New Physics scenarios within $2\nu\beta\beta$-decay

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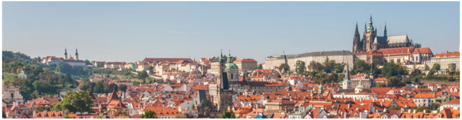
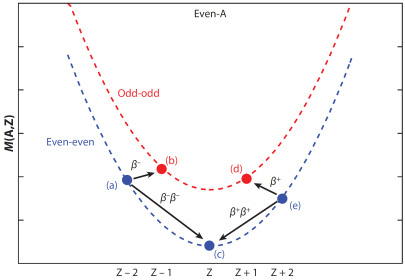
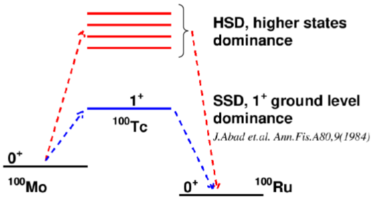


Outlook

$2\nu\beta\beta$-decay is the rarest observed decay in nature, $T^{2\nu}_{1/2} > 10^{18}$ yrs.

$$2\nu\beta\beta : (A,Z) \rightarrow (A, Z+2) + 2e^- + 2\overline{\nu}_e$$





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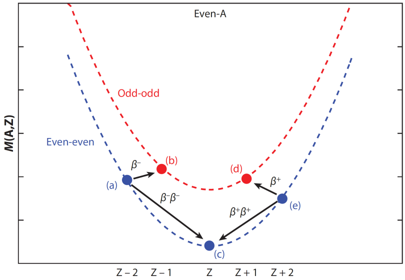

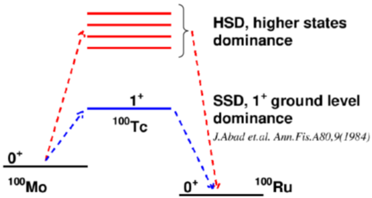

There is an increasing interest in the process due to the possible observation of $0\nu\beta\beta$ decay.

$$0\nu\beta\beta : (A, Z) \to (A, Z+2) + 2e^-$$

ν_e Majorana particle

GERDA (^{76}Ge), CUPID-0 (^{82}Se), NEMO-3(^{100}Mo), CUORE (^{130}Te), EXO (^{136}Xe), KamlandZEN (^{136}Xe)

What is measured?

The inverse half-live is defined as

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$$\frac{d\Gamma^{2\nu}}{d(\cos\theta)} = \frac{1}{2}\Gamma^{2\nu}(1 + K^{2\nu}\cos\theta), \quad \frac{d\Gamma^{2\nu}}{d(\cos\theta)dE_{e_1}dE_{e_2}} = \frac{\Gamma^{2\nu}}{2}\frac{d\Gamma^{2\nu}_{norm}}{dE_{e_1}dE_{e_2}}(1 + \kappa^{2\nu}(E_{e_1}, E_{e_2})\cos\theta)$$

where we define the angular correlation coefficient

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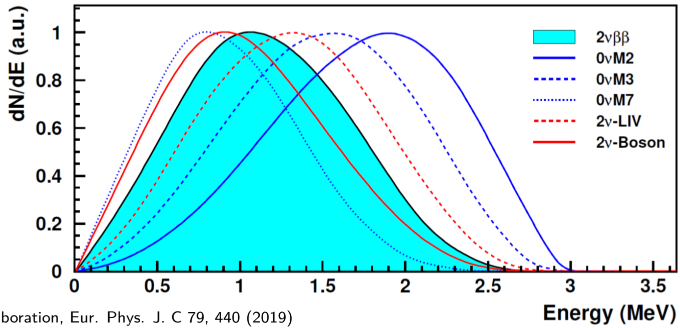
where we define the angular correlation coefficient

