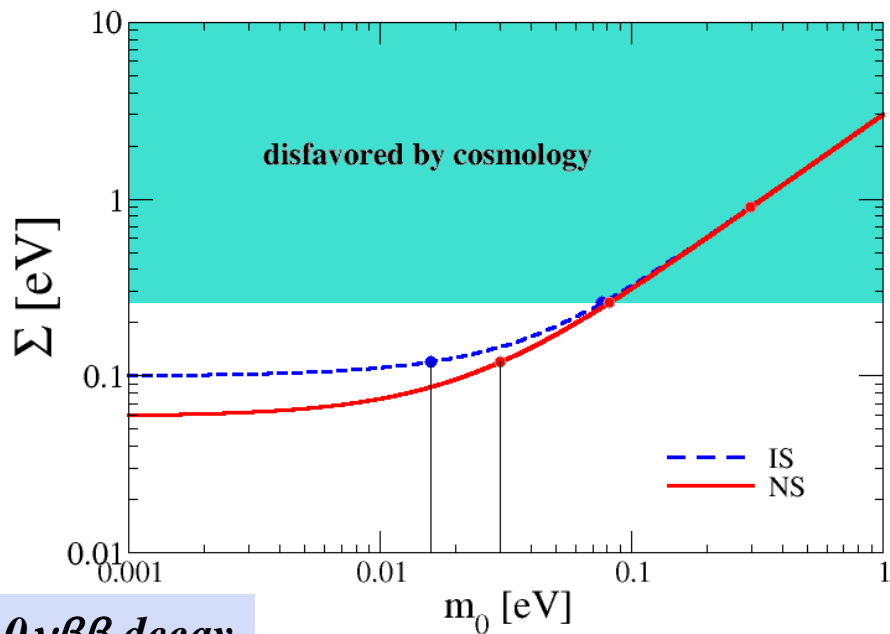


$$U^{PMNS} = \begin{pmatrix} c_{12}c_{13} & c_{13}s_{12} & e^{-i\delta}s_{13} \\ -c_{23}s_{12} - e^{i\delta}c_{12}s_{13}s_{23} & c_{12}c_{23} - e^{i\delta}s_{12}s_{13}s_{23} & c_{13}s_{23} \\ s_{12}s_{23} - e^{i\delta}c_{12}c_{23}s_{13} & -e^{i\delta}c_{23}s_{12}s_{13} - c_{12}s_{23} & c_{13}c_{23} \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



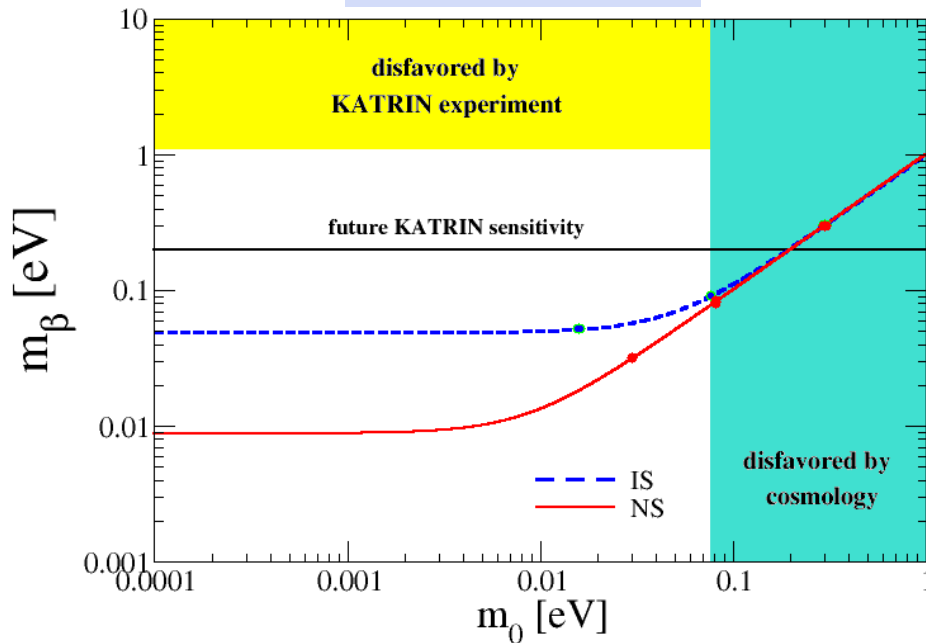
# Cosmological measurements

$$\Sigma = m_1 + m_2 + m_3$$



$\Sigma < 0.90 \text{ eV}$   
 $< 0.26 \text{ eV (Planck coll.)}$   
 $< 0.12 \text{ eV}$

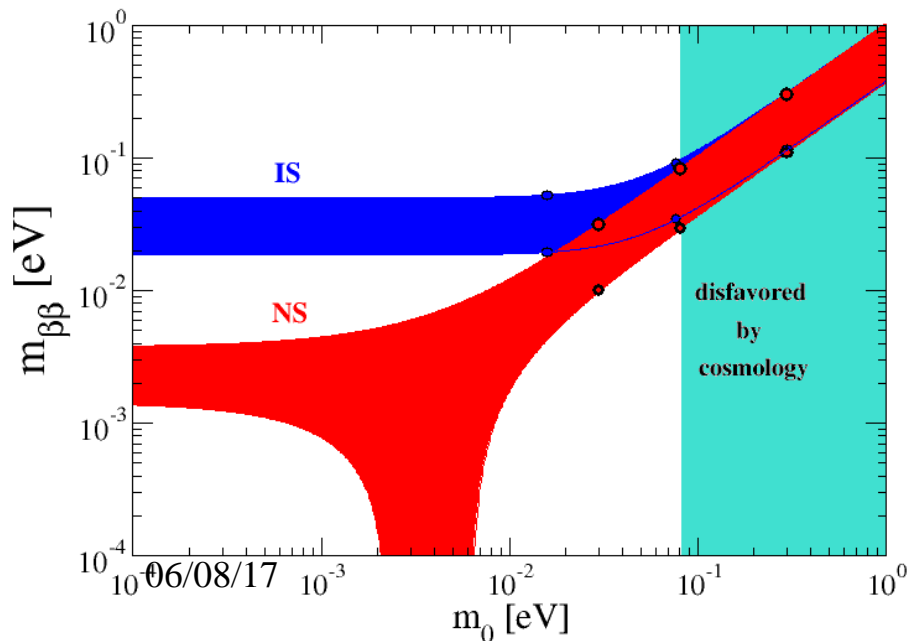
# Tritium $\beta$ -decay

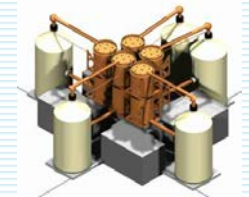


$$m_\beta = \sqrt{c_{13}^2 c_{12}^2 m_1^2 + c_{13}^2 s_{12}^2 m_2^2 + s_{13}^2 m_3^2}$$

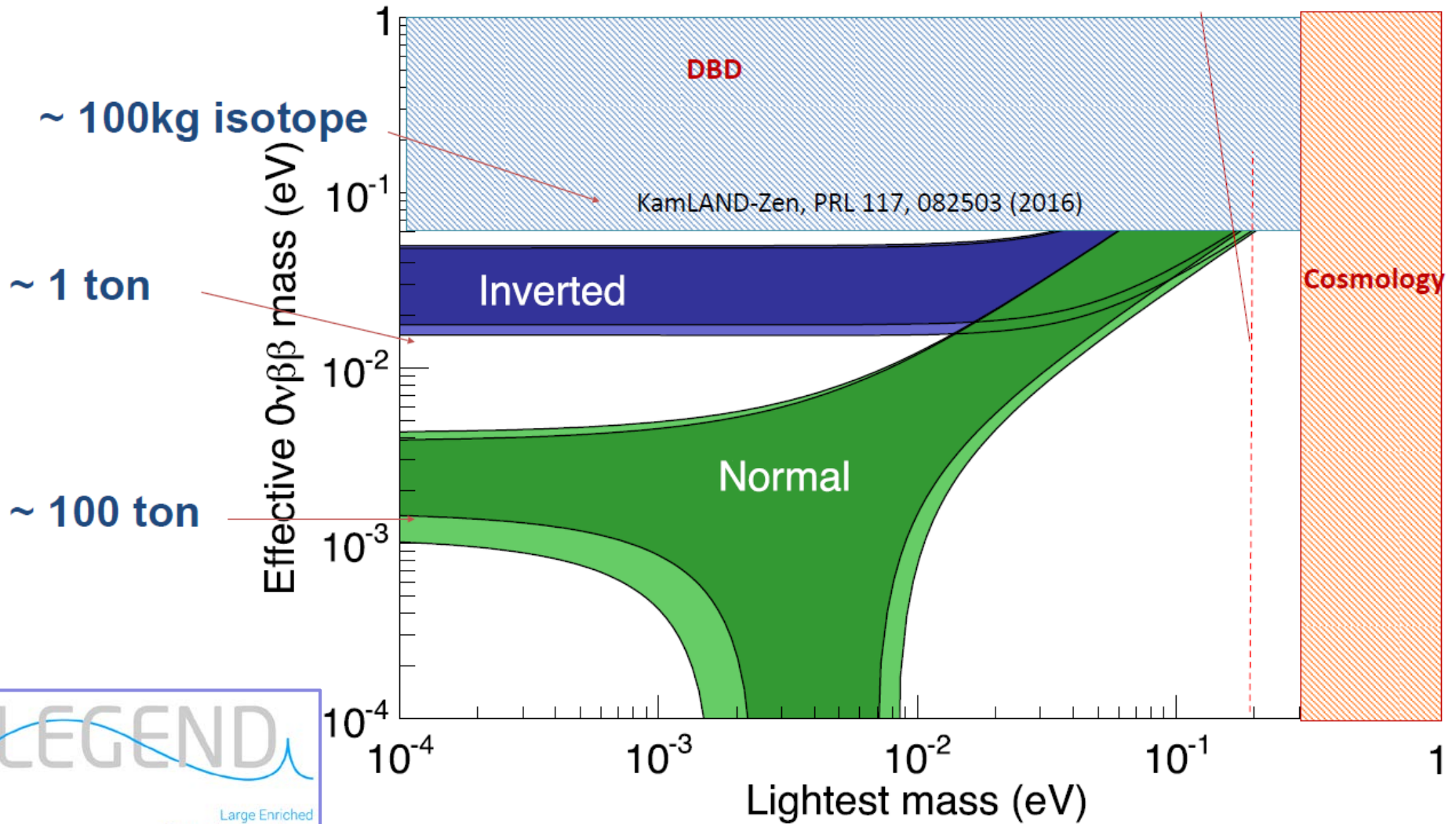
$m_\beta < 1.1 \text{ eV (KATRIN)}$

# $0\nu\beta\beta$ -decay

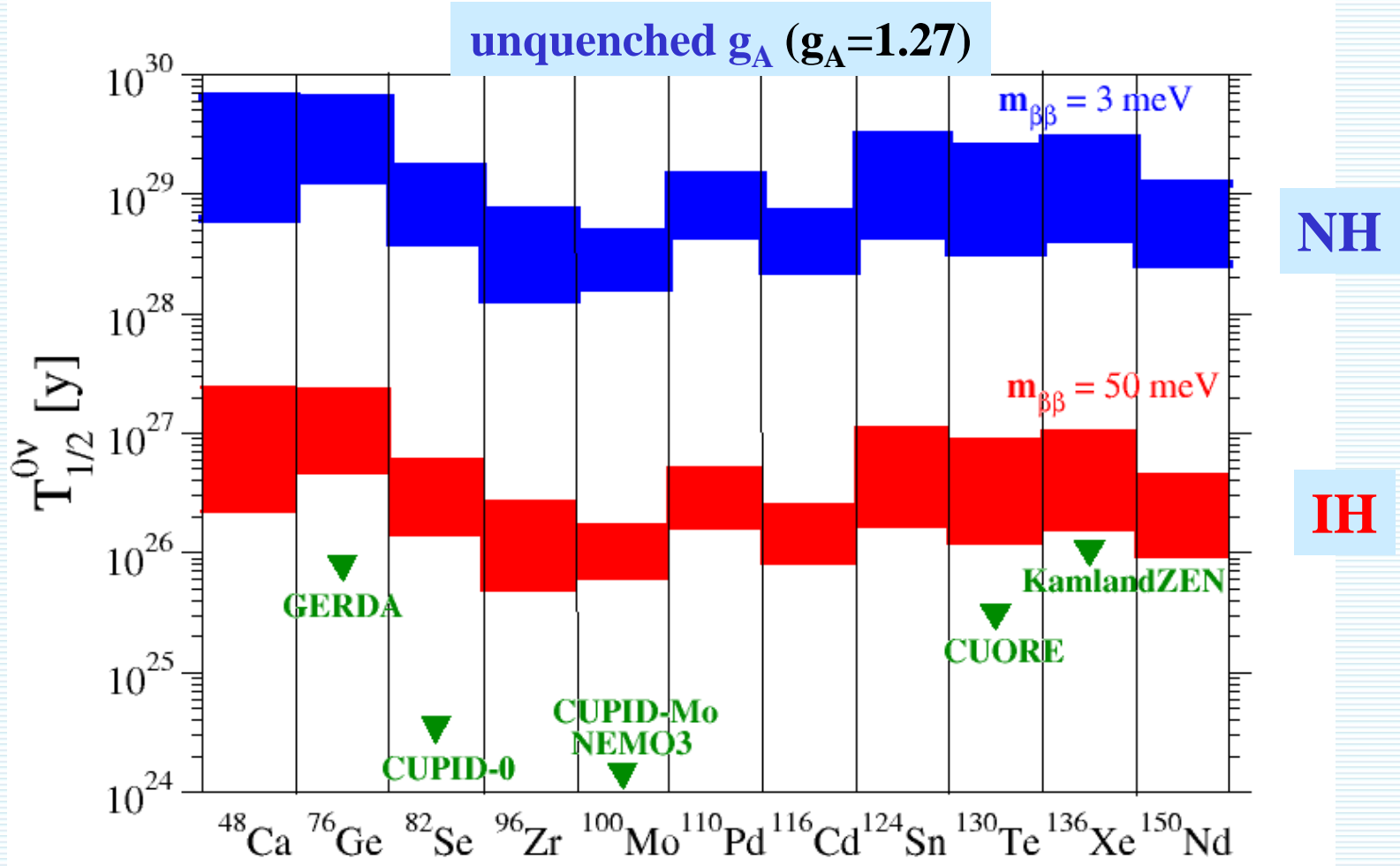




## Estimated KATRIN Sensitivity



# 0νββ –half lives for NH and IH with included uncertainties in NMEs



**NH:**  $m_1 \ll m_2 \ll m_3 \quad m_3 \simeq \sqrt{\Delta m^2}$

**IH:**  $m_3 \ll m_1 < m_2 \quad m_1 \simeq m_2 \simeq \sqrt{\Delta m^2}$

$m_1 \ll \sqrt{\delta m^2}, \quad m_2 \simeq \sqrt{\delta m^2}$

$m_3 \ll \sqrt{\Delta m^2}$

$1.4 \text{ meV} \leq m_{\beta\beta} \leq 3.6 \text{ meV}$

**Lightest ν-mass equal to zero**

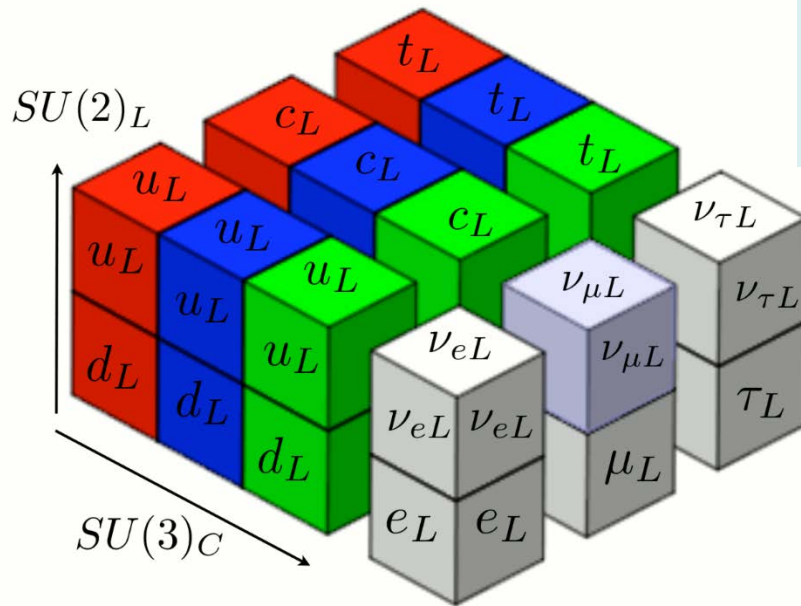
$20 \text{ meV} \leq m_{\beta\beta} \leq 49 \text{ meV}$

