The effect of quantum decoherence in astrophysical neutrino oscillations

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Based on:
[1] K.Stankevich, A.Studenikin, Phys.Rev.D 101 (2020)
[2] K.Stankevich, A.Studenikin, PoS ICHEP2020 (2021)

Astroneutrino Theory Workshop 2021
20 September to 1 October, 2021
Prague, Czech Republic

Outline

- 1. Introduction

  neutrino quantum decoherence
  Lindblad master equations

- 2. Manifestations of neutrino quantum decoherence

  neutrino reactor fluxes
  neutrino solar fluxes
  neutrino supernova fluxes

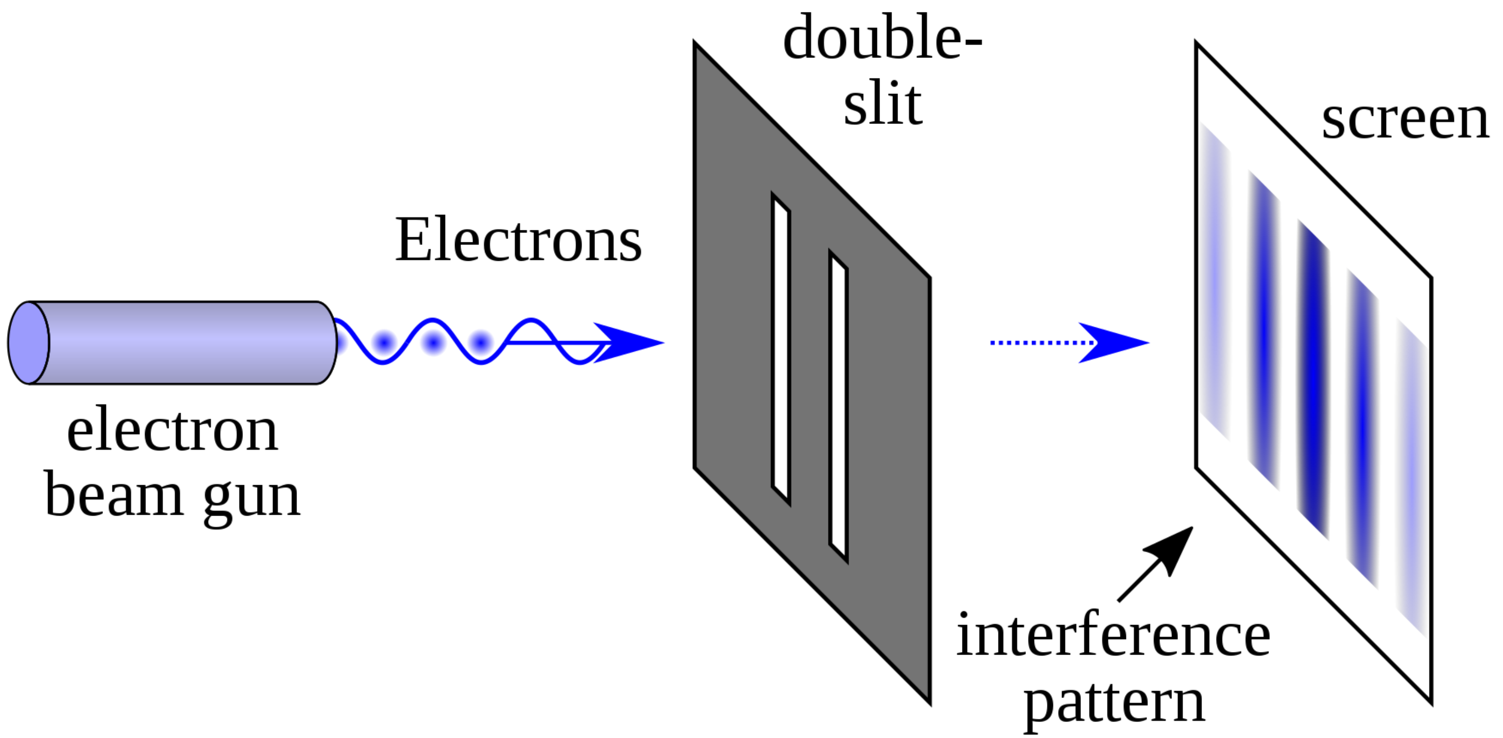
- 3. Mechanisms of neutrino quantum decoherence

  matter and field fluctuations
  neutrino radiative decay (electromagnetic interaction)
  nonforward neutrino scattering processes (weak interaction)