$$K^{2\nu} = \frac{\Lambda^{2\nu}}{\Gamma^{2\nu}}.$$

$$\begin{Bmatrix} \Gamma^{2\nu} \\ \Lambda^{2\nu} \end{Bmatrix} = \frac{m_e(G_\beta m_e^2)^4}{8\pi^7}(g_A^{\text{eff}})^4\frac{1}{m_e^{11}}\int_{m_e}^{E_i-E_f-m_e} p_{e_1}E_{e_1}\int_{m_e}^{E_i-E_f-E_{e_1}} p_{e_2}E_{e_2}$$
$$\times\int_0^{E_i-E_f-E_{e_1}-E_{e_2}} E_{\nu_1}^2E_{\nu_2}^2\begin{Bmatrix} \mathscr{A}^{2\nu}F_{ss}(E_{e_1})F_{ss}(E_{e_2}) \\ \mathscr{B}^{2\nu}E_{ss}(E_{e_1})E_{ss}(E_{e_2}) \end{Bmatrix} dE_{\nu_1}dE_{e_2}dE_{e_1}$$

Outlook

With the increasing statistics for $2\nu\beta\beta$-decay, the idea of searching for New Physics in $2\nu\beta\beta$-decay becomes reliable.

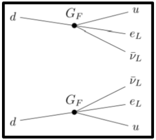


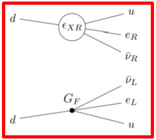
NEMO-3 Collaboration, Eur. Phys. J. C 79, 440 (2019)

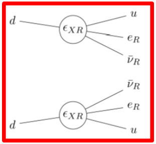
spectral index n

$$\frac{d\Gamma}{dE_{e_1} dE_{e_2}} \propto (Q - E_{e_1} - E_{e_2})^n p_{e_1} E_{e_1} F_{ss}(E_{e_1}) p_{e_2} E_{e_2} F_{ss}(E_{e_2})$$

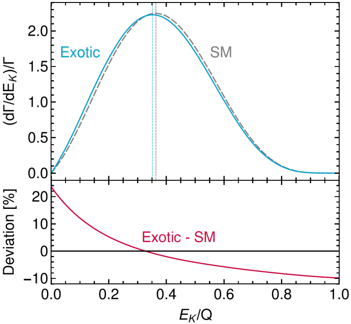
$$\mathscr{L} = \frac{G_F \cos\theta_C}{\sqrt{2}} \left[(1 + \delta_{\mathrm{SM}} + \epsilon_{LL})\, j_L^{\mu} J_{L\mu} + \epsilon_{RL} j_L^{\mu} J_{R\mu} + \epsilon_{LR} j_R^{\mu} J_{L\mu} + \epsilon_{RR} j_R^{\mu} J_{R\mu} \right] + \mathrm{H.c.}$$

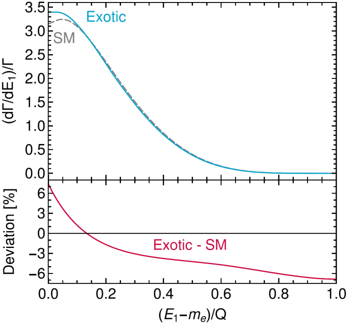






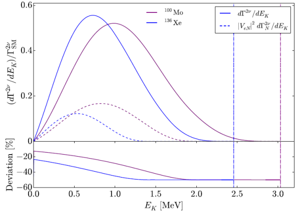
F.F. Deppisch, L. Graf, F. Šimkovic, Phys. Rev. Lett. 125, 171801 (2020)

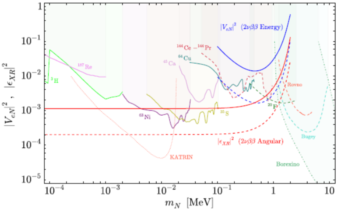




$$\mathscr{L} = \frac{G_F \cos\theta_C}{\sqrt{2}} \left[(1+\delta_{\rm SM}) j_L^{\mu} J_{L\mu} + V_{eN} j_L^{N\mu} J_{L\mu} + \epsilon_{LR} j_R^{N\mu} J_{L\mu} + \epsilon_{RR} j_R^{N\mu} J_{R\mu} \right] + \text{H.c.}$$