Collaboration	Isotope	After 84 years ...	mass (0 $\nu\beta\beta$ isotope)	Status
CANDLES	Ca-48	305 kg CaF <sub>2</sub> crystals - liq. scint	0.3 kg	Construction
CARVEL	Ca-48	<sup>48</sup> CaWO <sub>4</sub> crystal scint.	~ ton	R&D
GERDA I	Ge-76	Ge diodes in LAr	15 kg	Complete
GERDA II	Ge-76	Point contact Ge in LAr	31	Operating
MAJORANA DEMONSTRATOR	Ge-76	Point contact Ge	25 kg	Operating
<b>LEGEND</b>	Ge-76	Point contact with active veto	~ ton	R&D
NEMO3	Mo-100 Se-82	Foils with tracking	6.9 kg 0.9 kg	Complete
SuperNEMO Demonstrator	Se-82	Foils with tracking	7 kg	Construction
SuperNEMO	Se-82	Foils with tracking	100 kg	R&D
LUCIFER (CUPID)	Se-82	ZnSe scint. bolometer	18 kg	R&D
AMoRE	Mo-100	CaMoO <sub>4</sub> scint. bolometer	1.5 - 200 kg	R&D
LUMINEU (CUPID)	Mo-100	ZnMoO <sub>4</sub> / Li <sub>2</sub> MoO <sub>4</sub> scint. bolometer	1.5 - 5 kg	R&D
COBRA	Cd-114,116	CdZnTe detectors	10 kg	R&D
CUORICINO, CUORE-0	Te-130	TeO <sub>2</sub> Bolometer	10 kg, 11 kg	Complete
<b>CUORE</b>	Te-130	TeO <sub>2</sub> Bolometer	<b>206 kg</b>	Operating
<b>CUPID</b>	Te-130	TeO <sub>2</sub> Bolometer & scint.	~ ton	R&D
<b>SNO+</b>	Te-130	0.3% <sup>nat</sup> Te suspended in Scint	<b>160 kg</b>	Construction
EXO200	Xe-136	Xe liquid TPC	79 kg	Operating
<b>nEXO</b>	Xe-136	Xe liquid TPC	~ ton	R&D
<b>KamLAND-Zen (I, II)</b>	Xe-136	2.7% in liquid scint.	<b>380 kg</b>	Complete
<b>KamLAND2-Zen</b>	Xe-136	2.7% in liquid scint.	<b>750 kg</b>	Upgrade
NEXT-NEW	Xe-136	High pressure Xe TPC	5 kg	Operating
<b>NEXT-100</b>	Xe-136	High pressure Xe TPC	100 kg - ton	R&D
<b>PandaX - III</b>	Xe-136	High pressure Xe TPC	~ ton	R&D
DCBA	Nd-150	Nd foils & tracking chambers	20 kg	R&D

*A generation of Majorana neutrino mass,  
 $0\nu\beta\beta$ -decay mechanisms*

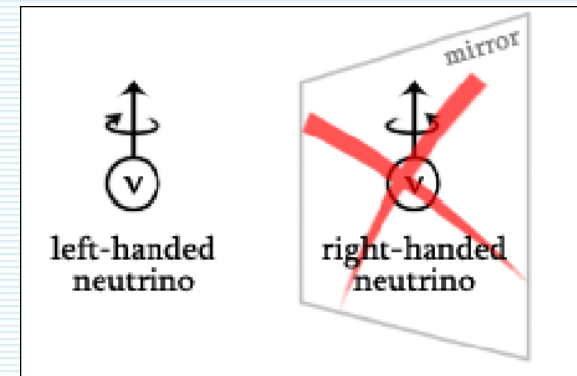
# Standard Model

(an astonishing successful theory, based on few principles)

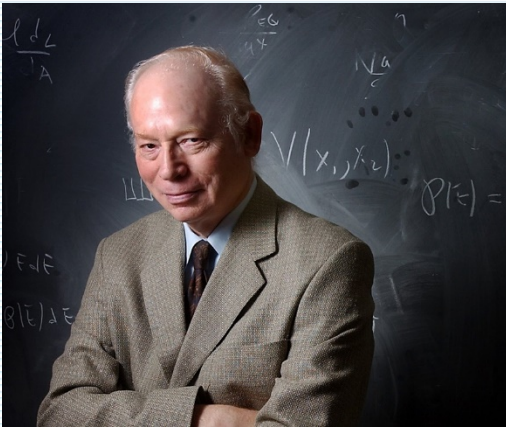


## Neutrino is a special particle in SM:

- It is the only fermion that does not carry electric charge (like bosons  $\gamma, g, H^0$ ) !
- In the SM, the only left-handed neutrinos  $\nu_L$  appears in the theory.
- One cannot obtain a mass for  $\nu_L$  with any renormalizable coupling with the Higgs fields through SSB.



However, we know that  $\nu$ 's do have mass from the  $\nu$ -oscillation experiments!  
=> Thus the neutrino mass indicates that there is something new = **BSM physics!**

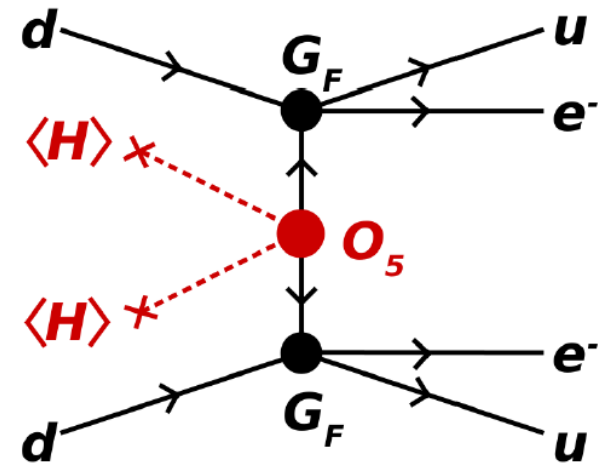


Weinberg, 1979:  $d=5$

$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda} (L_i H)(L_j H)$$

Weinberg does not take credit for predicting neutrino masses, but he thinks it's the right interpretation. What's more, he says, the non-renormalisable interaction that produces the neutrino masses is probably also accompanied with non-renormalisable interactions that produce proton decay and other things that haven't been observed, such as violation of baryon-number conservations. "We don't know anything about the details of those terms, but I'll swear they are there."

$0\nu\beta\beta$  decay:

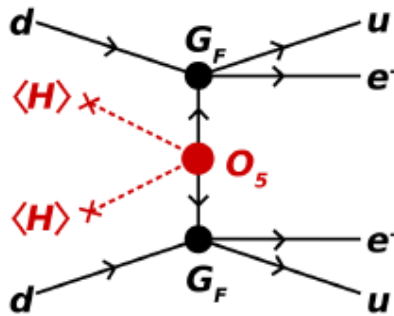


$$\mathcal{L} = \mathcal{L}_{SM}^{(4)} + \frac{1}{\Lambda} \sum_i c_i^{(5)} \mathcal{O}_i^{(5)} + \frac{1}{\Lambda^2} \sum_i c_i^{(6)} \mathcal{O}_i^{(6)} + O\left(\frac{1}{\Lambda^3}\right)$$

## Beyond the SM physics

**Amplitude for**  
 **$(A,Z) \rightarrow (A,Z+2) + 2e^-$**   
**can be divided into:**

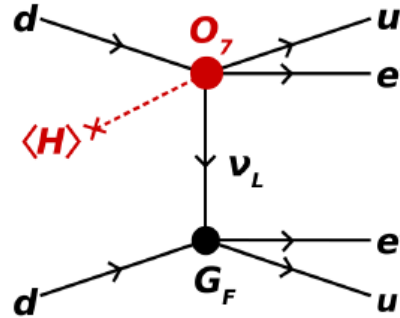
**mass mechanism:  $d=5$**



$$\mathcal{O}_W \propto \frac{c_{ij}}{\Lambda} (L_i H)(L_j H)$$

Weinberg, 1979

**long range:  $d=7$**



$$\mathcal{O}_2 \propto LLLe^c H$$

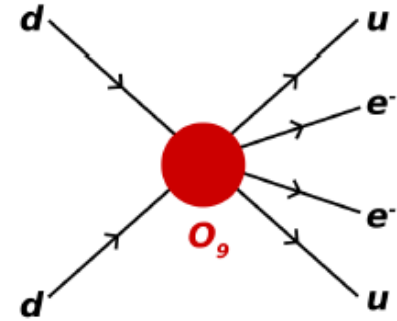
$$\mathcal{O}_3 \propto LLQd^c H$$

$$\mathcal{O}_4 \propto LL\bar{Q}\bar{u}^c H$$

$$\mathcal{O}_8 \propto L\bar{e}^c \bar{u}^c d^c H$$

Babu, Leung: 2001  
 de Gouvea, Jenkins: 2007

**short range:  $d=9$  ( $d=11$ )**



$$\mathcal{O}_5 \propto LLQd^c HHH^\dagger$$

$$\mathcal{O}_6 \propto LL\bar{Q}\bar{u}^c HHH^\dagger H$$

$$\mathcal{O}_7 \propto LQ\bar{e}^c \bar{Q}HHH^\dagger$$

$$\mathcal{O}_9 \propto LLLe^c Le^c$$

$$\mathcal{O}_{10} \propto LLLe^c Qd^c$$

$$\mathcal{O}_{11} \propto LLQd^c Qd^c$$

.....

Valle



# Quark Condensate Seesaw Mechanism for Neutrino Mass

A. Babič, S. Kovalenko, M.I. Krivoruchenko, F.Š., arXiv:1911.12189

This operator contributes to the **Majorana-neutrino mass matrix** due to chiral symmetry breaking via the **light-quark condensate**.

The SM gauge-invariant effective operators

$$O_7^{u,d} = \frac{\tilde{g}_{\alpha\beta}^{u,d}}{\Lambda^3} \overline{L}_\alpha^C L_\beta H \left\{ (\overline{Q} u_R), (\overline{d}_R Q) \right\}$$

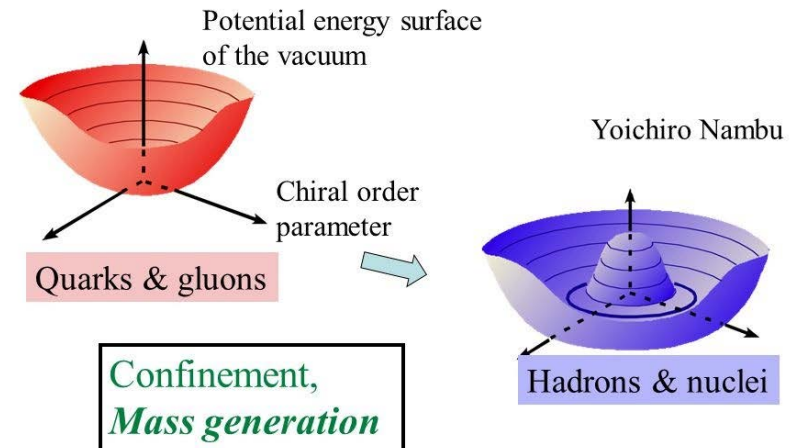
After the **EWSB** and **ChSB** one arrives at the Majorana mass matrix of active neutrinos

$$m_{\alpha\beta}^\nu = g_{\alpha\beta} v \frac{\langle \overline{q}q \rangle}{\Lambda^3} = g_{\alpha\beta} v \left( \frac{\omega}{\Lambda} \right)^3$$

$$g_{\alpha\beta} = g_{\alpha\beta}^u + g_{\alpha\beta}^d, \quad v/\sqrt{2} = \langle H^0 \rangle$$

$$\omega = -\langle \overline{q}q \rangle^{1/3}, \quad \langle \overline{q}q \rangle^{1/3} \approx -283 \text{ MeV}$$

Spontaneous breaking of **chiral ( $\chi$ ) symmetry**



$\Lambda \sim$  a few TeV  
we get the neutrino mass in the **sub-eV** ballpark

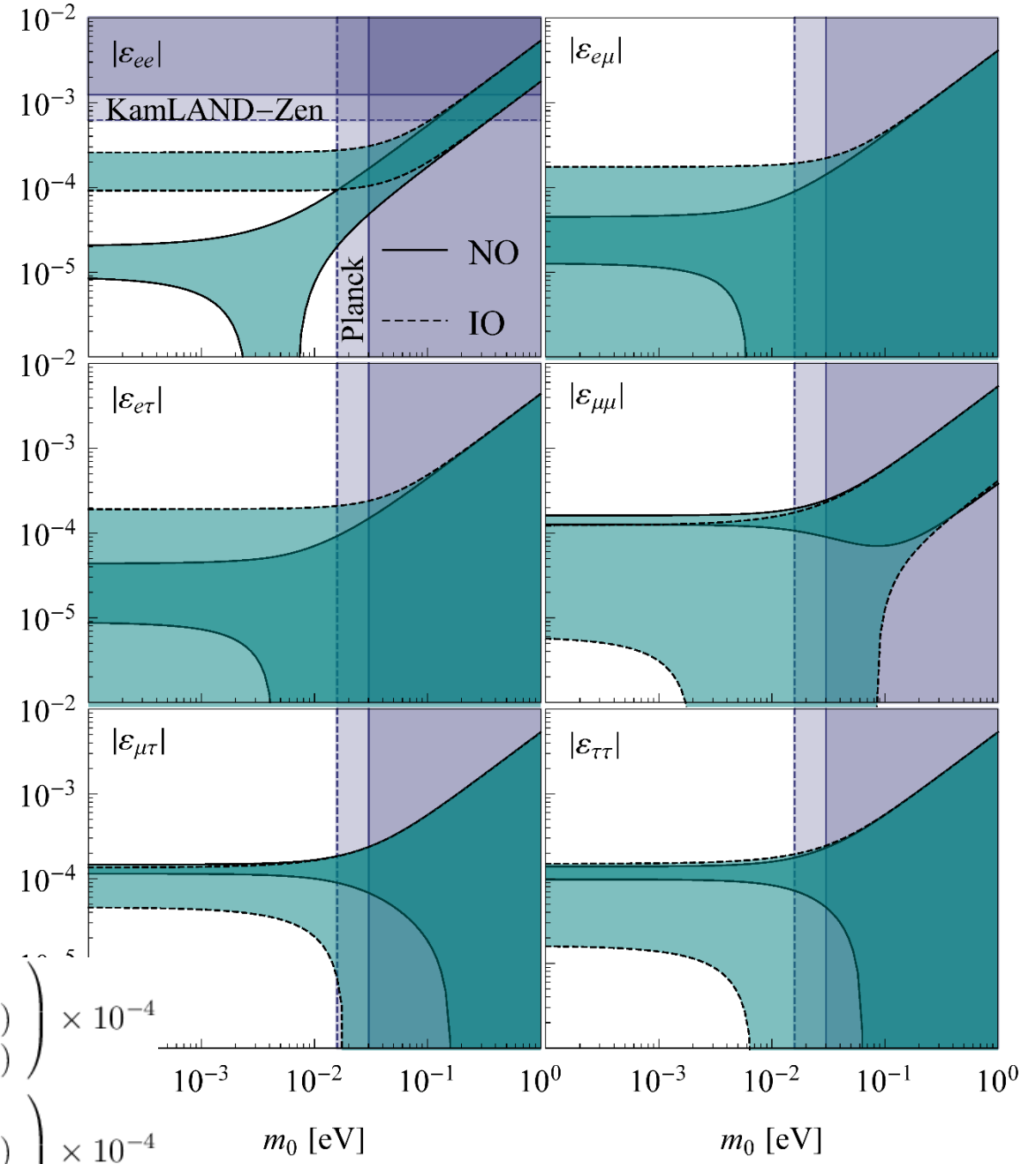
The literature lacks the limits on this class of non-standard interactions



$$\epsilon_{\alpha\beta} = \frac{g_{\alpha\beta}^2 / \Lambda_{\text{LNV}}^2}{G_F / \sqrt{2}} \approx \frac{M_{\alpha\beta} / \langle \bar{q}q \rangle_0}{G_F / \sqrt{2}}$$

$$|\epsilon_{\alpha\beta}^{\text{NH}}| = \begin{pmatrix} (0, 1.7) & (0, 1.3) & (0, 1.5) \\ & (0.9, 2.4) & (0.7, 2.4) \\ & & (0.5, 2.3) \end{pmatrix} \times 10^{-4}$$

$$|\epsilon_{\alpha\beta}^{\text{NH}}| = \begin{pmatrix} (0.9, 2.7) & (0, 1.9) & (0, 2.1) \\ & (0, 1.7) & (0.1, 1.8) \\ & & (0, 1.9) \end{pmatrix} \times 10^{-4}$$



