広島大学

Time evolution of lepton number from relativistic regime to non-relativistic regime

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The work based on

- Time evolution of Lepton Number carried by Majorana neutrinos (PTEP (2021)ptab025)2101.07751
- Insight into neutrino mass phenomenology exploring the non-relativistic regime in quantum field theory . : 2106.02783

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Introduction

- **Until now , all the fermions discovered are Dirac fermion.**
- **The exceptions are neutrinos, which may have good chance to be Majorana fermions.**
- **However the present experiment with neutrino oscillations, one can not distinguish Majorana neutrinos and Dirac neutrinos.**
- **Lepton family numbers as observable, one can discriminate Majorana neutrinos from Dirac neutrinos.**

PMNS matrix for Dirac and Majorana neutrinos

- Charged current interaction in the (charged lepton) flavor basis $\alpha = e, \mu, \tau$

Majorana case:

- $\frac{g}{\sqrt{2}} \overline{l_{L\alpha}} \gamma_\mu \nu_{L\alpha} W^- - \frac{1}{2} \overline{(\nu_{L\alpha})^c}\, m_{\nu\alpha\beta} \nu_{L\beta} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + \mathrm{h.c.}$

Dirac case:

- $\frac{g}{\sqrt{2}} \overline{l_{L\alpha}} \gamma_\mu \nu_{L\alpha} W^- - \frac{1}{2} \overline{(\nu_{R\alpha})}\, m_{D\alpha\beta} \nu_{L\beta} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + \mathrm{h.c.}$

Number of angles and CPV phases in PMNS matrix V

In the mass basis $i = 1, 2, 3$

- **Majorana case:** $V^T_{i\alpha} m_{\nu\alpha\beta} V_{\beta j} = m_i \delta_{ij}$ $\nu_{L\alpha} = V_{\alpha i} \nu_{Li}$
- $\frac{g}{\sqrt{2}} \overline{l_{L\alpha}} \gamma_\mu V_{\alpha i} \nu_{Li} W^- - \frac{1}{2} \overline{(\nu_{Li})^c}\, m_i \nu_{Li} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$

$l_\alpha \rightarrow e^{i\theta_\alpha} l_\alpha$

$V_{\alpha i} \rightarrow e^{-i(\theta_\alpha)} V_{\alpha i}$: **3 angles+3 CPV(=6-3)**

- **Dirac case :** $U^\dagger_{i\alpha} m_{D\alpha\beta} V_{\beta j} = m_i \delta_{ij}$
- $\frac{g}{\sqrt{2}} \overline{l_{L\alpha}} \gamma_\mu V_{\alpha i} \nu_{Li} W^- - \frac{1}{2} \overline{\nu_{Ri}}\, m_i \nu_{Li} - \overline{l_{R\alpha}}\, m_{l\alpha}\, l_{L\alpha} + h.c.$

$l_\alpha \rightarrow e^{i\theta_\alpha} l_\alpha$, $\nu_i \rightarrow e^{i\theta_i} \nu_i$

$V_{\alpha i} \rightarrow e^{i(\theta_i - \theta_\alpha)} V_{\alpha i}$:**3 angles +1CPV(=6-5)**

Parametrization of PMNS matrix with angles and phases

Dirac case

$$V_{\sigma i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

Dirac (KM)phase

Majorana case

$$V_{\sigma i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix},$$

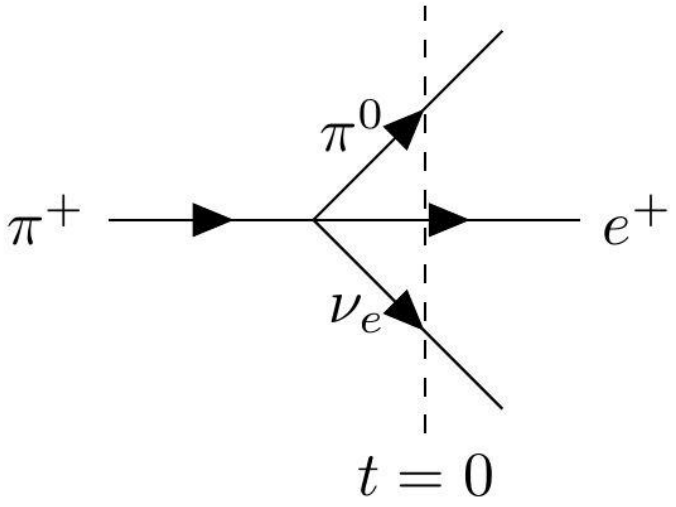
Two Majorana phases

Lepton Number with Majorana mass term

- We consider the following situation.
- (t<0:massless, t=0, mass term included, t>0 massive)