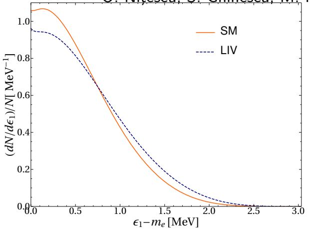
P.D. Bolton, F.F. Deppisch, L. Gráf, F. Šimkovic, Phys. Rev. D 103, 055019 (2021)

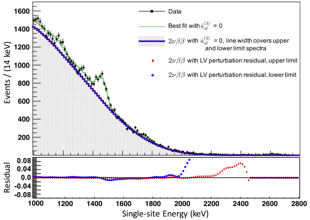




$$\mathscr{H} = (g^{\mu\nu} + \chi^{\mu\nu}) \left[\bar{\psi}_p(x)\gamma_\mu (C_V + C_A\gamma_5)\psi_n(x)\right] \left[\bar{\psi}_e(x)\gamma_\nu (1-\gamma_5)\psi_\nu(x)\right] + \text{H.c.}$$

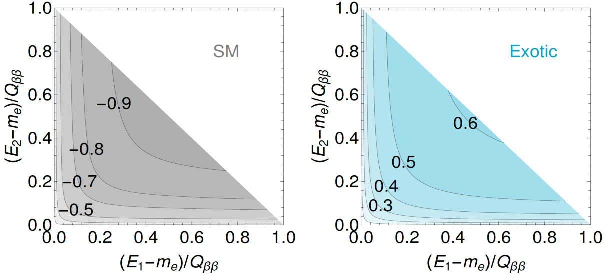
O. Nițescu, S. Ghinescu, M. Mirea, S. Stoica, Phys. Rev. D 103, L031701 (2021)





differential $$\frac{d\Gamma^{2\nu}}{d(\cos\theta)dE_{e_1}dE_{e_2}} = \frac{\Gamma^{2\nu}}{2}\frac{d\Gamma^{2\nu}_{norm}}{dE_{e_1}dE_{e_2}}(1+\kappa^{2\nu}(E_{e_1},E_{e_2})\cos\theta)$$

^{100}Mo



global $$\frac{d\Gamma^{2\nu}}{d(\cos\theta)} = \frac{1}{2}\Gamma^{2\nu}(1+K^{2\nu}\cos\theta) \qquad K^{2\nu} = K^{2\nu}_{\mathrm{SM}} + \alpha\epsilon^2_{XR}$$

$$\epsilon_{XR} \lesssim 2.7\times10^{-2} \text{ at NEMO-3} \qquad \epsilon_{XR} \lesssim 4.8\times10^{-3} \text{ at SuperNEMO}$$

F.F. Deppisch, L. Graf, F. Šimkovic, Phys. Rev. Lett. 125, 171801 (2020)

$$|\mathring{a}^{(3)}_{of}| \lesssim 1.04\times10^{-3} \text{ MeV at NEMO-3} \qquad |\mathring{a}^{(3)}_{of}| \lesssim 3.3\times10^{-5} \text{ MeV at SuperNEMO}$$

O. Nițescu, S. Ghinescu, M. Mirea, S. Stoica, Phys. Rev. D 103, L031701 (2021)

Outlook

$\mathscr{A}^{2\nu}$ and $\mathscr{B}^{2\nu}$ are products of Fermi and Gamow-Teller nuclear matrix elements

$$\mathscr{A}^{2\nu} = \frac{1}{4}\left|\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2\left(M_F^K + M_F^L\right) - \left(M_{GT}^K + M_{GT}^L\right)\right|^2 + \frac{3}{4}\left|\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2\left(M_F^K - M_F^L\right) + \frac{1}{3}\left(M_{GT}^K - M_{GT}^L\right)\right|^2$$