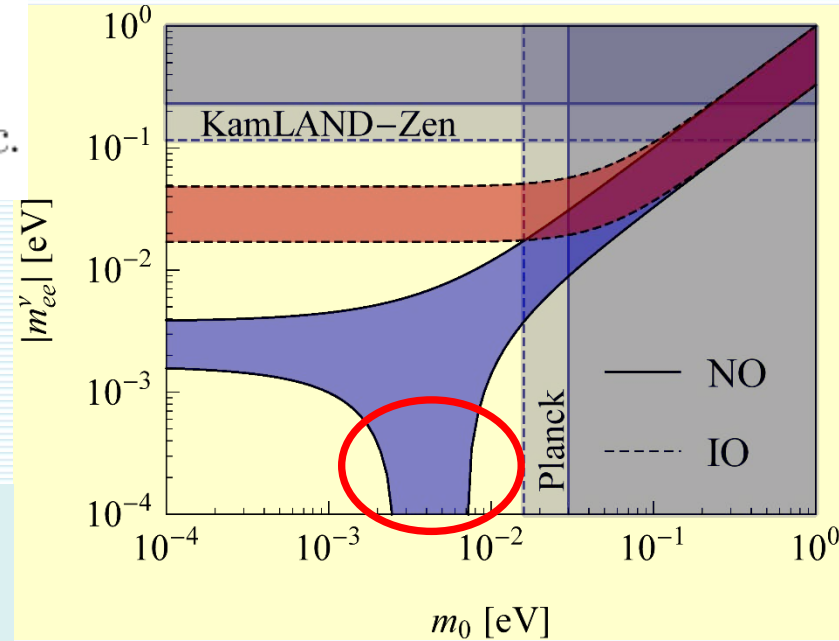
# The genuine QCSS scenario with no fine-tuning

A. Babič, S. Kovalenko,  
M.I. Krivoruchenko, F.Š.,  
arXiv: 1911.12189 [hep-ph]

$$\mathcal{L}_7 = \frac{G_F}{\sqrt{2}} \bar{e}_L \nu_L^C \left( \varepsilon^{u_{ee}} \bar{u}_R d_L - \varepsilon^{d_{ee}} \bar{u}_L d_R \right) + \frac{G_F}{\sqrt{2}} \bar{\nu}_L^C \nu_L \left( \varepsilon^{u_{ee}} \bar{u}_L u_R + \varepsilon^{d_{ee}} \bar{d}_R d_L \right) + \text{H.c.}$$

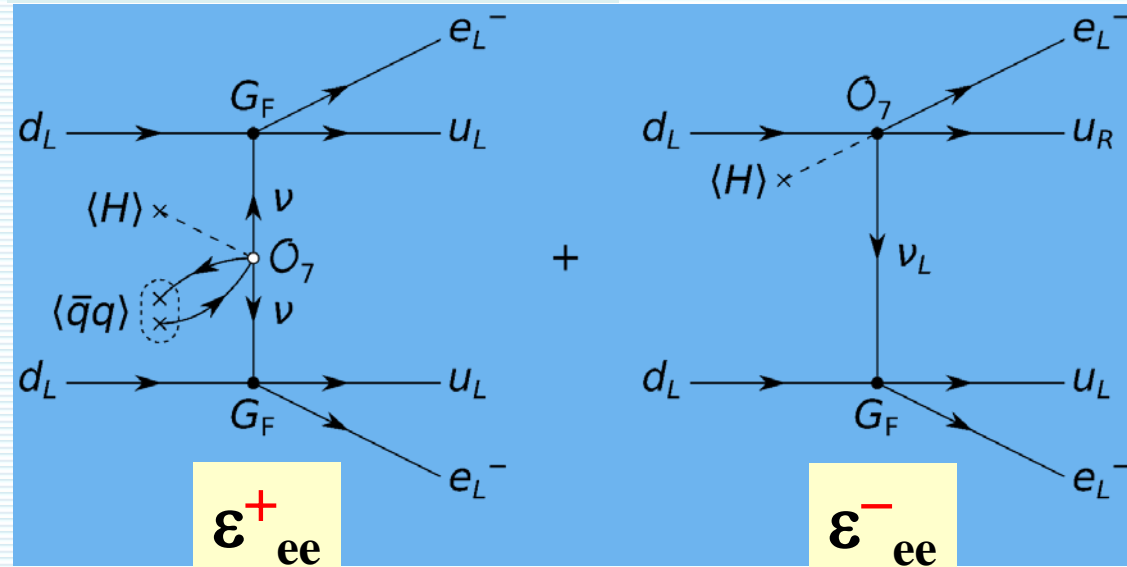
$$\varepsilon_{\alpha\beta}^{u,d} = \frac{g_{\alpha\beta}^{u,d} v / \Lambda^3}{G_F / \sqrt{2}}$$

$$\varepsilon_{\alpha\beta}^{\pm} = \varepsilon_{\alpha\beta}^u \pm \varepsilon_{\alpha\beta}^d$$



(a) discussed earlier in  
S.Kovalenko,  
M.I. Krivoruchenko, F.Š.,  
PRL 112, 142503 (2014).

(b) discussed earlier in  
H. Päs, M. Hirsch,  
S. Kovalenko  
PLB 453, 194 (1999).



## New features:

**IH excluded, limits  $\varepsilon < 10^{-8}$**

**$2 \text{ meV} < m_1 < 7 \text{ meV}$**

**$9 \text{ meV} < m_2 < 11 \text{ meV}$**

**$50 \text{ meV} < m_2 < 51 \text{ meV}$**

Neutrino  
spectrum

Prediction for Kathrin

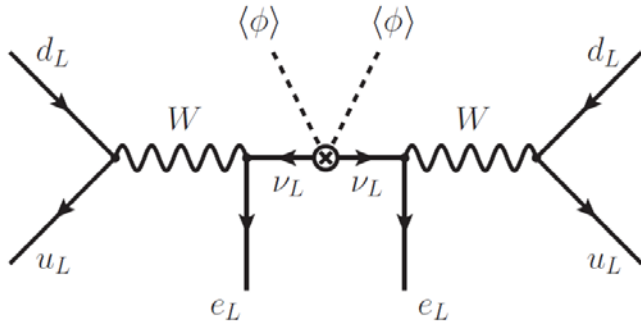
**$9 \text{ meV} < m_\beta < 12 \text{ meV}$**

Prediction for cosmology

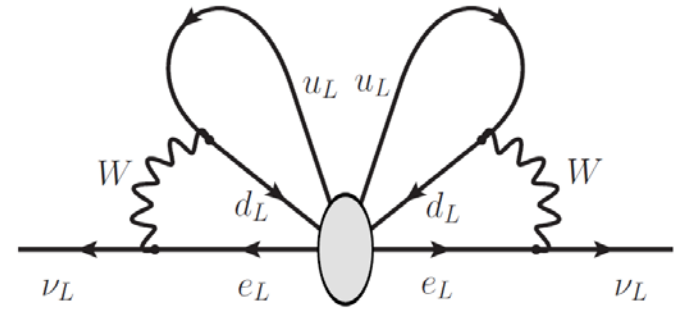
**$62 \text{ meV} < m_1 + m_2 + m_3 < 69 \text{ meV}$**

If  $0\nu\beta\beta$  is observed the  $\nu$  is  
a Majorana particle

Majorana  $m_\nu \implies 0\nu\beta\beta$



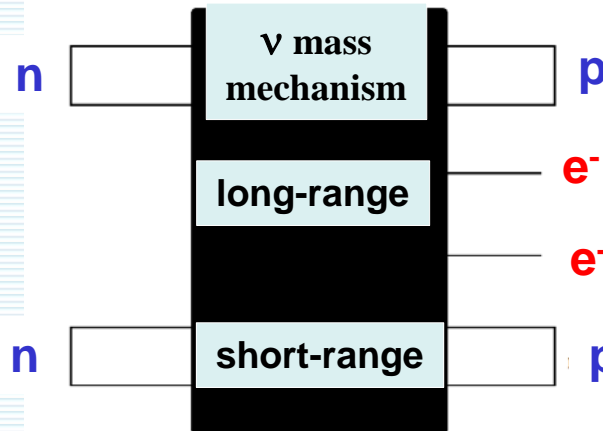
$0\nu\beta\beta \implies$  Majorana  $m_\nu$



Schechter, Valle: PRD 1982

## Different $0\nu\beta\beta$ -decay scenarios

Can we say  
something about  
content  
of the black box?



Considering

- i. Sterile  $\nu$
- ii. Different LNV scales
- iii. Right-handed currents
- iv. Non-standard  $\nu$ -interactions
- v. ....

# Left-handed neutrinos: Majorana neutrino mass eigenstate $N$ with arbitrary mass $m_N$

Faessler, Gonzales, Kovalenko, F. Š., PRD 90 (2014) 096010]

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \left| \sum_N (U_{eN}^2 m_N) m_p M'^{0\nu}(m_N, g_A^{\text{eff}}) \right|^2$$

## General case

$$M'^{0\nu}(m_N, g_A^{\text{eff}}) = \frac{1}{m_p m_e} \frac{R}{2\pi^2 g_A^2} \sum_n \int d^3x d^3y d^3p \times e^{ip \cdot (\mathbf{x}-\mathbf{y})} \frac{\langle 0_F^+ | J^{\mu\dagger}(\mathbf{x}) | n \rangle \langle n | J_\mu^\dagger(\mathbf{y}) | 0_I^+ \rangle}{\sqrt{p^2 + m_N^2} (\sqrt{p^2 + m_N^2} + E_n - \frac{E_I - E_F}{2})} M'^{0\nu}(m_N \rightarrow 0, g_A^{\text{eff}}) = \frac{1}{m_p m_e} M_\nu'^{0\nu}(g_A^{\text{eff}})$$

$$M'^{0\nu}(m_N \rightarrow \infty, g_A^{\text{eff}}) = \frac{1}{m_N^2} M_N'^{0\nu}(g_A^{\text{eff}})$$

## light $\nu$ exchange

## heavy $\nu$ exchange

## Particular cases

$$[T_{1/2}^{0\nu}]^{-1} = G^{0\nu} g_A^4 \times \begin{cases} \left| \frac{\langle m_\nu \rangle}{m_e} \right|^2 \left| M_\nu'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \ll p_F \\ \left| \langle \frac{1}{m_N} \rangle m_p \right|^2 \left| M_N'^{0\nu}(g_A^{\text{eff}}) \right|^2 & \text{for } m_N \gg p_F \end{cases}$$

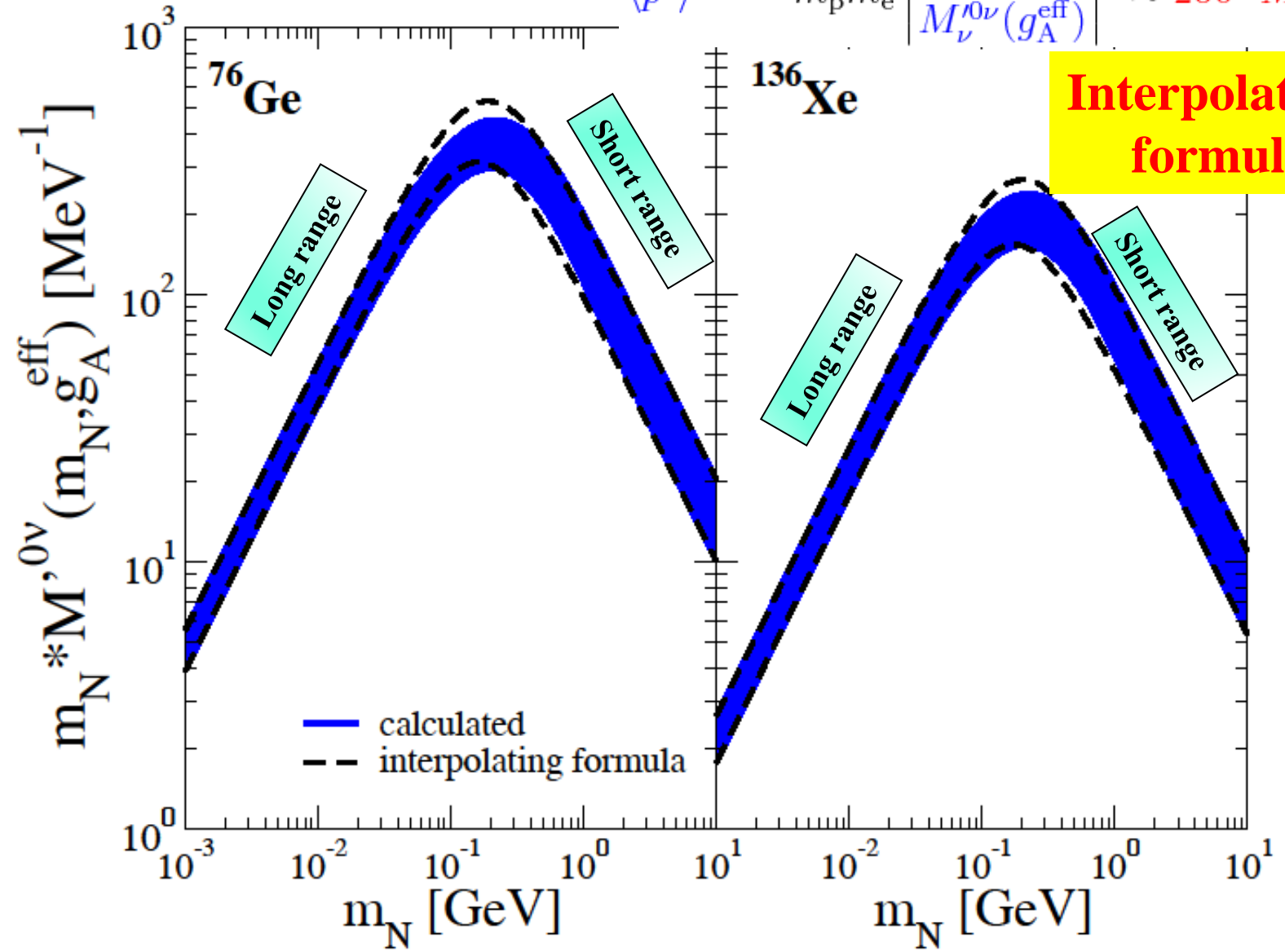
$$\langle m_\nu \rangle = \sum_N U_{eN}^2 m_N$$