- 4. Conclusion

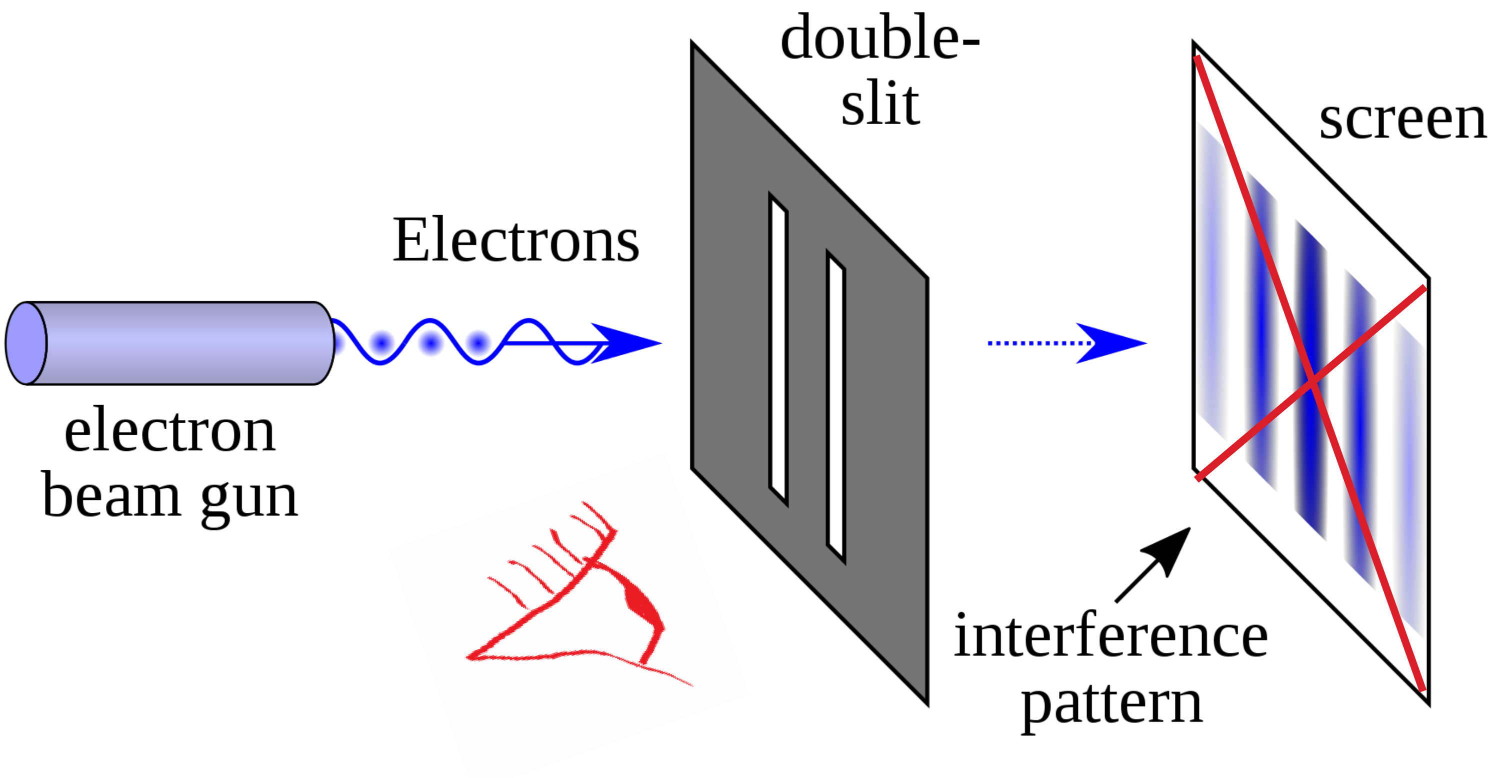
Introduction

- Quantum decoherence



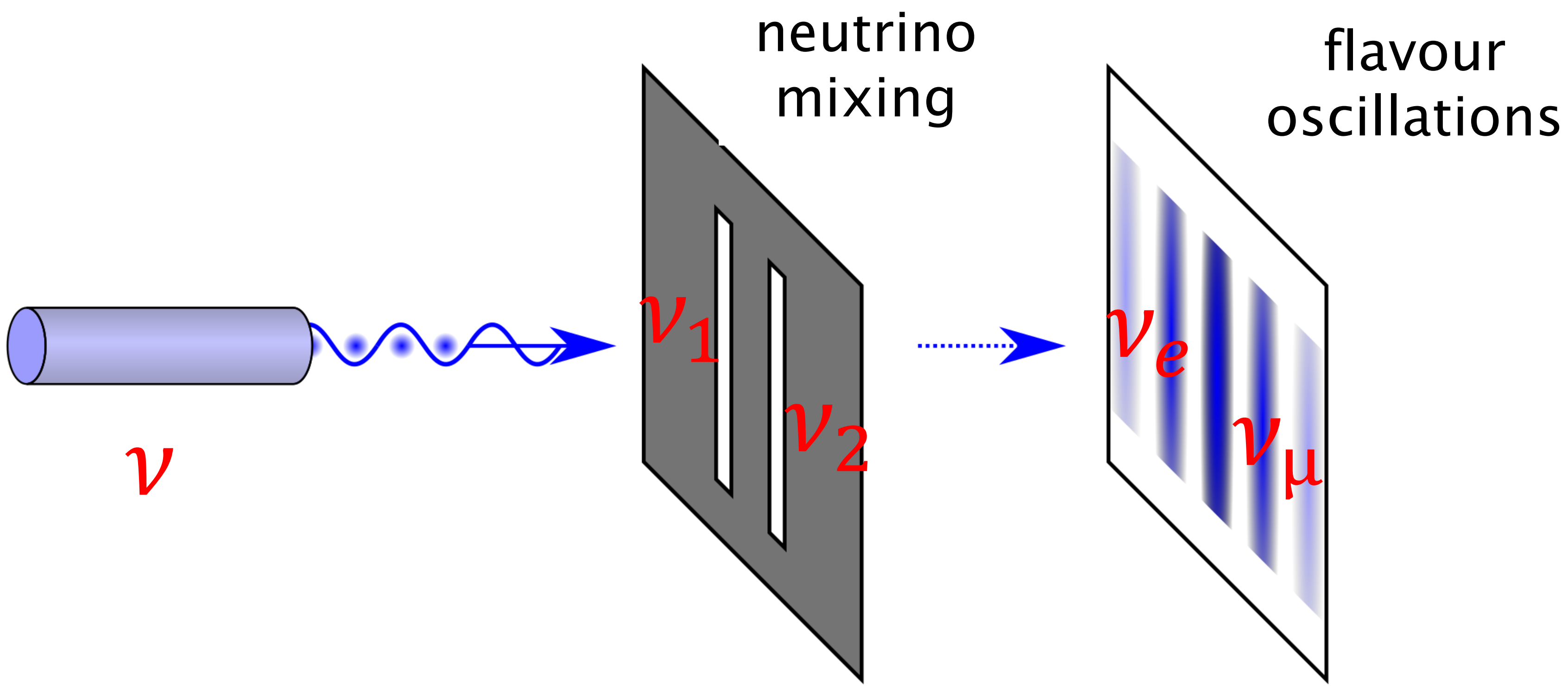
Introduction

- Quantum decoherence



Introduction

- Neutrino quantum decoherence



Lindblad equation (1976)