$$\mathscr{B}^{2\nu} = \frac{1}{4}\left|\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2\left(M_F^K + M_F^L\right) - \left(M_{GT}^K + M_{GT}^L\right)\right|^2 - \frac{1}{4}\left|\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2\left(M_F^K - M_F^L\right) + \frac{1}{3}\left(M_{GT}^K - M_{GT}^L\right)\right|^2$$

$$M_{F,GT}^{K,L} = m_e \sum_n M_{F,GT}(n) \frac{E_n - (E_i - E_f)/2}{[E_n - (E_i - E_f)/2]^2 - \epsilon_{K,L}^2}$$

$$M_F(n) = \left\langle 0_f^+ \right| \sum_m \tau_m^+ \left| 0_n^+ \right\rangle \left\langle 0_n^+ \right| \sum_{m'} \tau_{m'}^+ \left| 0_i^+ \right\rangle$$

$$M_{GT}(n) = \left\langle 0_f^+ \right| \sum_m \tau_m^+ \boldsymbol{\sigma}_m \left| 1_n^+ \right\rangle \left\langle 1_n^+ \right| \sum_{m'} \tau_{m'}^+ \boldsymbol{\sigma}_{m'} \left| 0_i^+ \right\rangle$$

$$\epsilon_K = \left(E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1}\right)/2$$

$$\epsilon_L = \left(E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1}\right)/2$$

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$$\mathscr{B}^{2\nu} = \frac{1}{4}\left|\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2\left(M_F^K + M_F^L\right) - \left(M_{GT}^K + M_{GT}^L\right)\right|^2 - \frac{1}{4}\left|\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2\left(M_F^K - M_F^L\right) + \frac{1}{3}\left(M_{GT}^K - M_{GT}^L\right)\right|^2$$

$$M_{F,GT}^{K,L} = m_e \sum_n M_{F,GT}(n) \frac{E_n - (E_i - E_f)/2}{\left[E_n - (E_i - E_f)/2\right]^2 - \epsilon_{K,L}^2}$$

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$$\epsilon_K = \left(E_{e_2} + E_{\nu_2} - E_{e_1} - E_{\nu_1}\right)/2$$

$$\epsilon_L = \left(E_{e_1} + E_{\nu_2} - E_{e_2} - E_{\nu_1}\right)/2$$

Understanding of the $2\nu\beta\beta$-decay NMEs is of crucial importance for correct evaluation of the $0\nu\beta\beta$-decay NMEs

There is no reliable calculation of the $2\nu\beta\beta$-decay NMEs yet

HSD

$$\mathscr{A}^{2\nu} = \mathscr{B}^{2\nu} = \left| \left(\frac{g_V}{g_A^{\text{eff}}} \right)^2 M_F - M_{GT} \right|,$$

where Fermi and Gamow-Teller matrix elements are given by

$$M_F = m_e \sum_n \frac{\left\langle 0_f^+ \middle\| \sum_m \tau_m^+ \middle\| 0_n^+ \right\rangle \left\langle 0_n^+ \middle\| \sum_m \tau_m^+ \middle\| 0_i^+ \right\rangle}{E_n - (E_i + E_f)/2}$$

$$M_{GT} = m_e \sum_n \frac{\left\langle 0_f^+ \middle\| \sum_m \tau_m^+ \sigma_m \middle\| 1_n^+ \right\rangle \left\langle 1_n^+ \middle\| \sum_m \tau_m^+ \sigma_m \middle\| 0_i^+ \right\rangle}{E_n - (E_i + E_f)/2}.$$

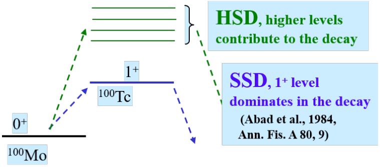
SSD

$$M_{GT}^{K,L} = m_e \frac{M_{GT}(n=1)\left(E_{n=1}(1_1^+) - (E_i - E_f)/2\right)}{[E_{n=1}(1^+) - (E_i - E_f)/2]^2 - \varepsilon_{K,L}^2},$$

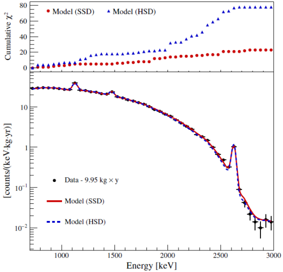
and $M_F^{K,L} = 0$.