$$\left\langle \frac{1}{m_N} \right\rangle = \sum_N \frac{U_{eN}^2}{m_N}$$

$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2,$$

$$\mathcal{A} = G^{0\nu} g_A^4 \left| M_N^{0\nu}(g_A^{\text{eff}}) \right|^2,$$

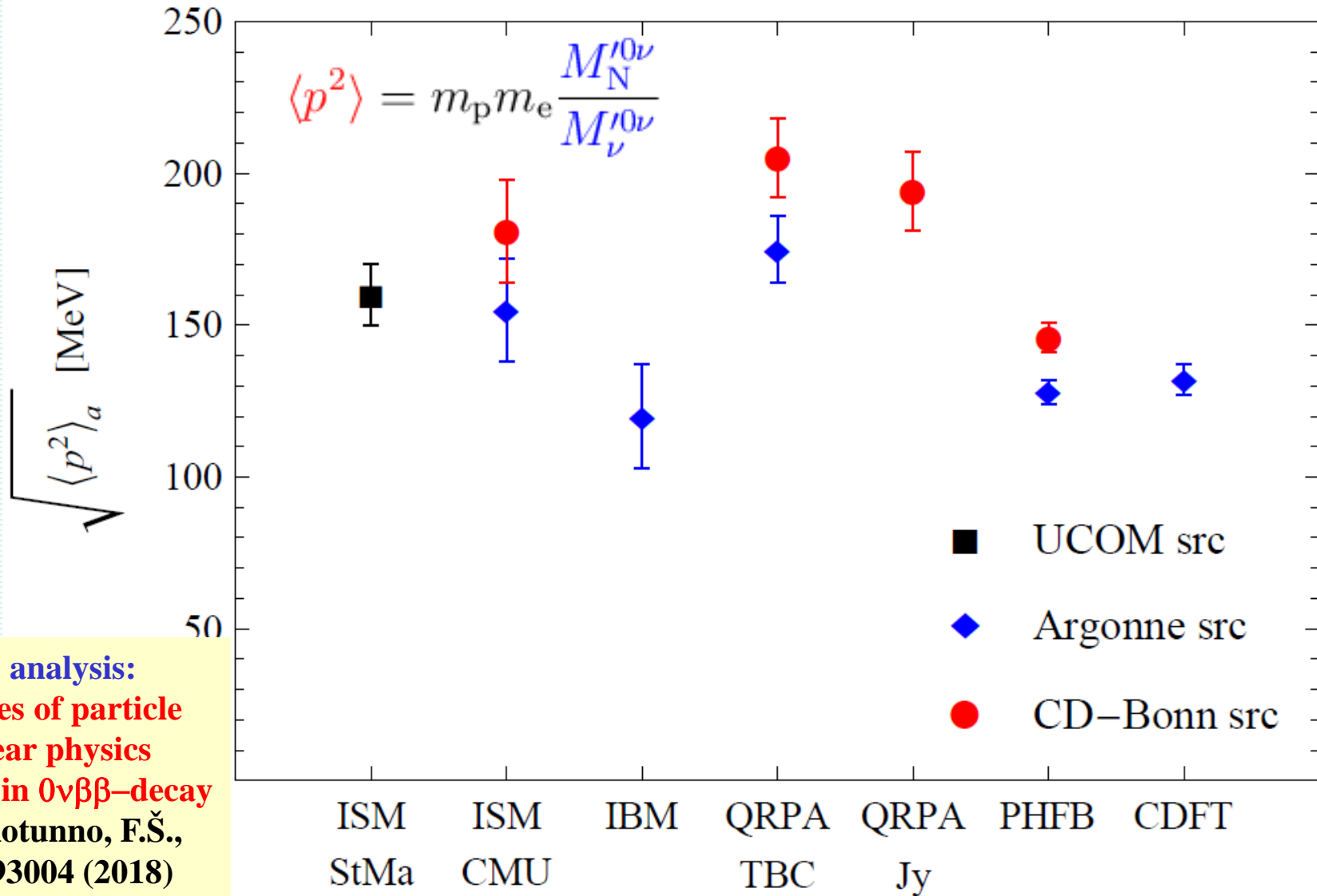
$$\langle p^2 \rangle = m_p m_e \left| \frac{M_N^{0\nu}(g_A^{\text{eff}})}{M_\nu^{0\nu}(g_A^{\text{eff}})} \right| \approx 200 \text{ MeV}$$



**Interpolating formula is justified  
by practically no dependence  $\langle p^2 \rangle$  on  $A$**

A. Babič, S. Kovalenko, M.I. Krivoruchenko,  
F.Š., PRD 98, 015003 (2018)

$$[T_{1/2}^{0\nu}]^{-1} = \mathcal{A} \cdot \left| m_p \sum_N U_{eN}^2 \frac{m_N}{\langle p^2 \rangle + m_N^2} \right|^2,$$



Detailed analysis:

Degeneracies of particle  
and nuclear physics

uncertainties in  $0\nu\beta\beta$ -decay

E. Lisi, A. Rotunno, F.Š.,  
PRD 92, 093004 (2018)

# The $0\nu\beta\beta$ -decay within L-R symmetric theories

(interpolating formula)

(D-M mass term, see-saw, V-A and V+A int., exchange of heavy neutrinos)

A. Babič, S. Kovalenko, M.I. Krivoruchenko, F.Š., PRD 98, 015003 (2018)

$$[T_{1/2}^{0\nu}]^{-1} = \eta_{\nu N}^2 C_{\nu N}$$

$$C_{\nu N} = g_A^4 \left| M_{\nu}^{\prime 0\nu} \right|^2 G^{0\nu}$$

Mixing of light and heavy neutrinos

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix}$$

$$\nu_{eL} = \sum_{j=1}^3 \left( U_{ej} \nu_{jL} + S_{ej} (N_{jR})^C \right),$$

$$\nu_{eR} = \sum_{j=1}^3 \left( T_{ej}^* (\nu_{jL})^C + V_{ej}^* N_{jR} \right)$$

Effective LNV parameter within LRS model  
(due interpolating formula)

$$\langle p^2 \rangle = m_p m_e \frac{M_N^{\prime 0\nu}}{M_{\nu}^{\prime 0\nu}}$$

$$\eta_{\nu N}^2 = \left| \sum_{j=1}^3 \left( U_{ej}^2 \frac{m_j}{m_e} + S_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2 + \lambda^2 \left| \sum_{j=1}^3 \left( T_{ej}^2 \frac{m_j}{m_e} + V_{ej}^2 \frac{\langle p^2 \rangle_a}{\langle p^2 \rangle_a + M_j^2} \frac{M_j}{m_e} \right) \right|^2$$

**The dominance of light and heavy  $\nu$ -mass contributions to  $0\nu\beta\beta$ -decay rate can not be established by observing this process at different nuclei.**



# 6x6 PMNS see-saw $\nu$ -mixing matrix (the most economical one, prediction for mixing of heavy neutral leptons)

6x6 neutrino mass matrix

$$\mathcal{U} = \begin{pmatrix} U & S \\ T & V \end{pmatrix} \quad \text{Basis} \quad (\nu_L, (N_R)^c)^T \quad \mathcal{M} = \begin{pmatrix} M_L & M_D \\ M_D & M_R \end{pmatrix}$$

**6x6 matrix:** 15 angles, 10+5 CP phases  
**3x3 matrix:** 3 angles, 1+2 CP phases

**3x3 block matrices  $U, S, T, V$  are  
generalization of PMNS matrix**

## Assumptions:

- i) the see-saw structure
- ii) mixing between different generations is neglected

$$\mathcal{U}_{\text{PMNS}} = \begin{pmatrix} U_{\text{PMNS}} & \zeta \mathbf{1} \\ -\zeta \mathbf{1} & U_{\text{PMNS}}^\dagger \end{pmatrix} \quad \mathcal{U}_{\text{PMNS}} \mathcal{U}_{\text{PMNS}}^\dagger = \mathcal{U}_{\text{PMNS}}^\dagger \mathcal{U}_{\text{PMNS}} = \mathbf{1}$$

see-saw  
parameter

$$\zeta = \frac{m_D}{m_{\text{LNV}}}$$

**6x6 matrix:** 3 angles, 1+2 CP phases, 1 see-saw par.

# 6x6 PMNS see-saw $\nu$ -mixing matrix (the most economical one)

$$\mathcal{U} = \begin{pmatrix} U_0 & \zeta \mathbf{1} \\ -\zeta \mathbf{1} & V_0 \end{pmatrix}$$

$$U_0 = U_{\text{PMNS}}$$

A. Babič, S. Kovalenko, M.I. Krivoruchenko, F.Š., PRD 98, 015003 (2018)

$$V_0 = U_{\text{PMNS}}^\dagger =$$

$$\begin{pmatrix} c_{12} c_{13} e^{-i\alpha_1} & \left( -s_{12} c_{23} - c_{12} s_{13} s_{23} e^{-i\delta} \right) e^{-i\alpha_1} & \left( s_{12} s_{23} - c_{12} s_{13} c_{23} e^{-i\delta} \right) e^{-i\alpha_1} \\ s_{12} c_{13} e^{-i\alpha_2} & \left( c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i\delta} \right) e^{-i\alpha_2} & \left( -c_{12} s_{23} - s_{12} s_{13} c_{23} e^{-i\delta} \right) e^{-i\alpha_2} \\ s_{13} e^{i\delta} & c_{13} s_{23} & c_{13} c_{23} \end{pmatrix}$$

Assumption about heavy neutrino masses  $M_i$  (by assuming see-saw)

Inverse  
proportional

$$m_i M_i \simeq m_D^2$$

$$M_{\beta\beta}^{\text{R}} = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 m_j \right|$$

$$m_i \simeq \zeta^2 M_i$$

$$M_{\beta\beta}^{\text{R}} = \lambda \zeta^2 \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 \frac{\langle p^2 \rangle_a}{m_j} \right|$$

Proportional

$M_{\beta\beta}^{\text{R}}$  depends on  
“Dirac” CP phase  $\delta$   
unlike “Majorana”  
CP phases  $\alpha_1$  and  $\alpha_2$

Heavy Majorana mass  $M_{\beta\beta}^{\text{R}}$  depends on the “Dirac” CP violating phase  $\delta$  6

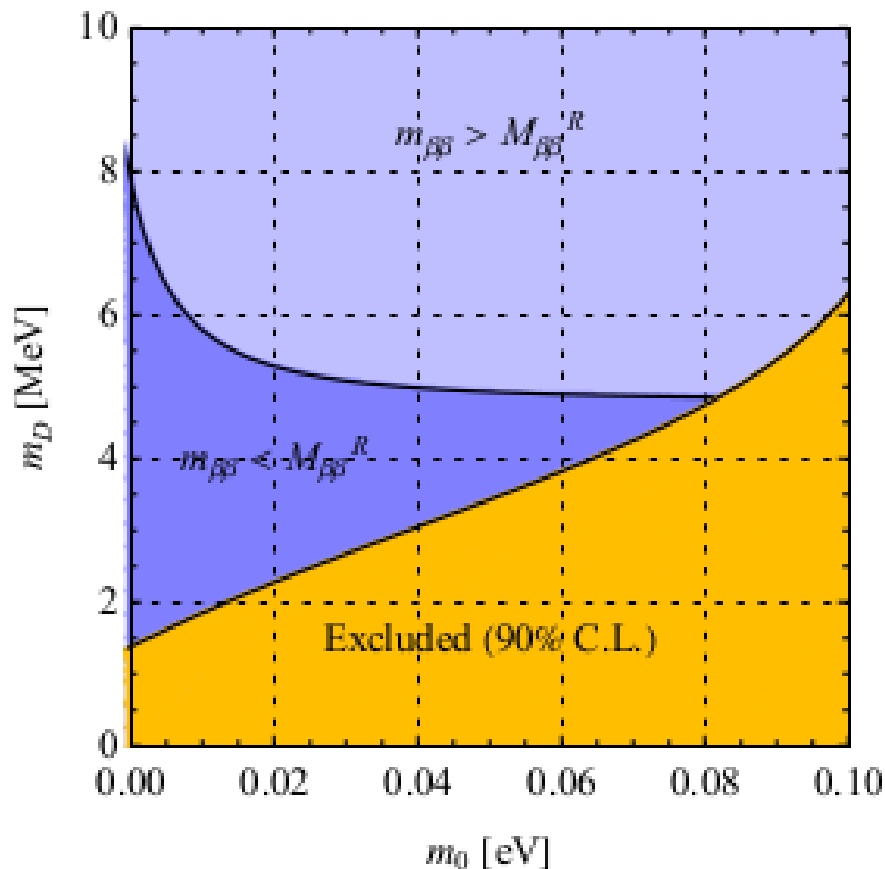
# See-saw scenario

$$m_i M_i \simeq m_D^2$$

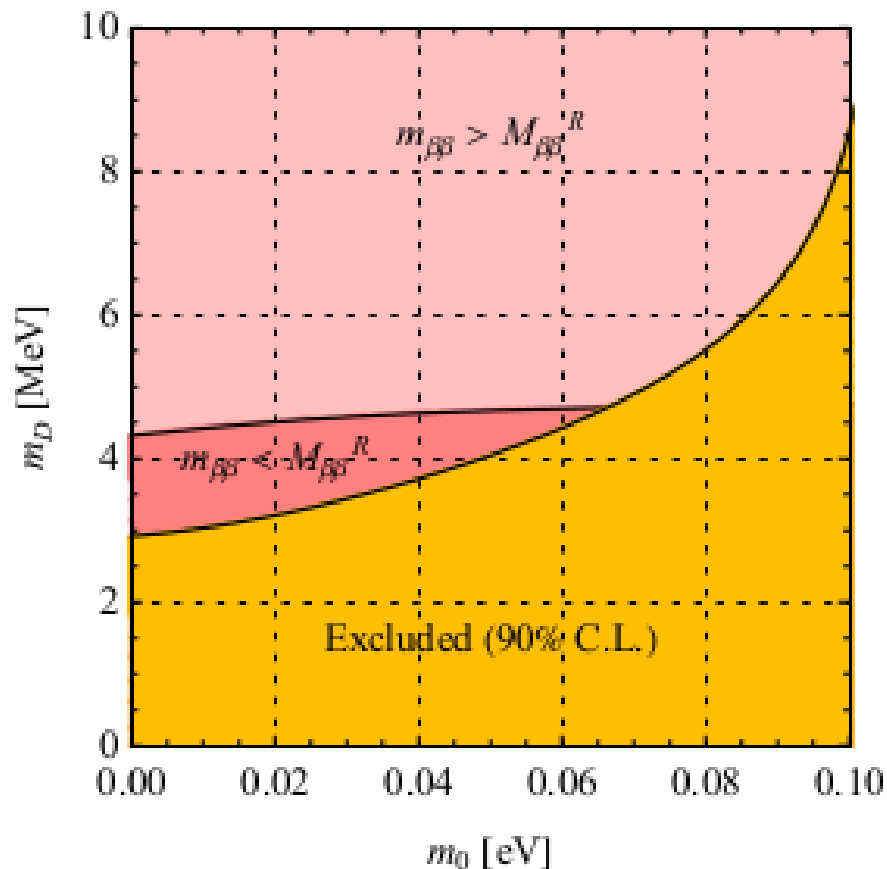
$$M_{\beta\beta}^R = \lambda \frac{\langle p^2 \rangle_a}{m_D^2} \left| \sum_{j=1}^3 (U_0^\dagger)_{ej}^2 m_j \right|$$

$$\eta_{\nu N}^2 = \frac{1}{m_e^2} \left( m_{\beta\beta}^2 + (M_{\beta\beta}^R)^2 \right)$$