Lindblad equation

$$\frac{\partial \rho_\nu(t)}{\partial t} = -i\left[H_S, \rho_\nu(t)\right] + D\left[\rho_\nu\right]$$

$\rho_\nu(t)$ is neutrino density matrix
H_s is Hamiltonian of the neutrino system

$$D_{ll} = -diag\{\Gamma_1, \Gamma_1, \Gamma_2\}$$

Γ_1 decoherence parameter
Γ_2 relaxation parameter

Dissipative operators

$$D\left[\rho_\nu(t)\right] = \frac{1}{2}\sum_{k=1}^{N^2-1}\left[V_k, \rho_\nu V_k^\dagger\right] + \left[V_k\rho_\nu, V_k^\dagger\right]$$

Lindblad equation can be decomposed to Pauli matrices σ_k :

$$\frac{\partial \rho_k(t)}{\partial t}\sigma_k = 2\epsilon_{ijk}H_i\rho_j(t)\sigma_k + D_{kl}\rho_l(t)\sigma_k$$

Lindblad equation (1976)

Lindblad equation

$$\frac{\partial \rho_\nu(t)}{\partial t} = -i\left[H_S, \rho_\nu(t)\right] + D\left[\rho_\nu\right]$$

$$D_{ll} = -diag\{\Gamma_1, \Gamma_1, \Gamma_2\}$$

Γ_1 decoherence parameter

Γ_2 relaxation parameter

Neutrino flavour oscillation probability accounting for the quantum decoherence of neutrino mass states

$$P_{\nu_\alpha \nu_\alpha} = \frac{1}{2}\left[1 + e^{-\Gamma_2 x}\cos^2 2\tilde{\theta} + e^{-\Gamma_1 x}\sin^2 2\tilde{\theta}\cos\left(\tilde{\Delta} x\right)\right]$$

Reactor and solar fluxes

Experimental studies of the neutrino quantum decoherence

Reactor neutrinos

[1] A.Capolupo, S.M.Giampaolo, G.Lambiase, *Phys.Lett.B* 792 (2019) 298
[2] J.A.B.Coelho, W.A.Mann, S.S.Bashar, *Phys.Rev.Lett.118* (2017) 221801
[3] Y.Farzan, T.Schwetz, A.Y.Smirnov, *J. High Energy Phys.* (2008) 067
[4] G.Barenboim et al, *Nucl.Phys.B* 758 (2006) 90
And many others…

Solar neutrinos

P.C. de Holanda, *JCAP 03* (2020) 012

Atmospheric neutrinos

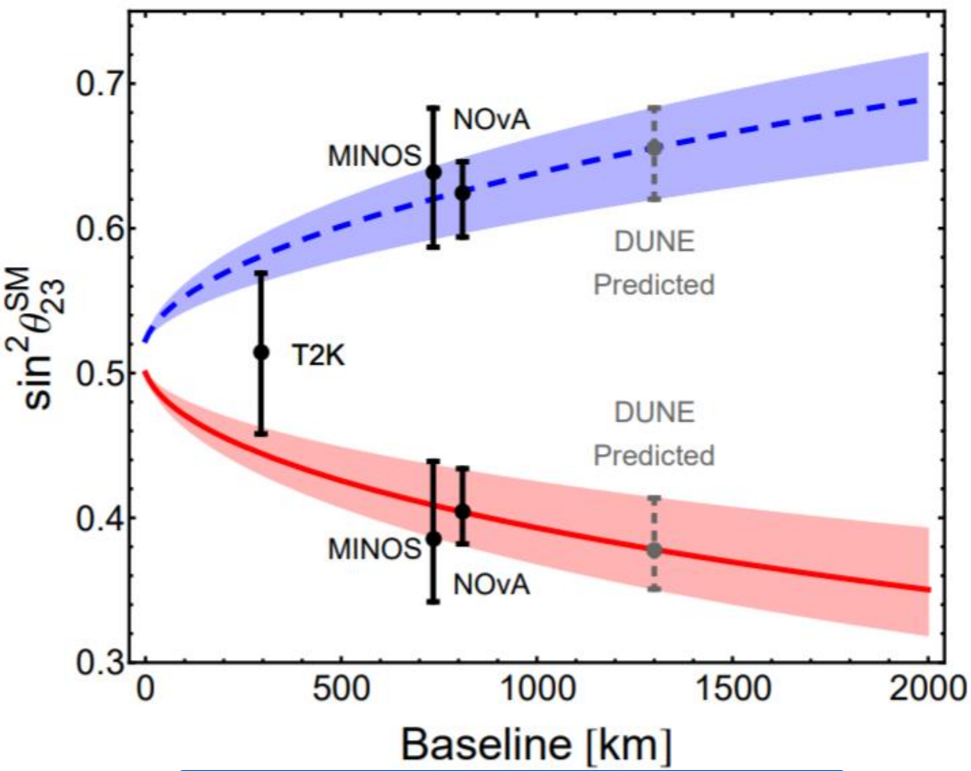
E.Lisi, A.Marrone, D.Montanino, *Phys.Rev.Lett.85* (2000) 1166