Look at energy distribution to see SSD/HSD effects

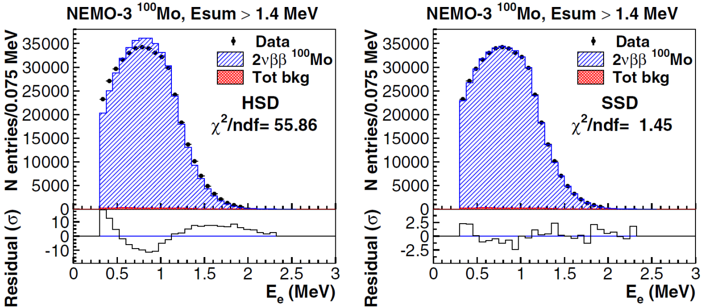
F. Šimkovic, P. Domin, S. Semenov J. Phys. G, 27, 2233, (2001)



CUPID-0 Collaboration, Phys. Rev. Lett. 123, 262501 (2019)



NEMO-3 Collaboration, Eur. Phys. J. C 79, 440 (2019)



Outlook

We get a more accurate expression for $\Lambda^{2\nu}$ and $\Gamma^{2\nu}$ by performing in the matrix elements the following Taylor expansion

$$
\begin{aligned}
M^{K,L}_{F,GT} &= m_e \sum_n M_{F,GT}(n) \frac{E_n - (E_i - E_f)/2}{[E_n - (E_i - E_f)/2]^2 - \epsilon^2_{K,L}} \\
&= m_e \sum_n M_{F,GT}(n) \frac{1}{E_n - (E_i - E_f)/2} \\
&\times \left\{ 1 + \left(\frac{\epsilon_{K,L}}{E_n - (E_i - E_f)/2} \right)^2 + \left(\frac{\epsilon_{K,L}}{E_n - (E_i - E_f)/2} \right)^4 + \dots \right\}
\end{aligned}
$$

F.Šimkovic, R. Dvornický, D. Štefánik, and A. Faessler, Phys. Rev. C 97, 034315 (2018)

which leads to

$$
\begin{aligned}
\Gamma^{2\nu} &\simeq \Gamma^{2\nu}_0 + \Gamma^{2\nu}_2 + \Gamma^{2\nu}_{22} + \Gamma^{2\nu}_4, \\
\Lambda^{2\nu} &\simeq \Lambda^{2\nu}_0 + \Lambda^{2\nu}_2 + \Lambda^{2\nu}_{22} + \Lambda^{2\nu}_4.
\end{aligned}
$$

The factorization of the decay rate is possible

$$\Gamma^{2\nu}(\xi_{31}^{2\nu},\xi_{51}^{2\nu}) = \ln(2)\;\; G_0^{2\nu}\;\left(g_A^{\text{eff}}\right)^4 \left|\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2 M_F - M_{GT}\right|^2$$
$$\times\left\{1+\xi_{31}\frac{G_2^{2\nu}}{G_0^{2\nu}}+\frac{1}{3}\xi_{31}^2\frac{G_{22}^{2\nu}}{G_0^{2\nu}}+\left(\frac{1}{3}\xi_{31}^2+\xi_{51}\right)\frac{G_4^{2\nu}}{G_0^{2\nu}}\right\},$$