## Normal spectrum



## Inverted spectrum



*The  $0\nu\beta\beta$ -decay NMEs – current status*

**2004 (factor 10)**

few groups, 2 nuclear structure methods:

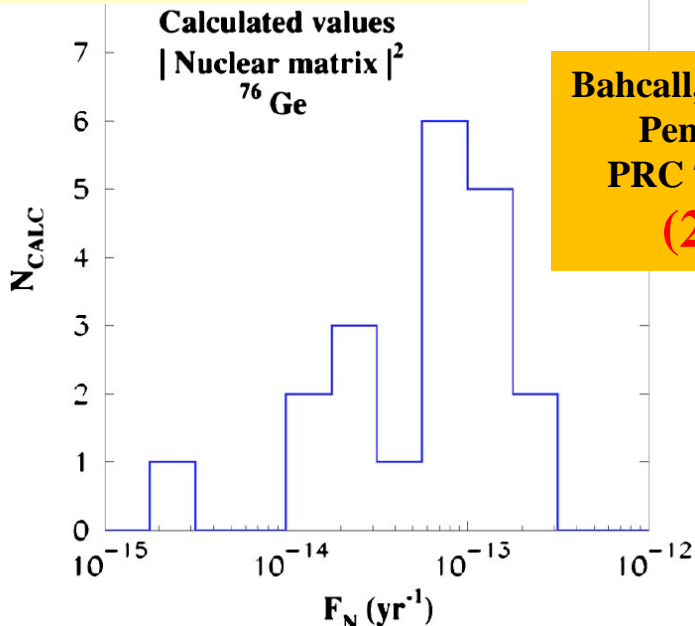
**Nuclear Shell Model, QRPA**

*$0\nu\beta\beta$  decay NMEs*

**2019 (factor 2-3)**

many groups, many nuclear structure methods:

**Nuclear Shell Model, QRPA, Interacting Boson Model, Energy Density Functional**



**Bahcall, Maruyama,  
Pena-Garay,  
PRC 70, 033012  
(2004)**

**Attempts (light nuclear systems):  
Ab initio calculations by different approaches – No Core Shell Model, Green’s Function Monte Carlo, Coupled Cluster Method, Lattice QCD**

***Nuclear Shell Model*** (Madrid-Strasbourg, Michigan, Tokyo): **Relatively small model space (1 shell), all correlations included, solved by direct diagonalization**

***QRPA*** (Tuebingen-Bratislava-Calltech, Jyvaskyla, Chapel Hill, Lanzhou, Prague): **Several shells, only simple correlations included**

***Interacting Boson Method*** (Yale-Concepcion): **Small space, important proton-neutron Pairing correlations missing**

***Energy Density Functional theory*** (Madrid, Beijing): **>10 shells, important proton-neutron pairing missing**

# Ab Initio Nuclear Structure

(Often starts with chiral effective-field theory)

*Nucleons, pions. Sufficient below chiral symmetry breaking scale. Expansion of operators in power of  $Q/\Lambda_\chi$ .  $Q=m_\pi$  or typical nucleon momentum.*

Physics of Hadrons

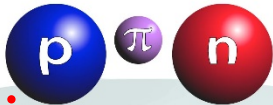
Degrees of Freedom



quarks, gluons

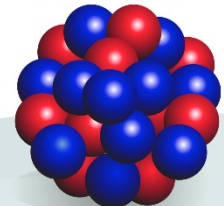


constituent quarks



baryons, mesons

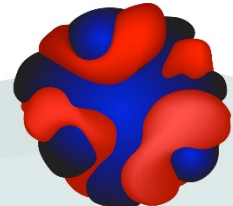
**ab initio**



protons, neutrons

Physics of Nuclei

**DFT**



nucleonic densities and currents

**collective models**



collective coordinates

Energy (MeV)

940  
neutron mass

$Q^0$   
LO

2N Force

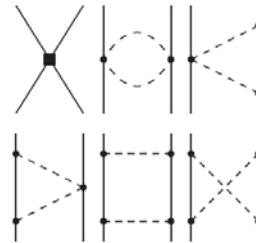


3N Force

4N Force

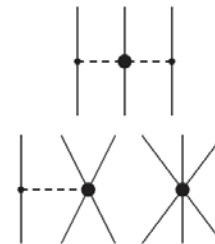
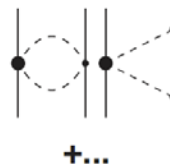
140  
pion mass

$Q^2$   
NLO



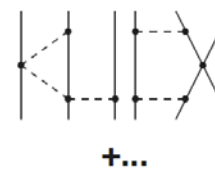
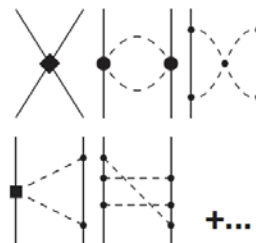
8  
proton separation energy in lead

$Q^3$   
NNLO



1.12  
vibrational state in tin

$Q^4$   
 $N^3LO$



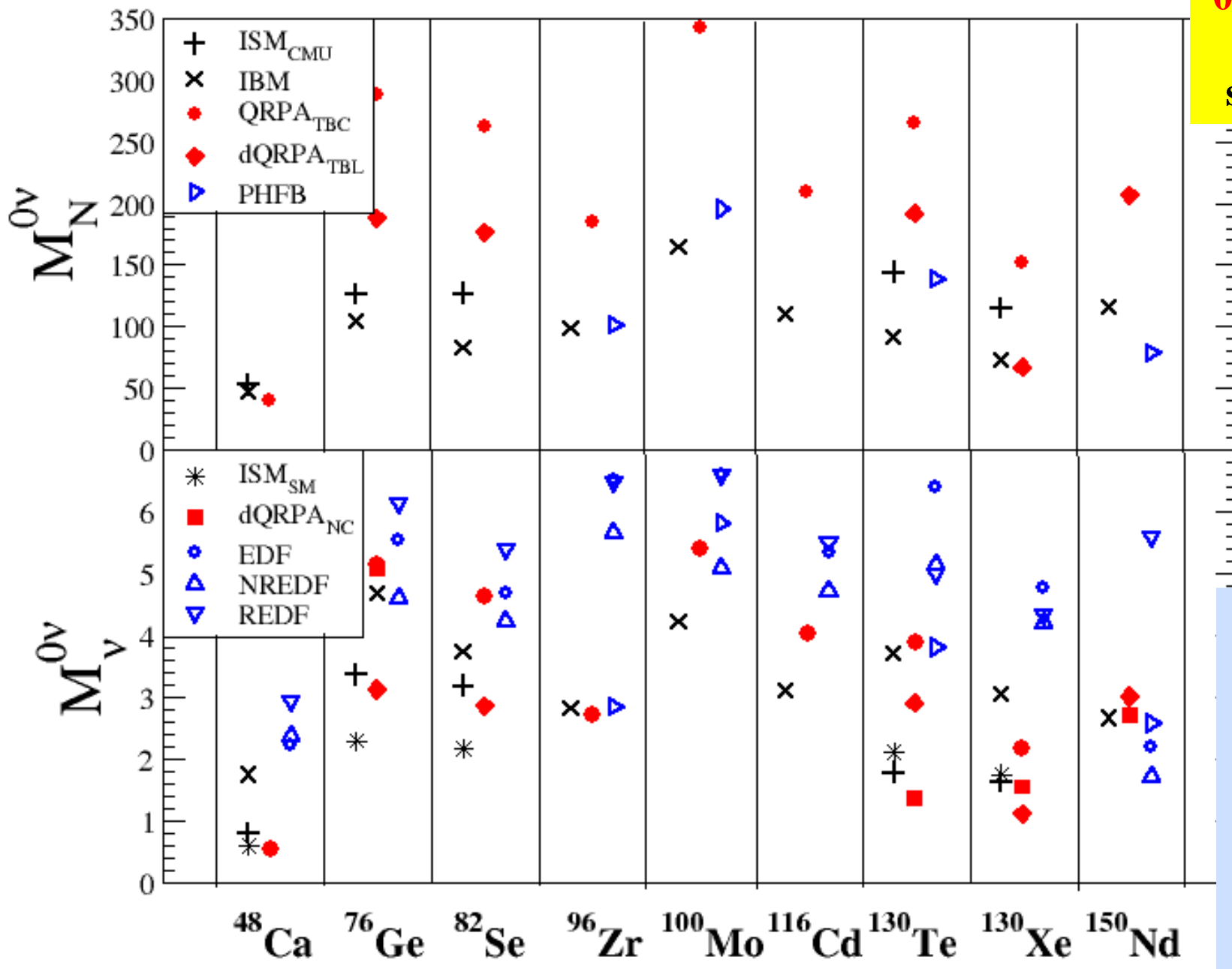
0.043  
rotational state in uranium

**Calculation for the hypothetical  $0\nu\beta\beta$  decay:**  
 $^{10}\text{He} \rightarrow ^{10}\text{Be} + e^- + e^-$   
**masses, spectra**

A. Schwenk,  
P. Navratil,  
J. Engel,  
J. Menendez

**Moore's law: exponential growth in computing power**

**$0\nu\beta\beta$ -decay  
NME  
status 2019**



*All models missing essential physics*

*Impossible to assign rigorous uncertainties*

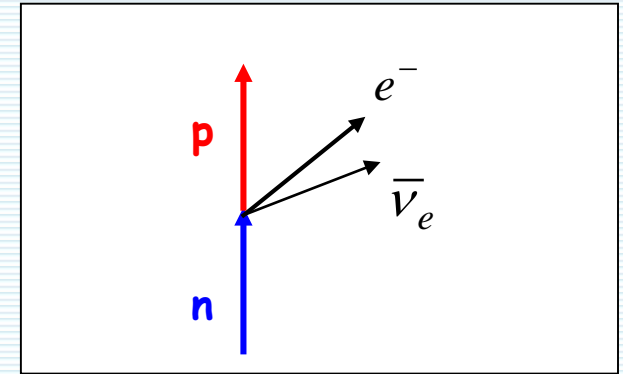
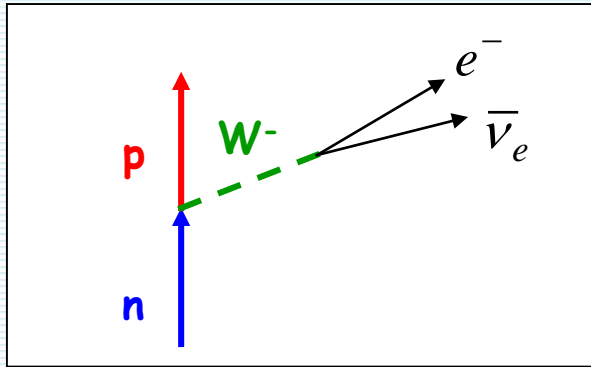
*unquenched  $g_A$*

## *Quenching of $g_A$ ( $q = g_A^{\text{eff}} / g_A^{\text{free}}$ )*

**Should  $g_A$  be quenched in medium?  
Missing wave-function correlations  
Renormalized operator?  
Neglected two-body currents?  
Model-space truncations?**



# Quenching in nuclear matter: $g_A^{\text{eff}} = q g_A^{\text{free}}$



$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}} [\bar{u}\gamma^\alpha(1-\gamma^5)d] [\bar{e}\gamma^\alpha(1-\gamma^5)\nu_e]$$

$$\mathcal{L} = -\frac{G_\beta}{\sqrt{2}} [\bar{p}\gamma^\alpha(g_V - g_A\gamma^5)n] [\bar{e}\gamma^\alpha(1-\gamma^5)\nu_e]$$

## CVC hypothesis

- $g_V = 1$  at the quark level
- $g_V = 1$  at the nucleon level
- $g_V = 1$  inside nuclei

## Quenching of $g_A$

- $g_A = 1$  at the quark level
- $g_A^{\text{free}} = 1.27$  at the nucleon level
- $g_A^{\text{eff}} = ?$  inside nuclei

**ISM:**  $(g_A^{\text{eff}})^4 \simeq 0.66$  ( $^{48}\text{Ca}$ ),  $0.66$  ( $^{76}\text{Ge}$ ),  $0.30$  ( $^{76}\text{Se}$ ),  $0.20$  ( $^{130}\text{Te}$ ) and  $0.11$  ( $^{136}\text{Xe}$ )

**QRPA:**  $(g_A^{\text{eff}})^4 = 0.30$  and  $0.50$  for  $^{100}\text{Mo}$  and  $^{116}\text{Cd}$

**IBM:**  $(g_A^{\text{eff}})^4 \simeq (1.269 A^{-0.18})^4 = 0.063$

Faessler, Fogli, Lisi, Rodin, Rotunno, F. Š,  
J. Phys. G 35, 075104 (2008).

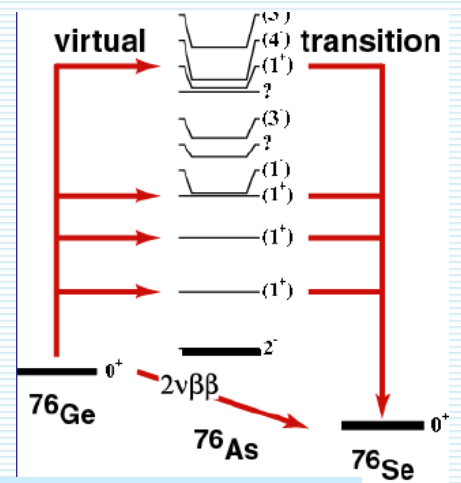
$g_A^4 = (1.269)^4 = 2.6$  **Quenching of  $g_A$**  (from exp.:  $T_{1/2}^{0\nu}$  up 2.5 x larger)

$(g_A^{\text{eff}})^4 = 1.0$

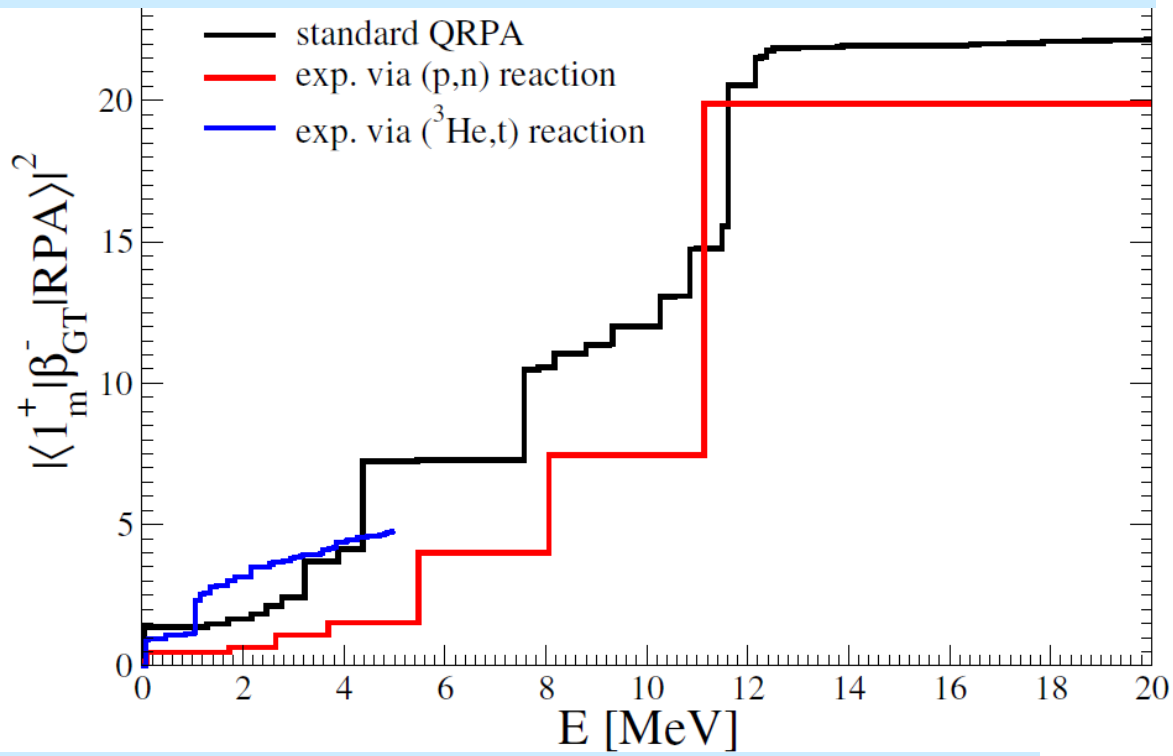
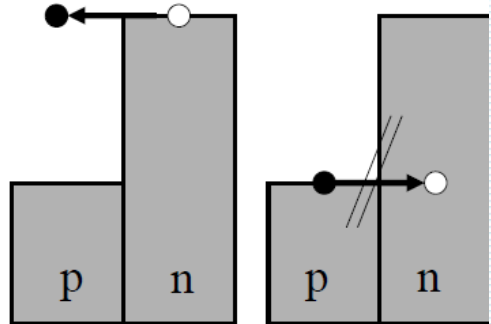
Strength of GT trans. (approx. given by **Ikeda sum rule** =  $3(N-Z)$ ) has to be quenched to reproduce experiment.