EXPERIMENTS:
Super-Kamiokande
MINOS
T2K
NoVA
IceCube
JUNO
DUNE

$\Gamma < 10^{-22} : 10^{-23}$ GeV

Most intriguing result

J.A.B.Coelho, W.A.Mann, S.S.Bashar, *Phys.Rev.Lett.* *118* (2017) 221801



$$\Gamma = (1.8 \pm 0.9) \times 10^{-23}\ \mathrm{GeV}$$

Interplay of neutrino quantum decoherence and collective oscillations

[1] K.Stankevich, A.Studenikin, PoS ICHEP2020 (2021)

Master equations for neutrino density matrix

$$i\frac{d\rho_f}{dt} = [H, \rho_f] + D[\rho_f]\,, \qquad i\frac{d\bar{\rho}_f}{dt} = [\bar{H}, \bar{\rho}_f] + D[\bar{\rho}_f]$$

Hamiltonian

$$H = H_{vac} + H_M + H_{\nu\nu}$$

Dissipative term (in the Lindblad form)

$$D\left[\rho_{\tilde{m}}(t)\right] = \frac{1}{2}\sum_{k=1}^{3}\left[V_k, \rho_{\tilde{m}} V_k^{\dagger}\right] + \left[V_k \rho_{\tilde{m}}, V_k^{\dagger}\right]$$

Linearized (in)stability analysis

[2] D.Väänänen, G.McLaughlin, *Phys.Rev.D 93* (2016) 1050

Supernovae toy-model

[3] C.J.Stapleford et al, Phys. Rev. D 94 (2016) 093007

Stability equation

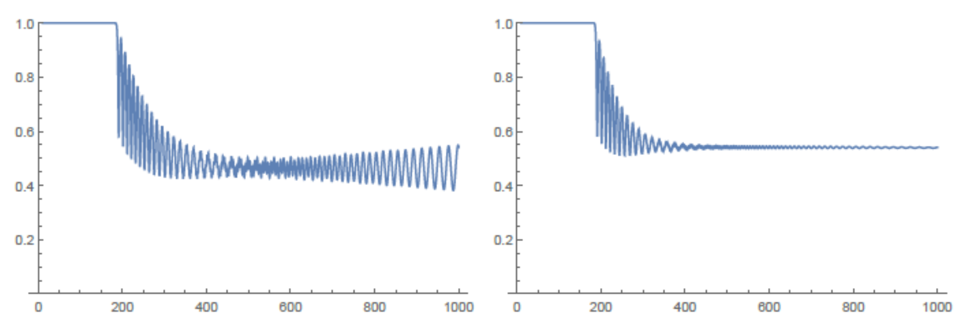
$$(\omega - i\Gamma_1)\begin{pmatrix}\rho'_{12}\\ \bar{\rho}'_{21}\end{pmatrix} = \begin{pmatrix}A_{12} & B_{12}\\ \bar{A}_{21} & \bar{B}_{21}\end{pmatrix}\begin{pmatrix}\rho'_{12}\\ \bar{\rho}'_{21}\end{pmatrix}$$

Elements of the stability matrix

$$A_{12} = (H^0_{11} - H^0_{22}) - \frac{\partial H_{12}}{\partial \rho_{12}}(\rho^0_{11} - \rho^0_{22}),$$
$$B_{12} = \frac{\partial H_{12}}{\partial \bar{\rho}^0_{21}}(\rho^0_{22} - \rho^0_{11}),$$
$$\bar{A}_{21} = (\bar{H}^0_{22} - \bar{H}^0_{11}) - \frac{\partial \bar{H}_{21}}{\partial \bar{\rho}_{21}}(\bar{\rho}^0_{22} - \bar{\rho}^0_{11}),$$
$$\bar{B}_{21} = \frac{\partial \bar{H}_{21}}{\partial \rho^0_{12}}(\bar{\rho}^0_{11} - \bar{\rho}^0_{22}).$$

Conditions for collective neutrino oscillations (bipolar/nutations)

$$\begin{cases}(A_{12} - \bar{A}_{21})^2 + 4B_{12}\bar{B}_{21} < 0,\\ \mathcal{I}m\left((A_{12} - \bar{A}_{21})^2 + 4B_{12}\bar{B}_{21}\right) > \Gamma_1.\end{cases}$$



a) b)

Figure 1: The survival probability of the electron neutrino in the absence of quantum decoherence (a) and for the case when the neutrino decoherence parameter is $\Gamma_1 = 10^{-21}$ GeV (b).

Interplay of neutrino quantum decoherence and collective oscillations

[1] K.Stankevich, A.Studenikin, PoS ICHEP2020 (2021)

Master equations for neutrino density matrix

$$i\frac{d\rho_f}{dt} = [H, \rho_f] + D[\rho_f]\,, \qquad i\frac{d\bar\rho_f}{dt} = [\bar H, \bar\rho_f] + D[\bar\rho_f]$$

Decomposition to Pauli matrices:

$$\frac{\partial P_k(t)}{\partial t}\sigma_k = 2\epsilon_{ijk}H_i P_j(t)\sigma_k + D_{kl}P_l(t)\sigma_k,$$

Linearization procedure:

$$P_k = P_k^0 + \delta P_k, \quad \text{where} \quad \delta P_k = P'_k e^{-i\omega t} + H.c.,$$
$$H_k = H_k^0 + \delta H_k, \quad \text{where} \quad \delta H_k = H'_k e^{-i\omega t} + H.c.,$$
$$H'_k = \frac{\partial H_k}{\partial P_k}P'_k + \frac{\partial H_k}{\partial \bar P_k}\bar P'_k.$$