with

$$\xi_{31}^{2\nu} = \frac{M_{GT-3}^{2\nu}}{M_{GT}-\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2 M_F},$$

$$\xi_{51}^{2\nu} = \frac{M_{GT-5}^{2\nu}}{M_{GT}-\left(\frac{g_V}{g_A^{\text{eff}}}\right)^2 M_F}.$$

$$M_{GT-1}^{2\nu} \equiv M_{GT}^{2\nu} = \sum_n M_{GT}(n)\frac{m_e}{E_n(1^+)-(E_i+E_f)/2},$$

$$M_{F-1}^{2\nu} \equiv M_{F}^{2\nu} = \sum_n M_{F}(n)\frac{m_e}{E_n(0^+)-(E_i+E_f)/2},$$

$$M_{GT-3}^{2\nu} = \sum_n M_{GT}(n)\frac{4\;\;m_e^3}{(E_n(1^+)-(E_i+E_f)/2)^{3}},$$

$$M_{GT-5}^{2\nu} = \sum_n M_{GT}(n)\frac{16\;\;m_e^5}{(E_n(1^+)-(E_i+E_f)/2)^{5}},$$

Quantities of experimental interest are functions of $\xi_{31}^{2\nu}$ and $\xi_{51}^{2\nu}$

$$\frac{d\Gamma^{2\nu}}{d(\cos\theta)} = \frac{1}{2}\Gamma^{2\nu}(\xi_{31}^{2\nu}, \xi_{51}^{2\nu})\left(1 + K^{2\nu}(\xi_{31}^{2\nu}, \xi_{51}^{2\nu})\cos\theta\right)$$

$$K^{2\nu}(\xi_{31}^{2\nu}, \xi_{51}^{2\nu}) = -\frac{H_0^{2\nu} + \xi_{31}H_2^{2\nu} + \frac{5}{9}\xi_{31}^2 H_{22}^{2\nu} + \left(\frac{2}{9}\xi_{31}^2 + \xi_{51}\right)H_4^{2\nu}}{G_0^{2\nu} + \xi_{31}G_2^{2\nu} + \frac{1}{3}\xi_{31}^2 G_{22}^{2\nu} + \left(\frac{1}{3}\xi_{31}^2 + \xi_{51}\right)G_4^{2\nu}}.$$

$$\frac{d\Gamma^{2\nu}}{dE_{e_1}\,dE_{e_2}\cos\theta} = \frac{1}{2}\frac{d\Gamma^{2\nu}}{dE_{e_1}\,dE_{e_2}}\left(1 + \kappa^{2\nu}(E_{e_1}, E_{e_2}, \xi_{31}^{2\nu}, \xi_{51}^{2\nu})\cos\theta\right).$$

O. Nițescu, S. Stoica, R. Dvornický and F.Šimkovic, Universe 2021, 7(5), 147

We fix $\xi_{51}^{2\nu}$ by the SSD relation

$$\xi_{51}^{2\nu} = \left(\frac{2m_e}{E_1(1^+) - (E_i + E_f)/2}\right)^4.$$

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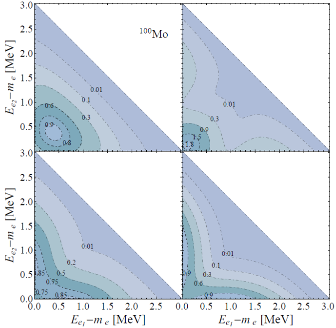
$$\left\{\begin{matrix} G_N^{2\nu} \\ H_N^{2\nu} \end{matrix}\right\} = \frac{m_e (G_\beta m_e^2)^4}{8\pi^7 \ln 2} \frac{1}{m_e^{11}} \int_{m_e}^{E_i - E_f - m_e} p_{e_1} E_{e_1} \int_{m_e}^{E_i - E_f - E_{e_1}} p_{e_2} E_{e_2}$$