$$\mathcal{L} = \overline{\nu_{\alpha L}} i\gamma^\mu \partial_\mu \nu_{\alpha L} - \theta(t) \left(\frac{m_{\alpha\beta}}{2} \overline{(\nu_{\alpha L})^c} \nu_{\beta L} + h.c. \right)$$
$$= \frac{1}{2} \overline{\psi_i} i\gamma^\mu \partial_\mu \psi_i - \theta(t) \frac{m_i}{2} \overline{\psi_i} \psi_i.$$

$(\alpha = e, \mu, \tau, i = 1, 2, 3)$. Majorana field, $\psi_i^c = \psi_i$.



Lepton family number of neutrino:

Lepton Family Number for t<0：

$$\mathrm{L}_\alpha(t=-\varepsilon)=\int d^3x :\bar{\nu}_{\alpha L}\gamma^0\nu_\alpha : =\int' \frac{d^3p}{(2\pi)^3|2\mathbf{p}|}\left(a_\alpha^\dagger(\mathbf{p})a_\alpha(\mathbf{p})-b_\alpha^\dagger(\mathbf{p})b_\alpha(\mathbf{p})\right).$$

$$\nu_{\alpha L}=\int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}\left(a_\alpha(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}}u_L(\mathbf{p})+b_\alpha^\dagger(\mathbf{p})e^{-i\mathbf{p}\cdot\mathbf{x}}v_L(\mathbf{p})\right)$$

After mass term is turned on: The mass matrix is diagonalized as; ⬇

$$\psi_i=\nu_{iL}+(\nu_{iL})^c, \quad \nu_{\alpha L}=V_{\alpha i}\nu_{iL},$$
$$m_i\delta_{ij}=\left(V^T\right)_{i\alpha}m_{\alpha\beta}V_{\beta j}.$$

Relations bet Creation and Annihilation operators

$$\nu_{\alpha L}(t=-0,\mathbf{x}) = P_L V_{\alpha i}\psi_i(t=+0,\mathbf{x}).$$

$$\frac{1}{\sqrt{2|\mathbf{p}|}}\begin{pmatrix} V^*_{\alpha j}a_\alpha(\mathbf{p}) \\ V_{\alpha j}a^\dagger_\alpha(-\mathbf{p})\end{pmatrix} = \frac{\sqrt{N_j(\mathbf{p})}}{2E_j(\mathbf{p})}\begin{pmatrix} 1 & \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} \\ \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} & 1\end{pmatrix}\begin{pmatrix} a_{Mj}(\mathbf{p},-) \\ a^\dagger_{Mj}(-\mathbf{p},-)\end{pmatrix},$$

$$\frac{1}{\sqrt{2|\mathbf{p}|}}\begin{pmatrix} V_{\alpha j}b_\alpha(\mathbf{p}) \\ V^*_{\alpha j}b^\dagger_\alpha(-\mathbf{p})\end{pmatrix} = \frac{\sqrt{N_j(\mathbf{p})}}{2E_j(\mathbf{p})}\begin{pmatrix} 1 & \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} \\ \frac{im_j}{E_j(\mathbf{p})+|\mathbf{p}|} & 1\end{pmatrix}\begin{pmatrix} a_{Mj}(\mathbf{p},+) \\ a^\dagger_{Mj}(-\mathbf{p},+)\end{pmatrix}.$$

$N_j(p) = E_j(p) + |p|$ $\lambda = \pm 1$ **(helicity)**

LHS= $$\int' \frac{d^3\mathbf{p}}{(2\pi)^3 2|\mathbf{p}|}\left(a_\alpha(\mathbf{p})e^{i\mathbf{p}\cdot\mathbf{x}}u_L(\mathbf{p}) + b^\dagger_\alpha(\mathbf{p})e^{-i\mathbf{p}\cdot\mathbf{x}}v_L(\mathbf{p})\right)$$