Supernovae toy-model

[3] C.J.Stapleford et al, Phys. Rev. D 94 (2016) 093007

Stability equation

$$(\omega - i\Gamma_1)\begin{pmatrix}\rho'_{12}\\ \bar\rho'_{21}\end{pmatrix} = \begin{pmatrix}A_{12} & B_{12}\\ \bar A_{21} & \bar B_{21}\end{pmatrix}\begin{pmatrix}\rho'_{12}\\ \bar\rho'_{21}\end{pmatrix}$$

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$$A_{12} = (H^0_{11} - H^0_{22}) - \frac{\partial H_{12}}{\partial \rho_{12}}(\rho^0_{11} - \rho^0_{22}),$$
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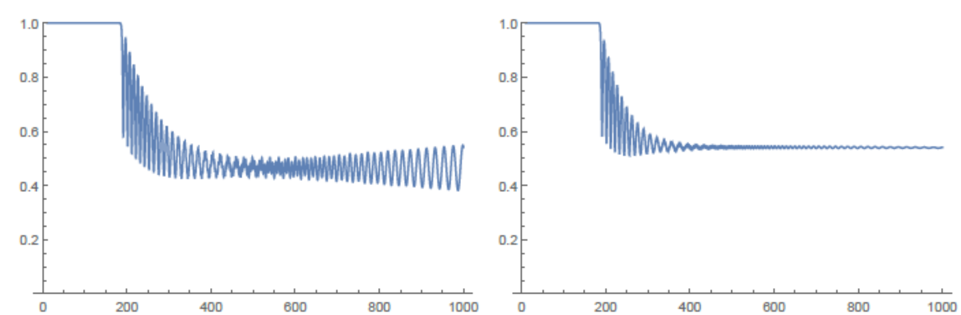
$$\begin{cases}(A_{12} - \bar A_{21})^2 + 4B_{12}\bar B_{21} < 0,\\ \mathcal{I}m\left((A_{12} - \bar A_{21})^2 + 4B_{12}\bar B_{21}\right) > \Gamma_1.\end{cases}$$



a) b)

Figure 1: The survival probability of the electron neutrino in the absence of quantum decoherence (a) and for the case when the neutrino decoherence parameter is $\Gamma_1 = 10^{-21}$ GeV (b).

PART 2

mechanisms of the neutrino quantum decoherence

1) Matter and field fluctuations

[1] F.N. Loreti, A.B. Balantekin, **Phys. Rev. D 50** (1994) 4762

[2] C.P.Burgess, D.Michaud, **Ann. Phys.** (1997) 256

[3] F.Benatti, R.Floriani, **Phys. Rev. D 71** (2005) 013003

2) Neutrino radiative decay

[4] K.Stankevich, A.Studenikin, **Phys. Rev. D 101** (2020)

[5] A.Lichkunov, K.Stankevich, A.Studenikin, M.Vialkov, **TAUP-2021**

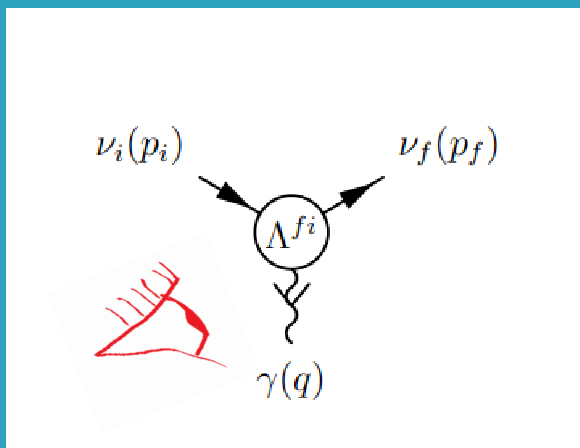
3) Nonforward neutrino scattering processes

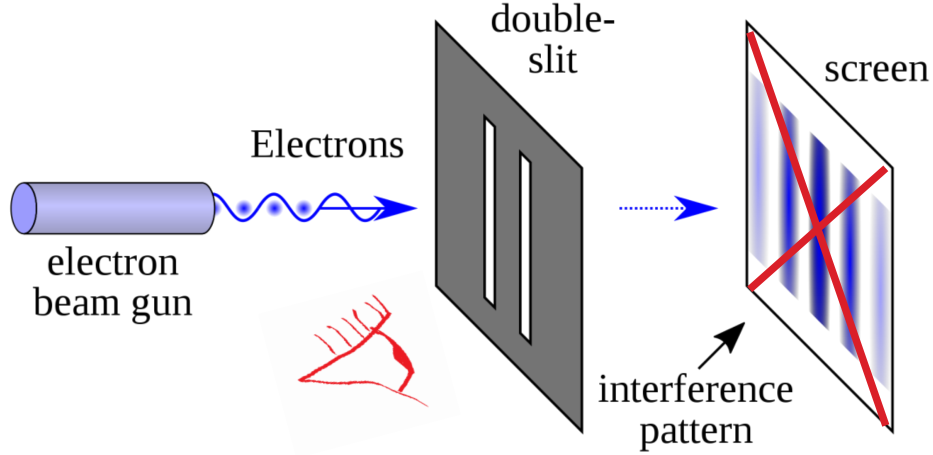
[6] J.F.Nieves, S.Sahu, Phys. Rev. D 102 (2020) 056007

[7] J.F.Nieves, S.Sahu, Phys. Rev. D 100 (2019) 115049

Neutrino radiative decay

Neutrino radiative decay





[1] K.Stankevich, A.Studenikin, Phys.Rev.D 101 (2020)