Using  $g_A^{\text{eff}} \approx 0.77 \times g_A^{\text{free}}$  agrees with data

${}^{76}_{32}\text{Ge}_{44} \Rightarrow$   
 $S_{\beta^-} - S_{\beta^+} = 3(N-Z) = 36$



**Pauli blocking**



**Cross-section for charge exchange reaction:**

$$\left[ \frac{d\sigma}{d\Omega} \right] = \left[ \frac{\mu}{\pi\hbar} \right]^2 \frac{k_f}{k_i} N_d |V_{\sigma\tau}|^2 |\langle f | \sigma\tau | i \rangle|^2$$

$q = 0!!$

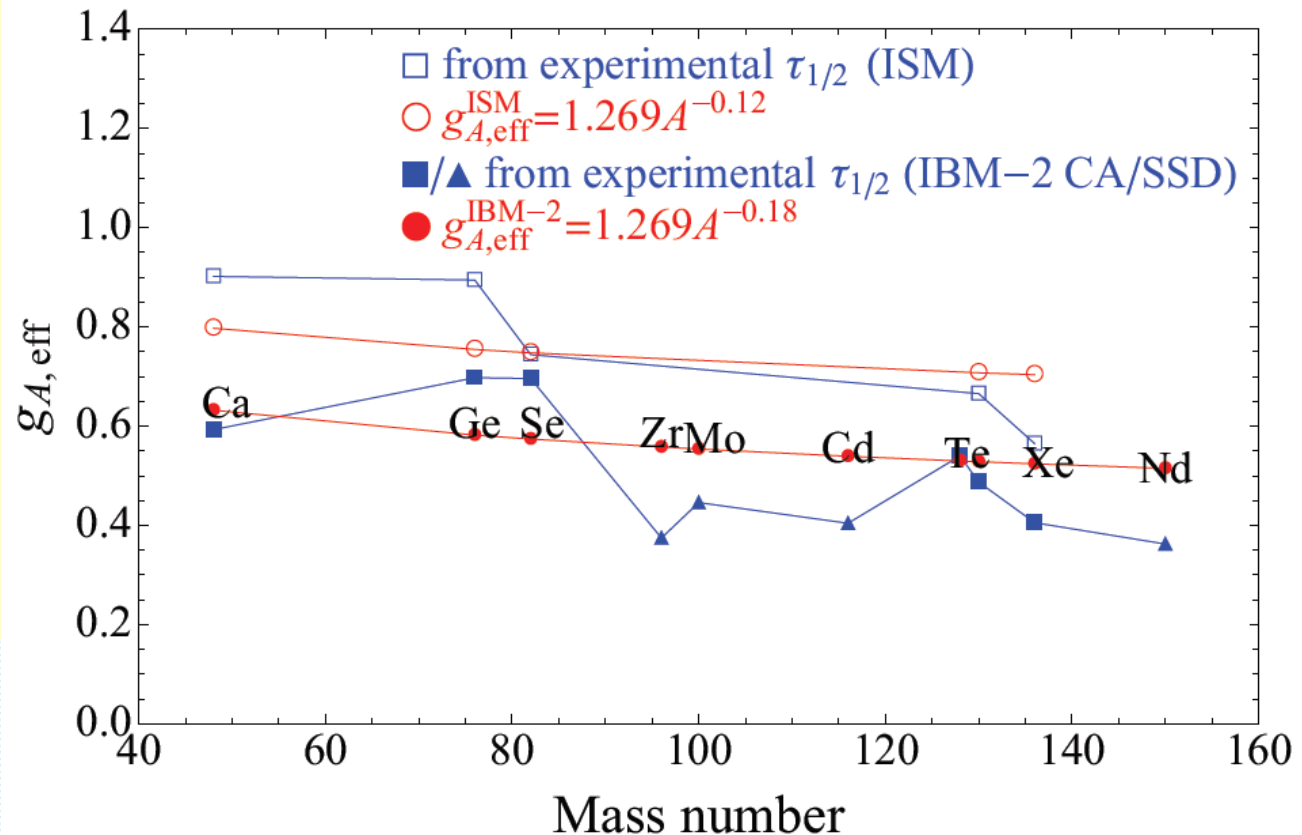
largest at 100 - 200 MeV/A

# Quenching of $g_A$ -IBM ( $T_{1/2}^{0\nu}$ suppressed up to factor 50)

$(g_A^{\text{eff}})^4 \simeq (1.269 A^{-0.18})^4 = 0.063$  (The Interacting Boson Model). This is an incredible result. The quenching of the axial-vector coupling within the IBM-2 is more like 60%.

It has been determined by theoretical prediction for the  $2\nu\beta\beta$ -decay half-lives, which were based on within closure approximation calculated Corresponding NMEs, with the measured half-lives.

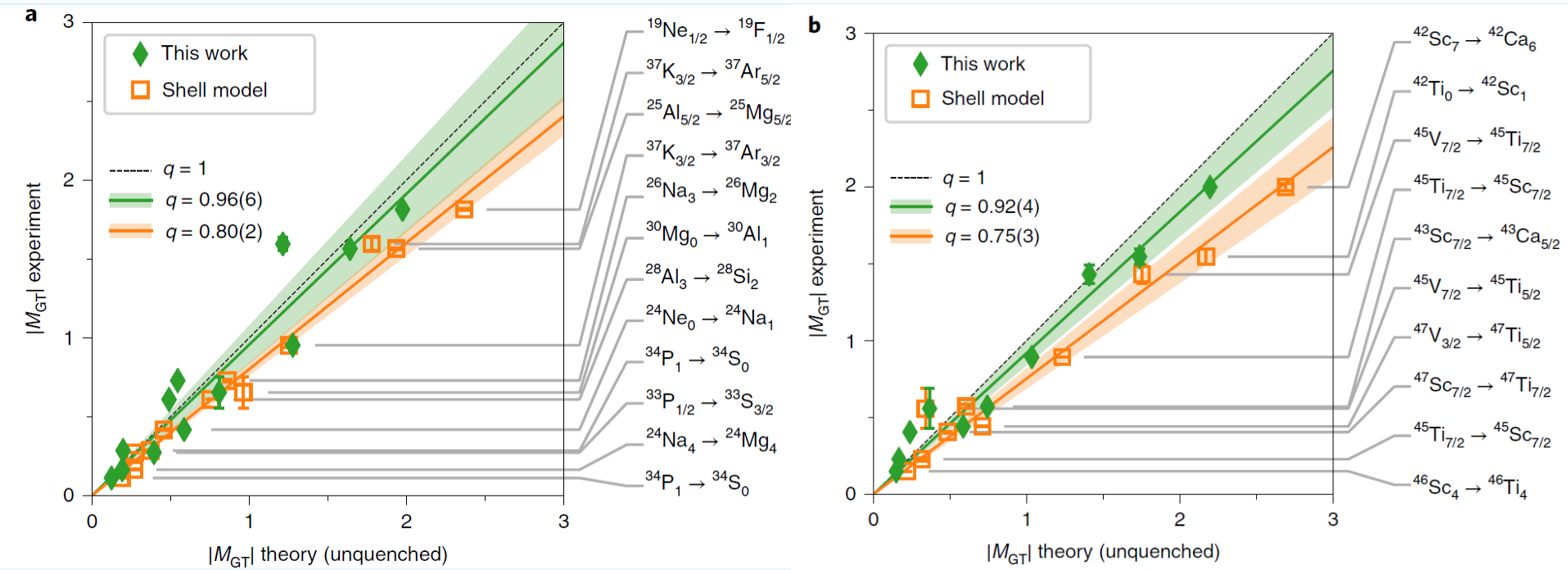
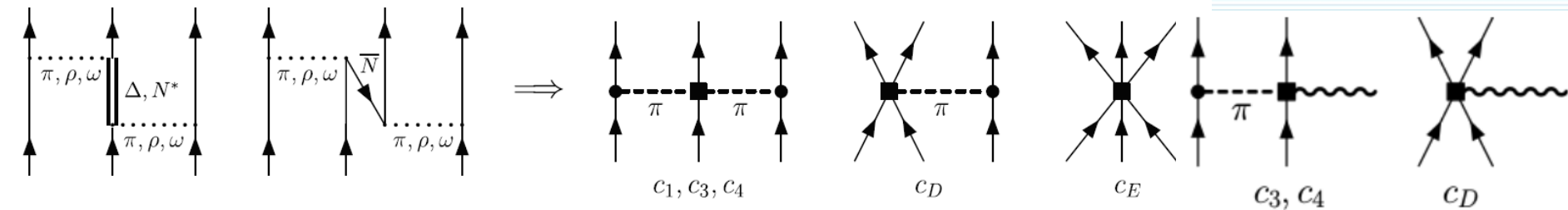
From F. Iachello



9/29/2021

**Ab initio calculations  
(light nuclear systems)  
including meson-  
exchange  
currents do not need  
any “quenching”**

# Discrepancy between experimental and theoretical $\beta$ -decay rates resolved from first principles



# Improved description of the $0\nu\beta\beta$ -decay rate (and novel approach of fixing $g_A^{\text{eff}}$ )

F. Š, R. Dvornický, D. Štefánik, A. Faessler, PRC 97, 034315 (2018).

Let perform  
Taylor expansion

$$M_{GT}^{K,L} = m_e \sum_n M_n \frac{E_n - (E_i + E_f)/2}{[E_n - (E_i + E_f)/2]^2 - \epsilon_{K,L}^2}$$

$\epsilon_{K,L}$

$$\frac{\epsilon_{K,L}}{E_n - (E_i + E_f)/2}$$

$$\epsilon_K = (E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1})/2$$

$$\epsilon_L = (E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1})/2$$

$$\epsilon_{K,L} \in \left(-\frac{Q}{2}, \frac{Q}{2}\right)$$

We get

$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\text{eff}}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{|\xi_{13}^{2\nu}|^2} \left(G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu}\right)$$

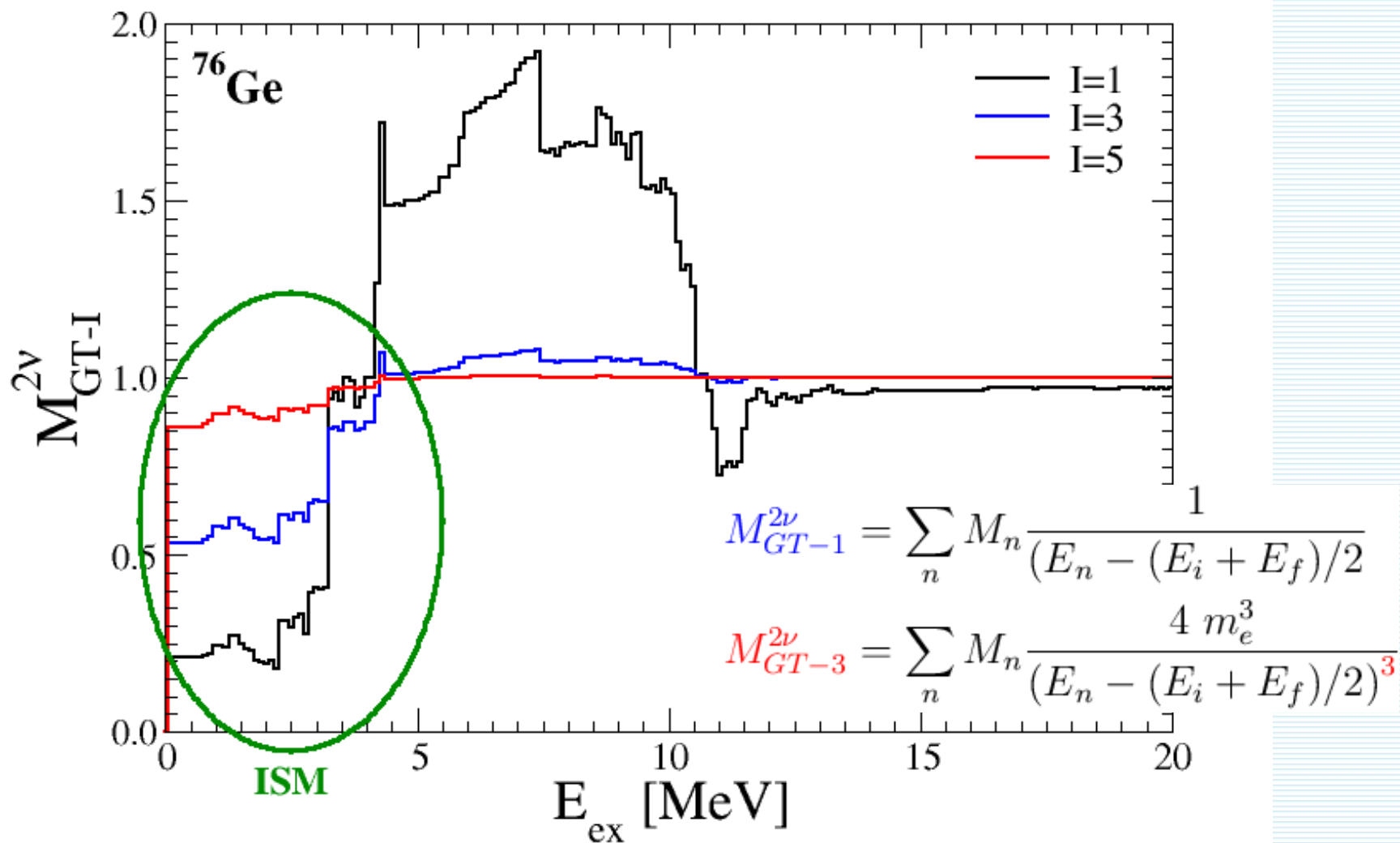
$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{E_n - (E_i + E_f)/2}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