RHS= $$V_{\alpha i}P_L\int'\sum_{\lambda=\pm1}\frac{d^3\mathbf{p}}{(2\pi)^3 2E(\mathbf{p})}\left(a_{Mi}(\mathbf{p},\lambda)u_i(\mathbf{p},\lambda)e^{i\mathbf{p}\cdot\mathbf{x}} + a^\dagger_{Mi}(\mathbf{p},\lambda)v_i(\mathbf{p},\lambda)e^{-i\mathbf{p}\cdot\mathbf{x}}\right).$$

Time evolution of operators for family basis

$$a_\alpha(\mathbf{p},t) = V_{\alpha j}V^*_{\gamma j}\left(\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}|\sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})}\right)a_\gamma(\mathbf{p}) - V_{\alpha j}V_{\gamma j}\frac{m_j\sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})}a^\dagger_\gamma(-\mathbf{p}),$$

$$b_\alpha(\mathbf{p},t) = V^*_{\alpha j}V_{\gamma j}\left(\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}|\sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})}\right)b_\gamma(\mathbf{p}) - V^*_{\alpha j}V^*_{\gamma j}\frac{m_j\sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})}b^\dagger_\gamma(-\mathbf{p}),$$

The time evolution is obtained through the evolution of operators for mass basis

$$\boldsymbol{a_{Mj}^{(+)}(q,t) = e^{\mp i\,E_j(q)t}\,a_{Mj}^{(+)}(q)}$$

Lepton family number $L_\alpha(t) = \int' \frac{d^3p}{(2\pi)^3|2\mathbf{p}|}\left(a_\alpha^\dagger(\mathbf{p},t)a_\alpha(\mathbf{p},t) - b_\alpha^\dagger(\mathbf{p},t)b_\alpha(\mathbf{p},t)\right).$

$$
\begin{aligned}
L_\alpha(t) = \int'_{\mathbf{p}\in A} \Bigg[& -\frac{m_i m_j \sin(E_i(\mathbf{p})t)\sin(E_j(\mathbf{p})t)}{E_i(\mathbf{p})E_j(\mathbf{p})} \{V^*_{\alpha i}V^*_{\beta i}V_{\alpha j}V_{\gamma j}(a^\dagger_\gamma(\mathbf{p})a_\beta(\mathbf{p}) + a^\dagger_\gamma(-\mathbf{p})a_\beta(-\mathbf{p})) \\
& \qquad - V_{\alpha i}V_{\beta i}V^*_{\alpha j}V^*_{\gamma j}(b^\dagger_\gamma(\mathbf{p})b_\beta(\mathbf{p}) + b^\dagger_\gamma(-\mathbf{p})b_\beta(-\mathbf{p}))\} \\
& + \left(\cos(E_i(\mathbf{p})t)\cos(E_j(\mathbf{p})t) + \frac{|\mathbf{p}|^2}{E_i(\mathbf{p})E_j(\mathbf{p})}\sin(E_i(\mathbf{p})t)\sin(E_j(\mathbf{p})t)\right. \\
& \qquad \left. + \frac{i|\mathbf{p}|}{E_i(\mathbf{p})}\sin(E_i(\mathbf{p})t)\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}|}{E_j(\mathbf{p})}\cos(E_i(\mathbf{p})t)\sin(E_j(\mathbf{p})t)\right) \\
& \times \left\{V^*_{\alpha i}V_{\beta i}V_{\alpha j}V^*_{\gamma j}\left(a^\dagger_\beta(\mathbf{p})a_\gamma(\mathbf{p}) + a^\dagger_\beta(-\mathbf{p})a_\gamma(-\mathbf{p})\right)\right. \\
& \qquad \left. - V_{\alpha i}V^*_{\beta i}V^*_{\alpha j}V_{\gamma j}\left(b^\dagger_\beta(\mathbf{p})b_\gamma(\mathbf{p}) + b^\dagger_\beta(-\mathbf{p})b_\gamma(-\mathbf{p})\right)\right\} \\
& - \left(\frac{m_j \sin(E_j(\mathbf{p})t)}{E_j(\mathbf{p})}\left(\cos(E_i(\mathbf{p})t) + \frac{i|\mathbf{p}|}{E_i(\mathbf{p})}\sin(E_i(\mathbf{p})t)\right)\right) \\
& \times \left\{V^*_{\alpha i}V_{\beta i}V_{\alpha j}V_{\gamma j}\left(a^\dagger_\beta(\mathbf{p})a^\dagger_\gamma(-\mathbf{p}) - a^\dagger_\beta(-\mathbf{p})a^\dagger_\gamma(\mathbf{p})\right)\right. \\
& \qquad \left. - V_{\alpha i}V^*_{\beta i}V^*_{\alpha j}V^*_{\gamma j}\left(b^\dagger_\beta(\mathbf{p})b^\dagger_\gamma(-\mathbf{p}) - b^\dagger_\beta(-\mathbf{p})b^\dagger_\gamma(\mathbf{p})\right)\right\} \\
& + \left(\frac{m_i \sin(E_i(\mathbf{p})t)}{E_i(\mathbf{p})}\left(\cos(E_j(\mathbf{p})t) - \frac{i|\mathbf{p}|}{E_j(\mathbf{p})}\sin(E_j(\mathbf{p})t)\right)\right) \\
& \times \{V^*_{\alpha i}V^*_{\beta i}V_{\alpha j}V^*_{\gamma j}(a_\beta(-\mathbf{p})a_\gamma(\mathbf{p}) - a_\beta(\mathbf{p})a_\gamma(-\mathbf{p})) \\
& \qquad - V_{\alpha i}V_{\beta i}V^*_{\alpha j}V_{\gamma j}(b_\beta(-\mathbf{p})b_\gamma(\mathbf{p}) - b_\beta(\mathbf{p})b_\gamma(-\mathbf{p}))\} \Bigg].
\end{aligned}
$$