Formalism

Liouville equation for the density matrix of a system composed of neutrinos and an electromagnetic field

$$\frac{\partial}{\partial t}\rho = -i \int d^3x \, [H(x), \rho]$$

ρ – density matrix of the full system

H – Hamiltonian of the full system

$H = H_\nu + H_{int} + H_\gamma$

H_ν – neutrino

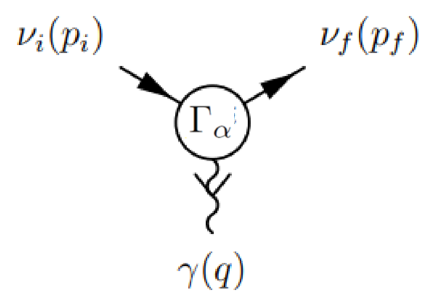
H_{int} – interaction

H_γ – electromagnetic field

Interaction is expressed as

$$H_{int}(x) = j_\alpha(x) A^\alpha(x)$$

$$j_\alpha(x) = \overline{\nu}_i(x) \Gamma_\alpha \nu_j(x)$$

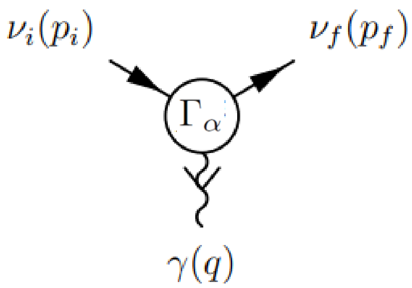


Based on: H.P. Breuer, F. Petruccione, "The theory of open quantum systems"

Elecromagnetic vertex

$$H_{int}(x) = j_\alpha(x) A^\alpha(x)$$

$$j_\alpha(x) = \overline{\nu}_i(x) \Gamma_\alpha \nu_j(x)$$



$$\Gamma_\mu(q) = \mathbb{f}_Q(q^2)\gamma_\mu - \mathbb{f}_M(q^2) i\sigma_{\mu\nu}q^\nu + \mathbb{f}_E(q^2)\sigma_{\mu\nu}q^\nu\gamma_5 + \mathbb{f}_A(q^2)(q^2\gamma_\mu - q_\mu \not{q})\gamma_5,$$

Formalism

Liouville equation

$$\frac{\partial}{\partial t}\rho = -i \int d^3x \, [H(x), \rho]$$

After integrating

$$\rho_\nu(t_f) = tr_F \left(Texp \left[\int_{t_i}^{t_f} d^4x \, [H(x), \rho(t_i)] \right] \right)$$

$\rho_\nu(t) = tr_F \rho(t)$ – density matrix for neutrino

We rewrite equation using the decomposition of the chronological time-ordering operator T into time-ordering operator for a matter current T_j and for electromagnetic fields T_A as $T = T_j T_A$

$$\rho_\nu(t_f) = T^j \left(\exp \left[\int_{t_i}^{t_f} d^4x \, [H_\rho(x), \rho(t_i)] \right] \right.$$
$$\left. tr_F \left\{ T^A \exp \left[\int_{t_i}^{t_f} d^4x \, [H_{int}(x), \rho(t_i)] \right] \right\} \right)$$

Formalism

$$\rho_\nu(t_f) = T^j \left(\exp\left[\int_{t_i}^{t_f} d^4x\,[H_\rho(x),\rho(t_i)]\right]\right.$$
$$\left. tr_F\left\{\underbrace{T^A \exp\left[\int_{t_i}^{t_f} d^4x\,[H_{int}(x),\rho(t_i)]\right]}_{\text{Wick theorem}}\right\}\right)$$

$$T^A \exp\left[\int_{t_i}^{t_f} d^4x\,[H_{int}(x),\rho(t)]\right] =$$
$$\exp\left[-\frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x'[A_\mu(x),A_\nu(x')][j^\mu(x)j^\nu(x'),\rho(t_i)]\Theta(t-t')\right]\exp\left[\int_{t_i}^{t_f} d^4x[H_{int}(x),\rho(t_i)]\right]$$

$\rho(t_i) = \rho_\nu(t_i) \otimes \rho_f$

$$tr_f\left\{\exp\left[\int_{t_i}^{t_f} d^4x[H_{int}(x),\rho(t_i)]\right]\right\} =$$
$$\exp\left[\frac{1}{2}\int_{t_i}^{t_f} d^4x \int_{t_i}^{t_f} d^4x'\{\langle A_\nu(x')A_\mu(x)\rangle_f j^\mu(x)j^\nu(x')\rho_\nu(t_i) + \langle A_\mu(x)A_\nu(x')\rangle_f \rho_\nu(t_i)j^\mu(x)j^\nu(x')\right.$$
$$\left. -\langle A_\nu(x')A_\mu(x)\rangle_f j^\mu(x)\rho_\nu(t_i)j^\nu(x') - \langle A_\mu(x)A_\nu(x')\rangle_f j^\nu(x')\rho_\nu(t_i)j^\mu(x)\}\right]$$

Neutrino Master Equation

The final master equation

$$\frac{\partial \rho_\nu(t)}{\partial t} = -i\left[H_S, \rho_\nu(t)\right] + D\left[\rho_\nu\right]$$

$$D[\rho_\nu] = \sum_{\omega_n} \Gamma(\omega_n)(1+N(\omega_n))\left(j(\omega_n)\rho_\nu j^\dagger(\omega_n) - \frac{1}{2}\{j^\dagger(\omega_n)j(\omega_n), \rho_\nu\}\right) + \\ + \sum_{\omega_n} \Gamma(\omega_n)N(\omega_n)\left(j^\dagger(\omega_n)\rho_\nu j(\omega_n) - \frac{1}{2}\{j(\omega_n)j^\dagger(\omega_n), \rho_\nu\}\right),$$