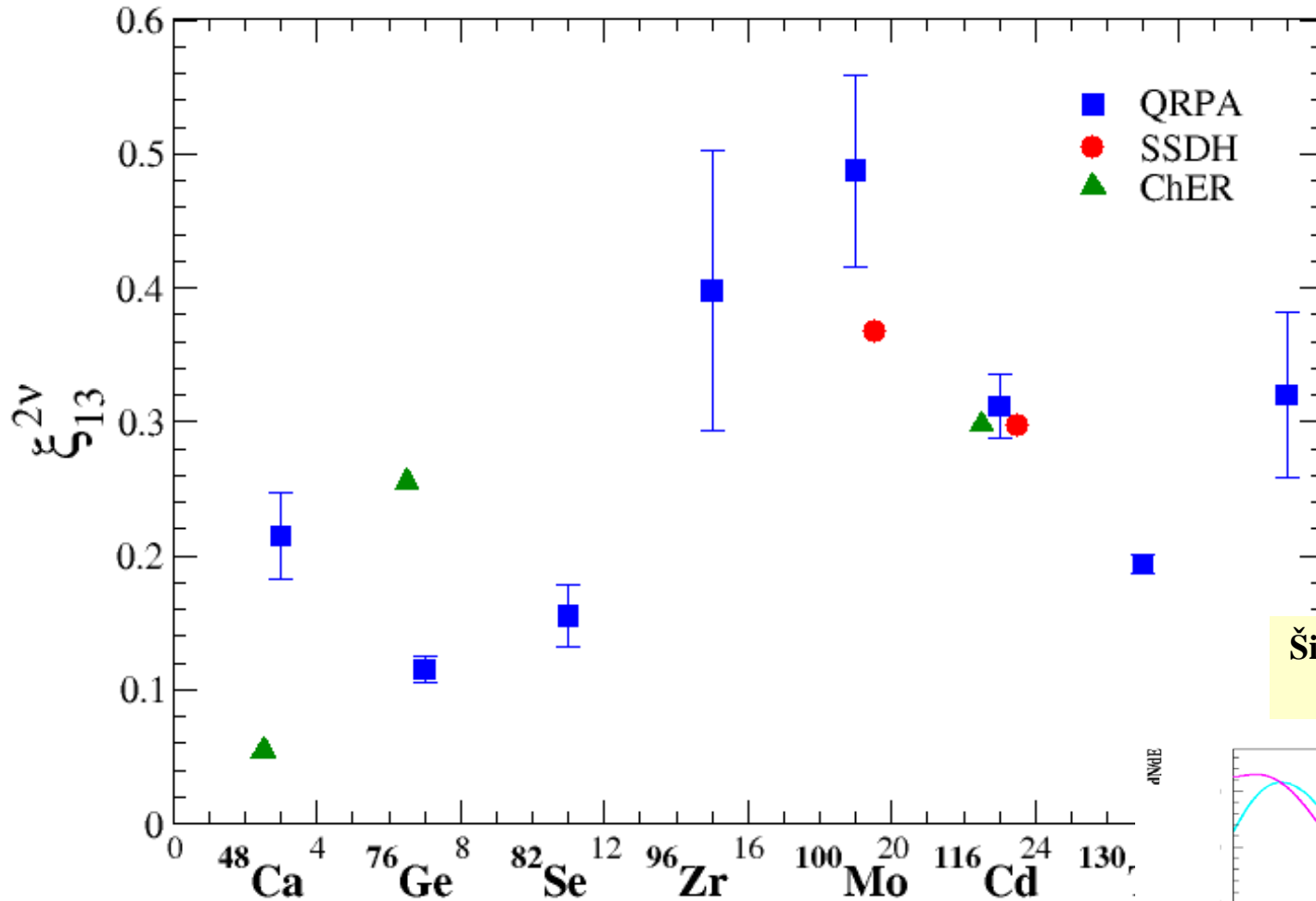
$$\xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

*The  $g_A^{\text{eff}}$  can be determined with measured half-life and ratio of NMEs and calculated NME dominated by transitions through low lying states of the intermediate nucleus (ISM)*

# The running sum of the $2\nu\beta\beta$ -decay NMEs (QRPA)



# $\xi_{13}$ tell us about importance of higher lying states of int. nucl.

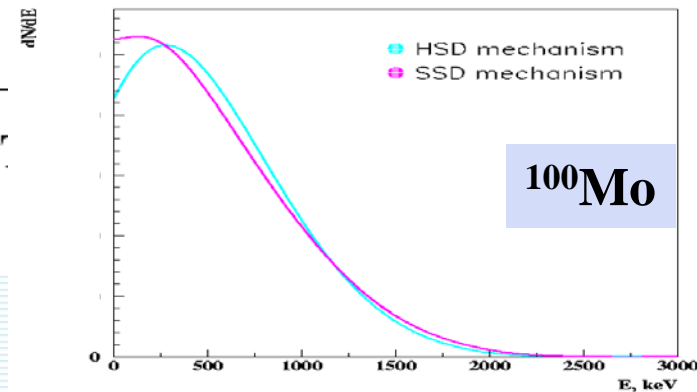


HSD:  $\xi_{13}=0$

$\xi_{13} \approx 1$  (large)  
Possible due to a large cancellation of contributions through lower and higher lying states of  $(A, Z+1)$ .

Šimkovic, Šmotlák, Semenov  
J. Phys. G, 27, 2233, 2001

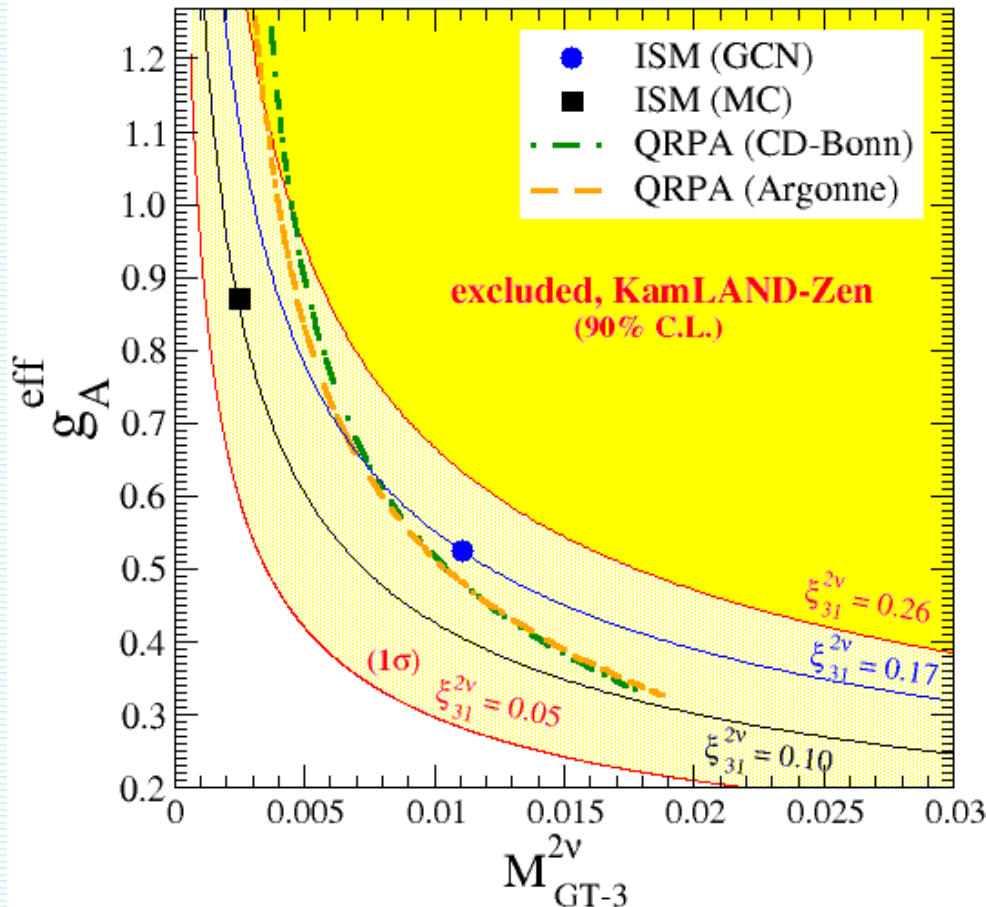
$\xi_{13}$  can be determined phenomenologically from the shape of energy distributions of emitted electrons



The  $g_A^{\text{eff}}$  can be determined with measured half-life and ratio of NMEs  $\xi_{31}^{2\nu}$  and calculated NME dominated by transitions through low lying states of the intermediate nucleus.

$M_{GT-3}$  have to be calculated by nuclear theory - ISM

$$(g_A^{\text{eff}})^2 = \frac{1}{|M_{GT-3}^{2\nu}|} \frac{|\xi_{13}^{2\nu}|}{\sqrt{T_{1/2}^{2\nu-\text{exp}} (G_0^{2\nu} + \xi_{13}^{2\nu} G_2^{2\nu})}}$$



$$M_{GT-1}^{2\nu} = \sum_n M_n \frac{1}{(E_n - (E_i + E_f)/2)}$$

$$M_{GT-3}^{2\nu} = \sum_n M_n \frac{4 m_e^3}{(E_n - (E_i + E_f)/2)^3}$$

$$\xi_{13}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT-1}^{2\nu}}$$

KamLAND-Zen Coll. (+J. Menendez, F.Š.),  
Phys.Rev.Lett. 122, 192501 (2019)



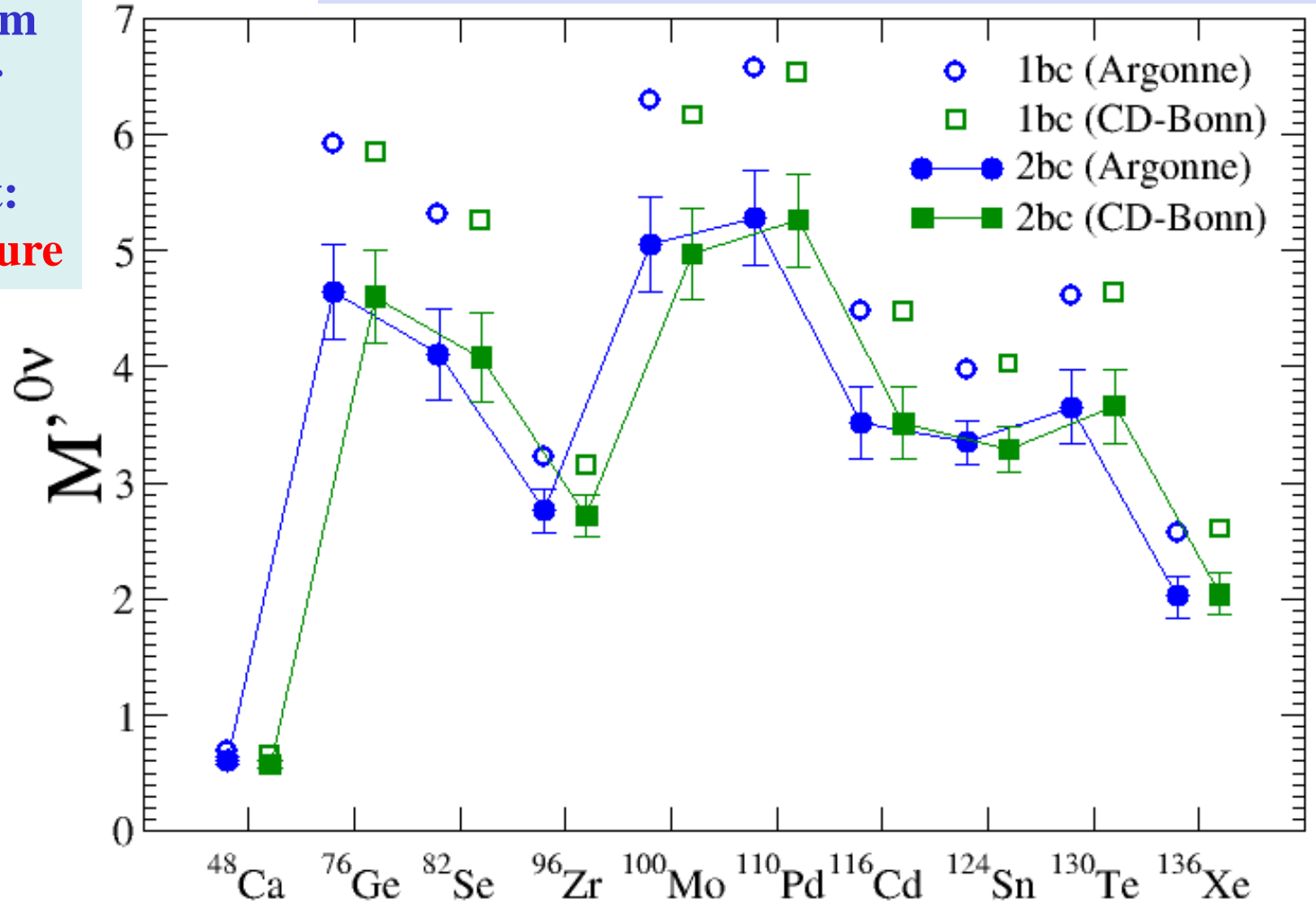
# Quenching of $g_A$ , two-body currents and QRPA

(Suppression of the  $0\nu\beta\beta$ -decay NME of about 20%)

Engel, Vogel, Faessler, F.Š., PRC 89 (2014) 064308  
 See also: Menendez, Gazit, Schwenk, PRL 107 (2011) 062501

Effect of momentum transfer

Relevant: muon capture



But, a strong suppression of  $2\nu\beta\beta$ -decay half-life, ( $g_A^{\text{eff}} = g_A \delta(p=0) = 0.7-1.0$ )

# Muon capture rates evaluated within QRPA

F. Š, R. Dvornický, P. Vogel, PRC 102, 034301 (2020).

TABLE I. The coefficients  $C_F$ ,  $C_{GT}$ , and  $C_T$  [see Eq. (23)] calculated within the present approach [see Eq. (18)] and in the Fujii-Primakoff approximation [see Eq. (19)].