independent on Majorana Phases

depend on Majorana Phases and masses

Expectation value of Lepton family Number with a specific initial state

1. Construct the Fock state on a vacuum annihilated by flavor basis operator:

$$a_\alpha(p)|0\rangle = b_\alpha(p)|0\rangle = 0 \quad \alpha = e, \mu, \tau$$

This vacuum differs from the one defined by $a_M(p,\pm)|0_M\rangle=0$.

2. One particle state with of $a^+{}_\alpha(p)|0\rangle, b^+{}_\alpha(p)|0\rangle$ are the lepton family number eigenstates with $L_\alpha=\pm 1$

$a^+{}_M(p)|0_M\rangle$=0does not have a definite lepton number.

Expectation value of Lepton Number with a specific initial state

$$
\begin{aligned}
\langle \sigma, q | L_\alpha(t) | \sigma, q \rangle = & \sum_i |V_{\alpha i}|^2 |V_{\sigma i}|^2 \frac{|\mathbf{q}|^2 + m_i^2 \cos(2E_i(\mathbf{q})t)}{E_i^2(\mathbf{q})} \\
& + \sum_{\{i,j\}} \boxed{\mathrm{Re}\left[V_{\alpha i}^* V_{\sigma i} V_{\alpha j} V_{\sigma j}^*\right]} \left[\cos((E_i(\mathbf{q}) - E_j(\mathbf{q}))t) \left(1 + \frac{|\mathbf{q}|^2 - \boxed{m_i m_j \,\mathrm{Re}\left[\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right]}}{E_i E_j} \right) \right. \\
& \left. + \cos((E_i(\mathbf{q}) + E_j(\mathbf{q}))t) \left(1 - \frac{|\mathbf{q}|^2 - \boxed{m_i m_j \,\mathrm{Re}\left[\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right]}}{E_i E_j} \right) \right] \\
& - \boxed{\mathrm{Im}\left[V_{\alpha 1}^* V_{\sigma 1} V_{\alpha 2} V_{\sigma 2}^*\right]} \sum_{\{i,j\}} \left[\left(\frac{|\mathbf{q}|}{E_i(\mathbf{q})} - \frac{|\mathbf{q}|}{E_j(\mathbf{q})} \right) \sin((E_i(\mathbf{q}) + E_j(\mathbf{q}))t) \right. \\
& + \left(\frac{|\mathbf{q}|}{E_i(\mathbf{q})} + \frac{|\mathbf{q}|}{E_j(\mathbf{q})} \right) \sin((E_i(\mathbf{q}) - E_j(\mathbf{q}))t) \\
& \left. - \boxed{\frac{m_i m_j}{E_i(\mathbf{q}) E_j(\mathbf{q})} \mathrm{Im}\left[\frac{V_{\sigma i}^* V_{\sigma j}}{V_{\sigma i} V_{\sigma j}^*}\right]} \Big(\cos((E_i(\mathbf{q}) - E_j(\mathbf{q}))t) - \cos((E_i(\mathbf{q}) + E_j(\mathbf{q}))t) \Big) \right],
\end{aligned}
$$