$\Gamma(\omega_n)$ is neutrino decay rates

ω_n is the energy difference between neutrino states

$\mathrm{N}(\omega) = \frac{1}{e^{\beta\omega}-1}$ is the Planck distribution function

eigenoperators of the neutrino Hamiltonian

$\mathrm{j}(\mathrm{t}, \vec{k}) = \sum_n e_n^{-i\omega t} j(\omega_n, \vec{k})$

$[H_\nu, j(\omega_n)] = \omega_n j(\omega_n)$

Example

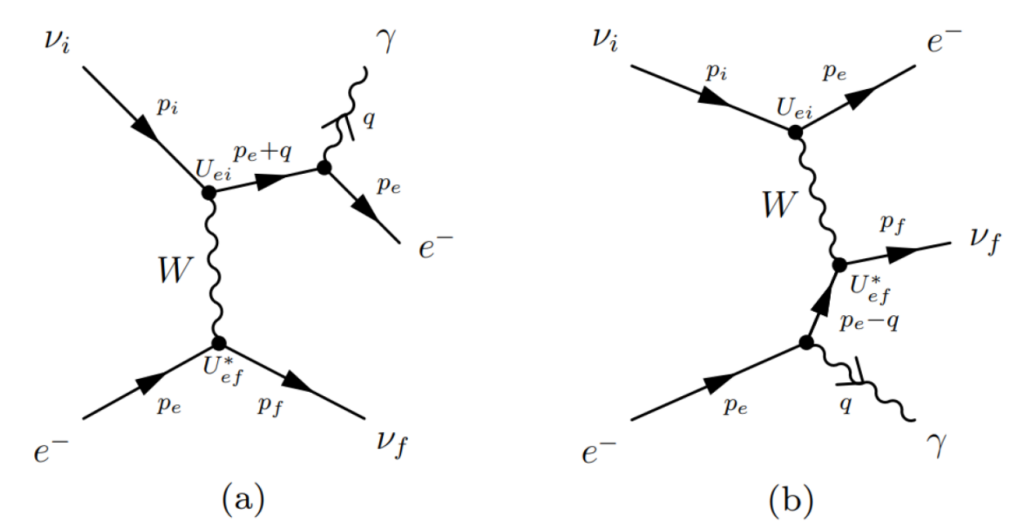
$$H_{int}(x) = j_\alpha(x) A^\alpha(x)$$

$$j_\alpha(x) = \overline{\nu}_i(x) \Gamma_\alpha \nu_j(x)$$

Electromagnetic vertex

$$\Gamma_\alpha = \sin 2\theta \, \tau \gamma_\alpha (1 - \gamma_5)$$

$$\tau = -\frac{e G_F T^2}{2\sqrt{2}}$$



J. C. D'Olivo, J. F. Nieves, P. B. Pal, Phys.Rev.Lett. 64 (1990) 1088.

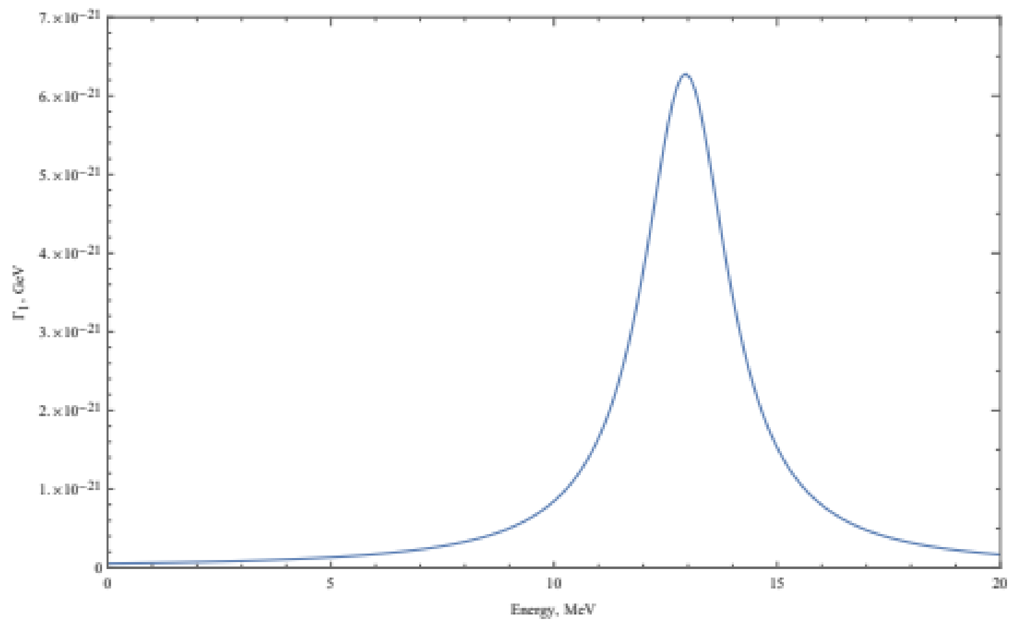
Results

[1] K.Stankevich, A.Studenikin, Phys.Rev.D 101 (2020)

Supernova environment

$T = 30\ MeV$
$T_\gamma = 100\ MeV$
$n = 10^{29}\ cm^{-3}$

[2] R. Bollig, H.-Th. Janka, et. al. Phys.Rev.Lett. 119 (2017)



Obtained values of decoherence parameters
$\Gamma_1 \approx 10^{-21} GeV$ for $\nu_e \nu_s$ oscillations
$\Gamma_1 \approx 10^{-31} GeV$ for $\nu_e \nu_\tau$ oscillations

Experimental constraints
$\Gamma_1 < 10^{-23} GeV$ for reactor neutrino fluxes
$\Gamma_1 < 10^{-28} GeV$ for solar neutrino fluxes