$$\times \int_0^{E_i - E_f - E_{e_1} - E_{e_2}} E_{\nu_1}^2 E_{\nu_2}^2 \mathscr{A}_N^{2\nu} \left\{\begin{matrix} F_{ss}(E_{e_1}) F_{ss}(E_{e_2}) \\ E_{ss}(E_{e_1}) E_{ss}(E_{e_2}) \end{matrix}\right\} dE_{\nu_1} dE_{e_2} dE_{e_1}$$

with $N = \{0, 2, 22, 4\}$. In the phase-space expressions,



$$\mathscr{A}_0^{2\nu} = 1, \quad \mathscr{A}_2^{2\nu} = \frac{\varepsilon_K^2 + \varepsilon_L^2}{(2m_e)^2},$$

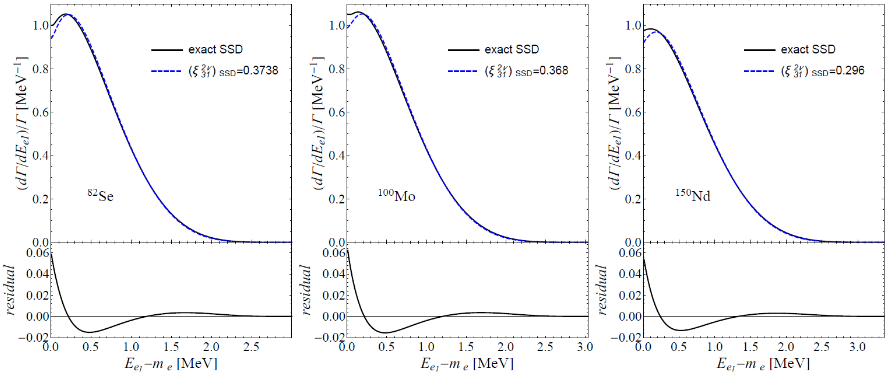
$$\mathscr{A}_{22}^{2\nu} = \frac{\varepsilon_K^2 \varepsilon_L^2}{(2m_e)^4}, \quad \mathscr{A}_4^{2\nu} = \frac{\varepsilon_K^4 + \varepsilon_L^4}{(2m_e)^4},$$

and

$$F_{ss}(E) = g_{-1}^2(E) + f_1^2(E),$$

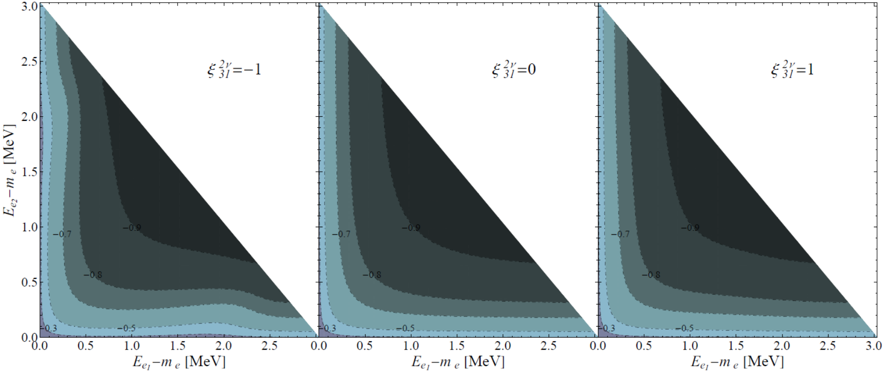
$$E_{ss}(E) = 2 g_{-1}(E) f_1(E).$$

Testing the Taylor expansion:

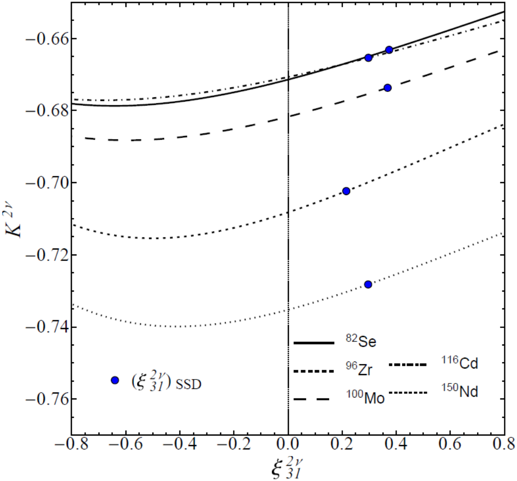


We can conclude that the improved formalism fixed in the SSD relation, is in an excelent agreement with the exact SSD formalism.

$$\frac{d\Gamma^{2\nu}}{dE_{e_1}\, dE_{e_2}\, \cos\theta} = \frac{1}{2}\,\frac{d\Gamma^{2\nu}}{dE_{e_1}\, dE_{e_2}} \left(1 + \kappa^{2\nu}(E_{e_1}, E_{e_2}, \xi_{31}^{2\nu}, \xi_{51}^{2\nu}) \cos\theta\right)$$



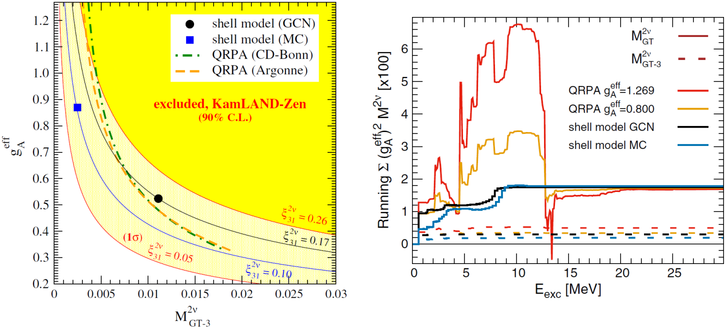
$$K^{2\nu}(\xi_{31}^{2\nu}, \xi_{51}^{2\nu}) = -\frac{H_0^{2\nu} + \xi_{31} H_2^{2\nu} + \frac{5}{9}\xi_{31}^2 H_{22}^{2\nu} + \left(\frac{2}{9}\xi_{31}^2 + \xi_{51}\right) H_4^{2\nu}}{G_0^{2\nu} + \xi_{31} G_2^{2\nu} + \frac{1}{3}\xi_{31}^2 G_{22}^{2\nu} + \left(\frac{1}{3}\xi_{31}^2 + \xi_{51}\right) G_4^{2\nu}}.$$



$$\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1} \simeq \left(g_A^{\rm eff}\right)^4 \left|M_{GT-3}^{2\nu}\right|^2 \frac{1}{\left|\xi_{31}^{2\nu}\right|^2} \times \left[G_0^{2\nu} + \xi_{31}^{2\nu} G_2^{2\nu} + \frac{1}{3}\left(\xi_{31}^{2\nu}\right)^2 G_{22}^{2\nu} + \left(\frac{1}{3}\left(\xi_{31}^{2\nu}\right)^2 + \xi_{51}^{2\nu}\right) G_4^{2\nu}\right].$$

- $\xi_{31}^{2\nu}$ determined from shape analyze of the energy distributions and/or angular correlation factor
- $\left[T_{1/2}^{2\nu\beta\beta}\right]^{-1}$ measured.
- $\left|M_{GT-3}^{2\nu}\right|^2$ have to be calculated by nuclear models.

KamLAND-Zen Collaboration, Phys. Rev. Lett. 122, 192501 (2019).



Outlook

Conclusions

- The $2\nu\beta\beta$-decay formalism was improved- a free (measurable) parameter model.
- The ratio of the nuclear matrix elements, $\xi_{31}^{2\nu}$ can be determined from the electron spectra and angular correlation factor.
- The parameter, $\xi_{31}^{2\nu}$ can be used as a tool to better understand nuclear systems involved in $2\nu\beta\beta$, $0\nu\beta\beta$-decay and neutrino physics.