Independent on Majorana phases

Depend on K.M. Dirac type CP phase

Dependent on Majorana phases

Properties of Lepton Family Number

1. **Dependence on the sum of the energy and differences;**

$\cos(E_i \pm E_j)t$; $\sin(E_i \pm E_j)t$

and for small momentum, they becom $\cos(m_i \pm m_j)t$ **..**

(Neutrino flavor oscillation probability depends only on the energy difference $E_i - E_j$**)**

2. **The explicit dependence on Majorana phases:** $V_{\sigma i} V^*_{\sigma j}$

(i,j mass eigen-states):

Example;

$$\boxed{\frac{V_{e1} V^*_{e2}}{V^*_{e1} V_{e2}} = e^{-i\,\alpha_{21}}}$$

Bounds on the expectation value:

- **The expectation value of the total lepton number**

$$\sum_{\alpha} \langle \mathbf{q}, \sigma | L_{\alpha}(t) | \mathbf{q}, \sigma \rangle = \sum_{i} |V_{\sigma i}|^2 \left(\frac{|\mathbf{q}|^2 + m_i^2 \cos 2(E_i(\mathbf{q})t)}{E_i^2(\mathbf{q})} \right).$$

$$-1 \leq \sum_{i} |V_{\sigma i}|^2 \frac{|\mathbf{q}|^2 - m_i^2}{|\mathbf{q}|^2 + m_i^2} \leq \sum_{\alpha} \langle \mathbf{q}, \sigma | L_{\alpha}(t) | \mathbf{q}, \sigma \rangle \leq 1.$$

When the momentum of the neutrino is smaller than the lightest neutrino mass, the total lepton number flips its sign from +1 to negative values.

Data used for numerical calculation

Input of neutrino parameters: (NuFIT 5.0(2020), I. Esteban et.al.).

	mass^2 differences	s_{12} s_{23}	s_{13} δ	$(\alpha_{21}, \alpha_{31} - 2\delta)$ $m_{ee}^{max}, m_{ee}^{min}$
normal	$\Delta m_{21}^2 = 7.4 \times 10^{-5}$ $\Delta m_{31}^2 = 2.52 \times 10^{-3}$	0.55 0.76	0.15 1.09π	$(0,0), (\pi,\pi)$ 0.012, 0.0018
inverted	$\Delta m_{21}^2 = 7.4 \times 10^{-5}$ $\Delta m_{23}^2 = 2.50 \times 10^{-3}$	0.55 0.76	0.15 1.57π	$(0,0),\ (\pi,\pi)$ 0.050 ,0.018

We assume the lightest neutrino mass to be 0.01[eV]. From that choice of the lightest neutrino mass each mass eigenvalue m_i[eV] is given by,

$$m_1 = 0.0100, \quad m_2 = 0.0132, \quad m_3 = 0.0512 \quad \text{(normal)},$$

$$m_1 = 0.0502, \quad m_2 = 0.0510, \quad m_3 = 0.0100 \quad \text{(inverted)}.$$

$$V_{\sigma i} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\frac{\alpha_{21}}{2}} & 0 \\ 0 & 0 & e^{i\frac{\alpha_{31}}{2}} \end{pmatrix}$$

Numerical Calculation |q|=0.2(eV) (Relativistic case)