$E_\nu$ (MeV)	$g_A^{\text{eff}}$	present approach			Fujii-Primakoff		
		$C_F$	$C_{GT}$	$C_T$	$C_F$	$C_{GT}$	$C_T$
75	0.80	0.976	0.797	-0.241	1.054	1.165	-0.333
	1.00	0.976	0.821	-0.197	1.054	1.091	-0.296
	1.27	0.976	0.847	-0.158	1.054	1.030	-0.265
85	0.80	0.965	0.805	-0.239	1.052	1.203	-0.359
	1.00	0.965	0.823	-0.197	1.052	1.117	-0.317
	1.27	0.965	0.844	-0.159	1.052	1.048	-0.282
95	0.80	0.955	0.818	-0.234	1.051	1.241	-0.385
	1.00	0.955	0.828	-0.195	1.051	1.145	-0.337
	1.27	0.955	0.844	-0.159	1.051	1.067	-0.298

**New formalism  
(derivation as  
by  $0\nu\beta\beta$ -decay)**

$$\Gamma = m_\mu \frac{(G_\beta m_\mu^2)^2}{2\pi} \times (g_A^{\text{eff}})^2 \left( C_F \frac{B_{\Phi F}}{(g_A^{\text{eff}})^2} + C_{GT} B_{\Phi GT} + C_T B_{\Phi T} \right)$$

$$B_{\Phi K}^k(p_{\nu_k}) = \frac{1}{\hat{J}_i} \sum_{M_i M_k} \int \frac{d\Omega_\nu}{4\pi} \times |\langle J_k M_k | \sum_{j=1}^A \tau_j^- e^{i\mathbf{p}_{\nu_k} \cdot \mathbf{r}_i} O_K \frac{\Phi_g(r_i)}{m_\mu^{3/2}} | J_i M_i \rangle|^2$$

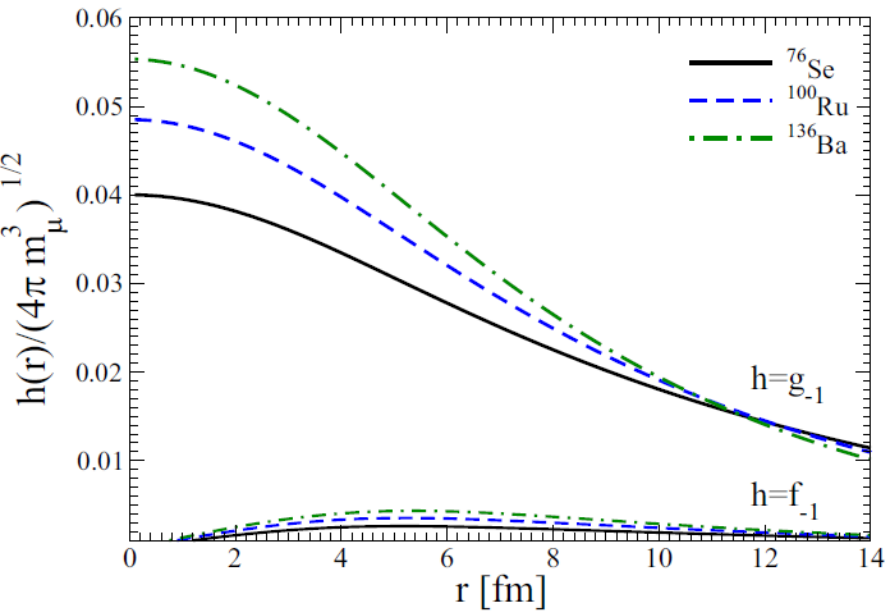
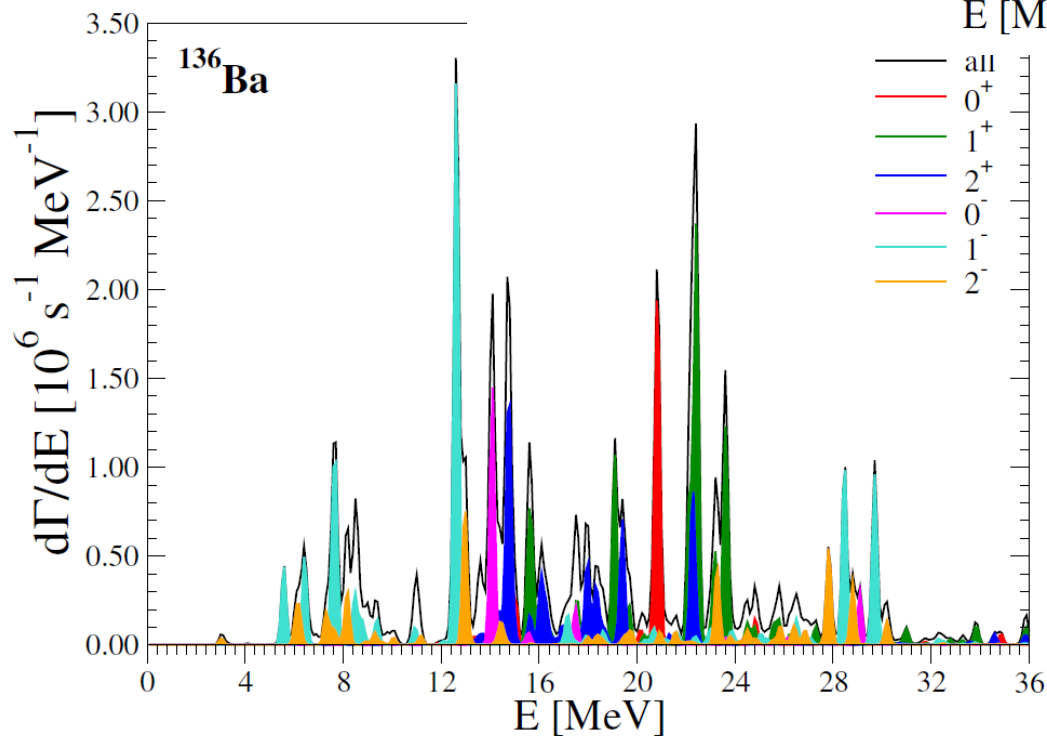
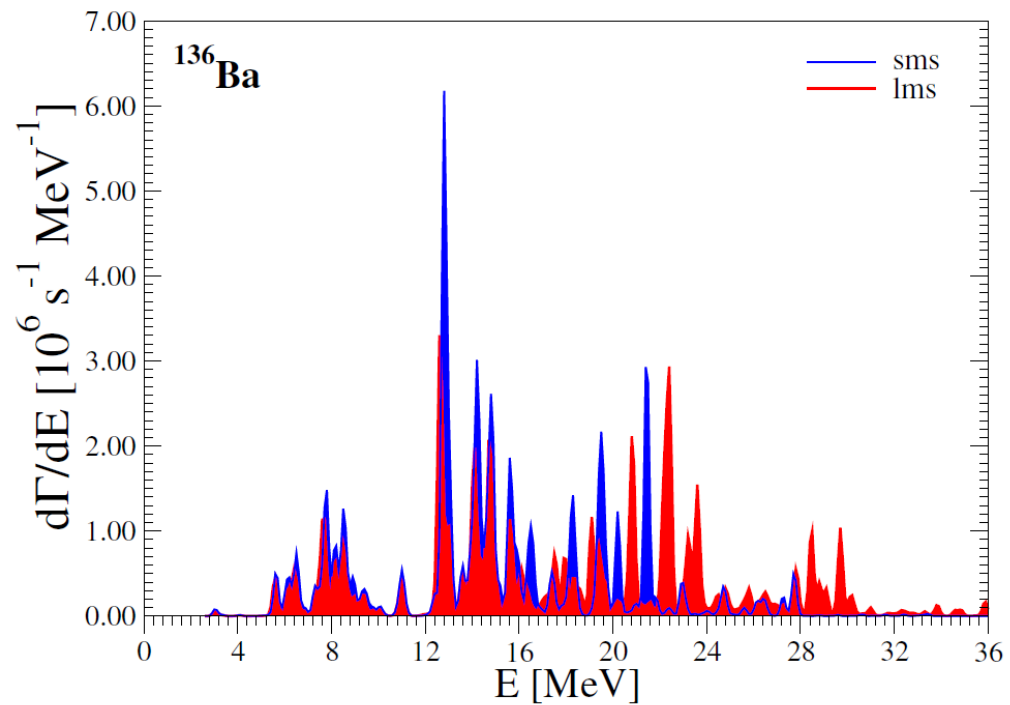
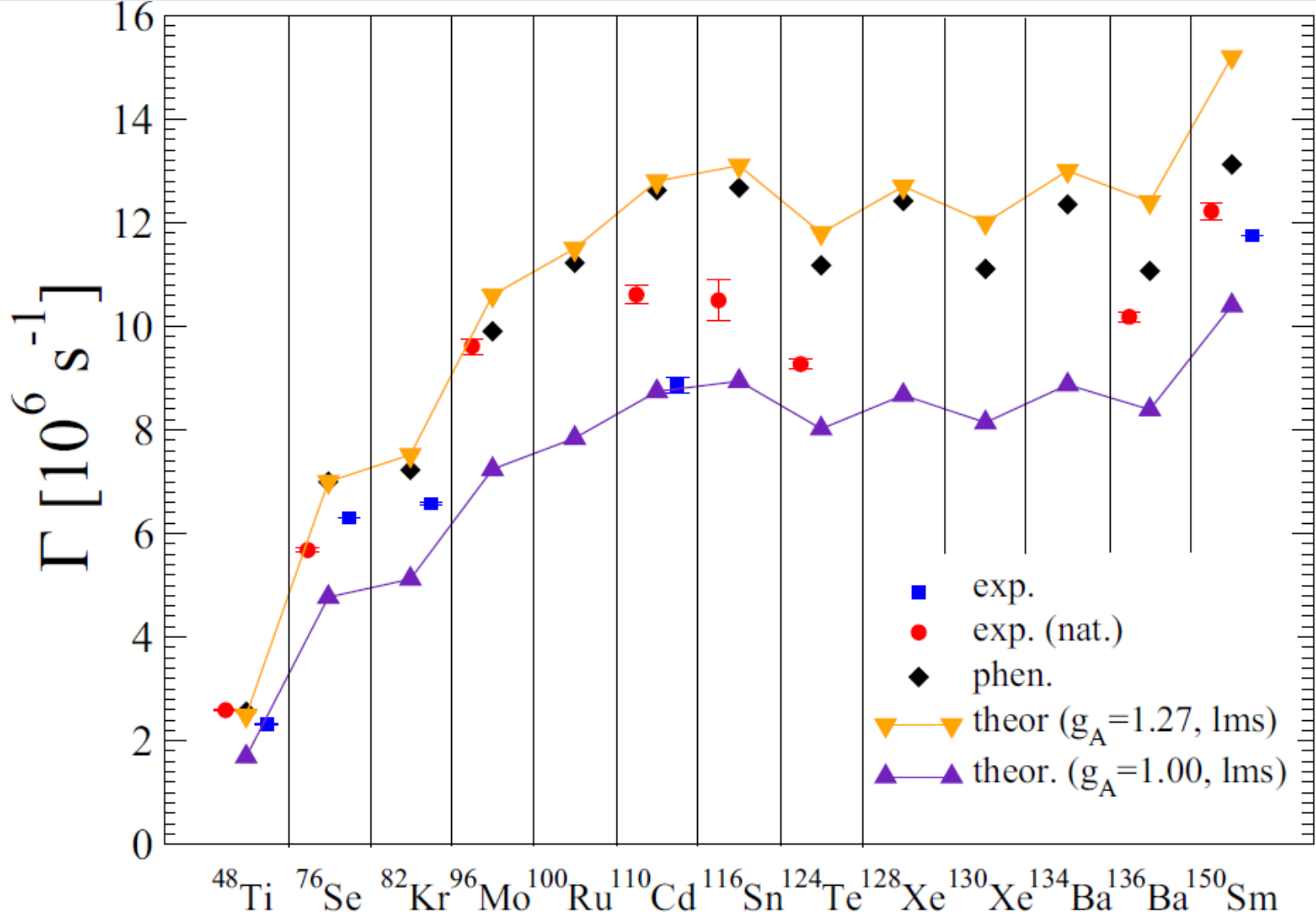


FIG. 1. The radial dependence of the functions  $g_{-1}(r)$  and  $f_{-1}(r)$  for  $^{76}\text{Se}$ ,





**In agreement with soft quenching**

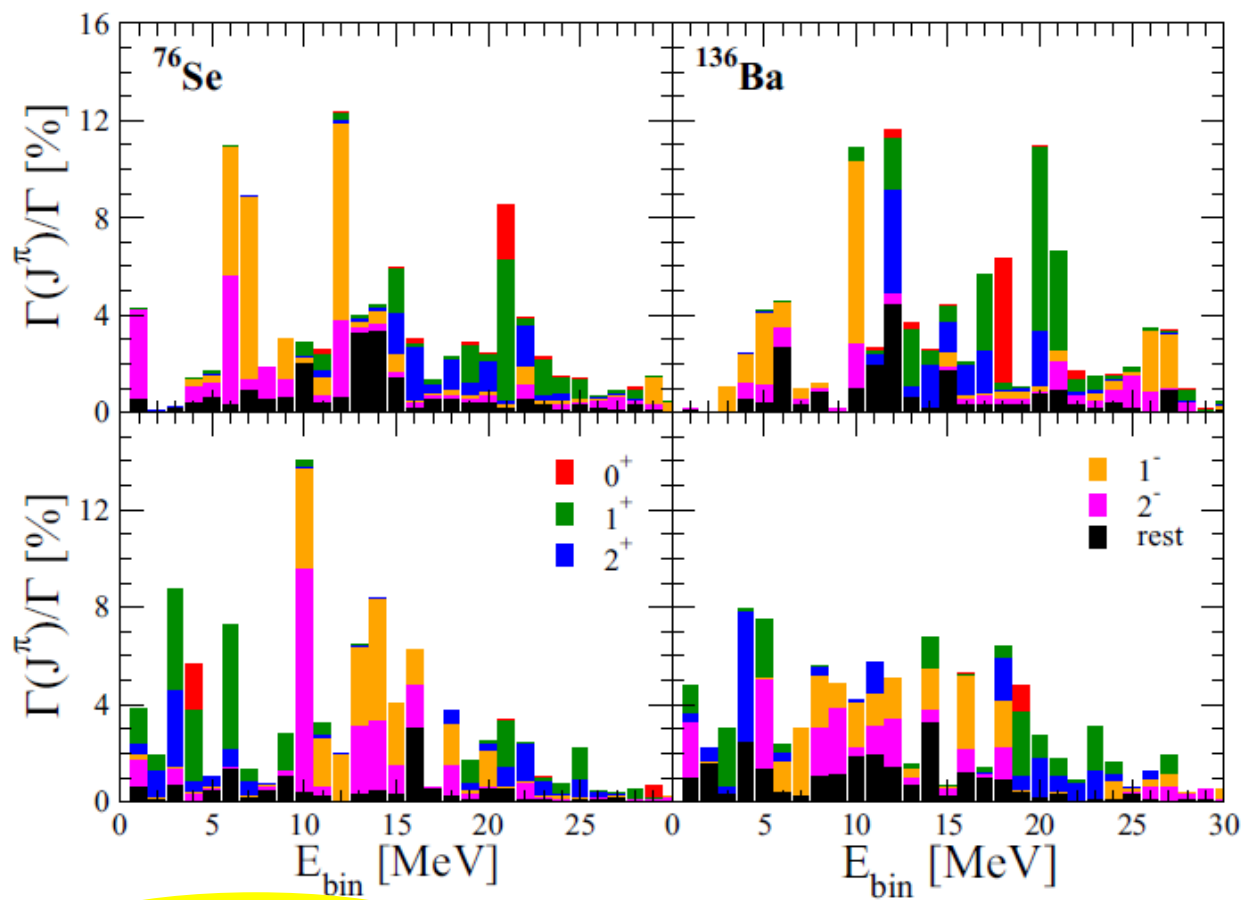
**Zinner, Langanke, Vogel PRC 74, 024326 (2006)**

**Marketin, Paar, Niksic, Vretenar PRC 79, 054323 (2009)**

**F.S., Dvornicky, Vogel**  
**PRC 100, 014619 (2019)**



**Jokiniemi, Suhonen,**  
**PRC 100, 014619 (2019)**  
**(Strong quenching needed)**



nucleus	$\Gamma_{GP}$	$\Gamma_{QRPA}$	$\Gamma_{pres}$	$\Gamma_{FP}$
$^{76}\text{Se}$	7.00	16.4	3.50	4.66
$^{82}\text{Kr}$	7.22	16.5	3.76	5.00
$^{96}\text{Mo}$	9.90	20.4	5.32	7.06
$^{100}\text{Ru}$	11.2	16.7	5.77	7.65
$^{116}\text{Sn}$	12.7	15.7	6.61	8.73
$^{128}\text{Xe}$	12.4	21.2	6.37	8.39
$^{130}\text{Xe}$	11.1	23.6	5.97	7.87
$^{136}\text{Ba}$	11.1	21.1	7.61	10.0

**Contradicting results:**  
**Strong quenching**  
**Jokiniemi, Suhonen,**  
**PRC 100, 014619 (2019)**

**Thank You!**



$\nu$ 's, the  
Standard  
Model  
misfits



$0\nu\beta\beta$

*WE are at  
the beginning  
of the **Beyond  
Standard Model  
Road...***

*people often **overestimate** what will happen in the next **two years**  
and **underestimate** what will happen **in ten** (Bill Gates)*