[3] R.L.N.Oliveira et al, Phys.Rev.D **89** (2014)
[4] P.C. de Holanda, *JCAP 03* (2020) 012

Neutrino decays

The final master equation

$$\frac{\partial \rho_\nu(t)}{\partial t} = -i\left[H_S, \rho_\nu(t)\right] + D\left[\rho_\nu\right]$$

$$D[\rho_\nu] = \sum_{\omega_n} \Gamma(\omega_n)(1 + N(\omega_n)) \left(j(\omega_n)\rho_\nu j^\dagger(\omega_n) - \frac{1}{2}\{j^\dagger(\omega_n)j(\omega_n), \rho_\nu\} \right) + \\ + \sum_{\omega_n} \Gamma(\omega_n)N(\omega_n) \left(j^\dagger(\omega_n)\rho_\nu j(\omega_n) - \frac{1}{2}\{j(\omega_n)j^\dagger(\omega_n), \rho_\nu\} \right),$$

$\Gamma(\omega_n)$ is neutrino decay rates

- Neutrino decay to

1) dark photons;

2) massless axions;

3) gravitons.

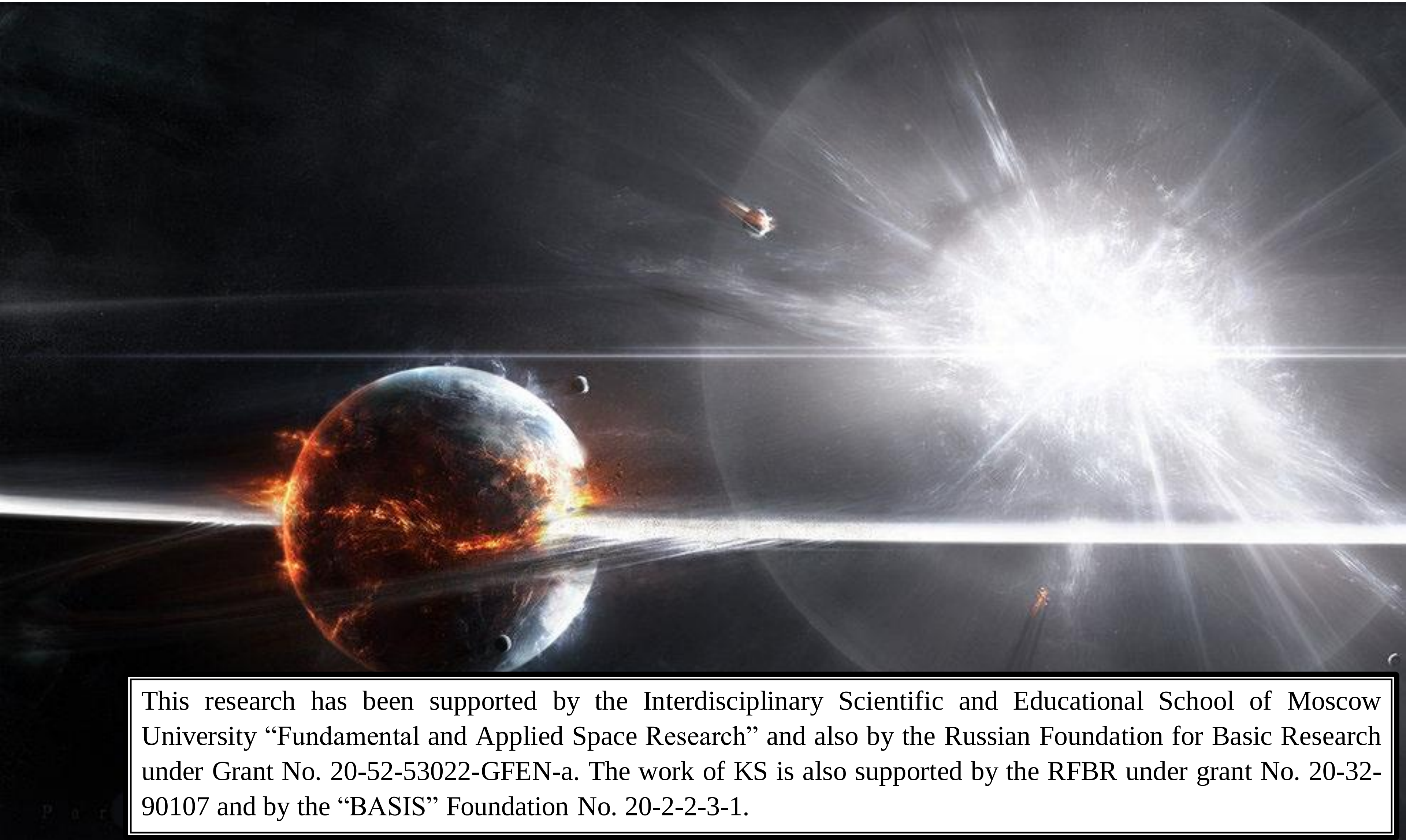
Conclusions

- We considered for the first time the interplay of two effects: neutrino quantum decoherence and collective neutrino oscillations. We derived new conditions of the existence of the collective bipolar neutrino oscillations that accounts the neutrino quantum decoherence.

- A new theoretical framework, based on the quantum field theory of open systems applied to neutrinos, has been developed to describe the neutrino evolution in external environments accounting for the effect of the neutrino quantum decoherence. We have used this approach to consider a new mechanism of the neutrino quantum decoherence engendered by the neutrino radiative decay.

- Neutrino quantum decoherence can serve as a signatures of the electromagnetic neutrino properties and physics beyond Standard Model

THANK YOU



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