Figure1 normal hierarchy

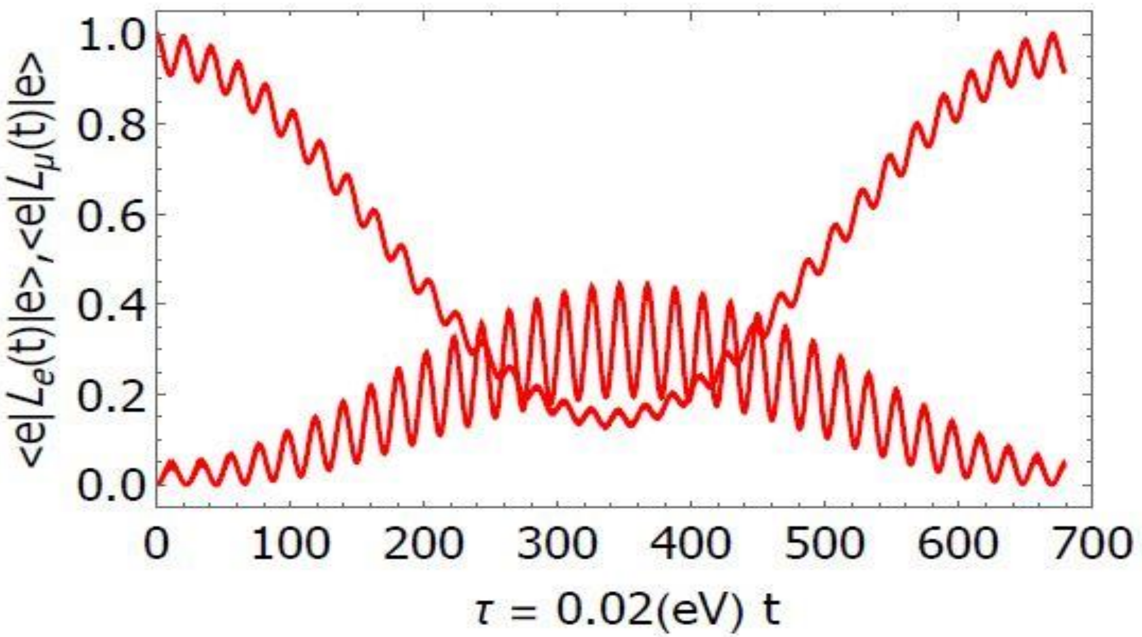
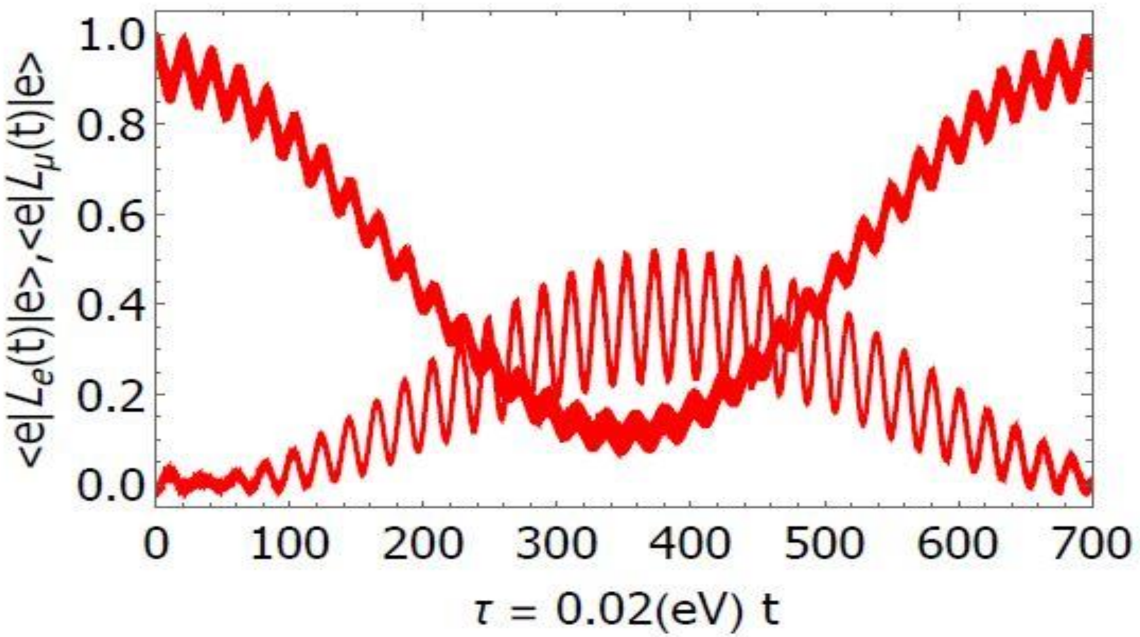


Figure2 inverted hierarchy



(1)The long period $\tau_L = \frac{2\pi}{E_2 - E_1} \simeq 680 \sim 700$, $\tau_{S_{1,2}} = 20.3, 20.9 = \frac{2\pi}{E_3 - E_{1,2}}$.

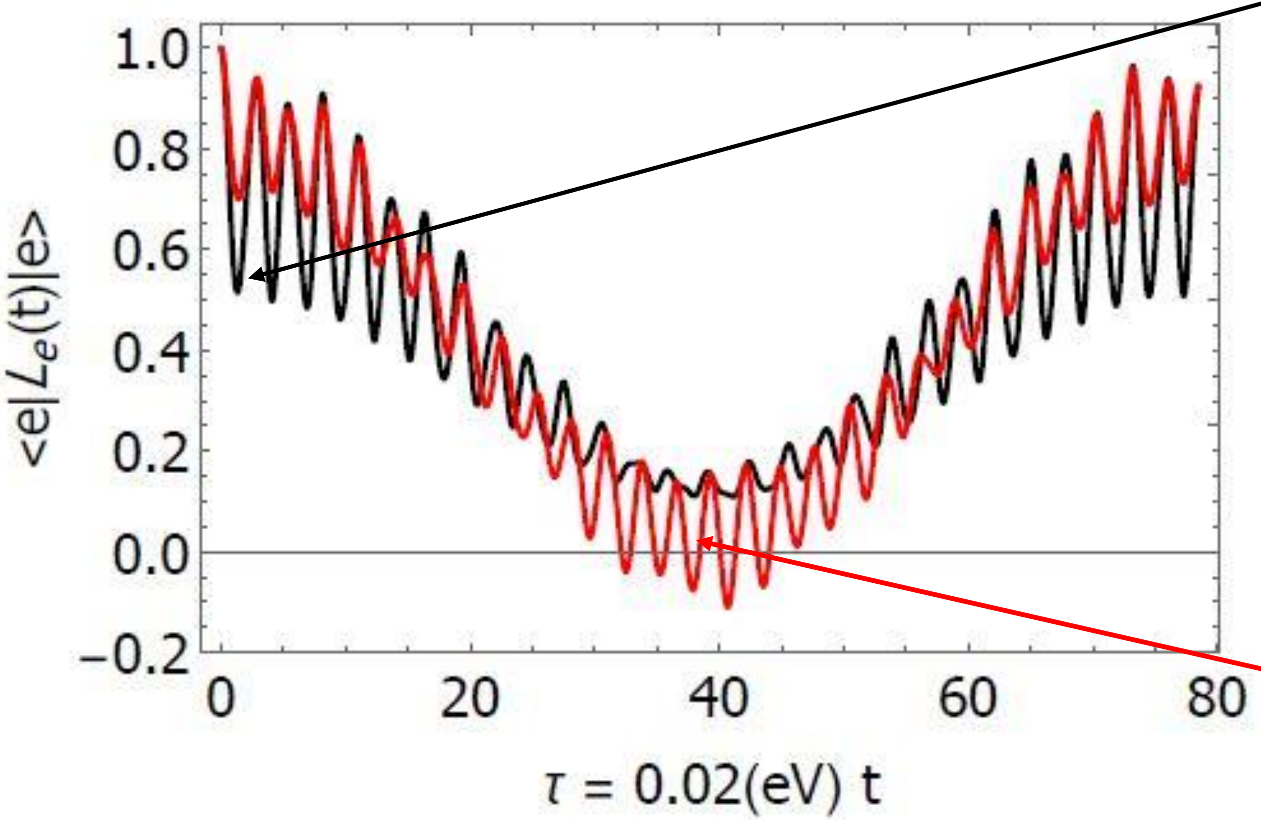
(2)The beat like behavior for muon number with $\tau_{beat} = \frac{\tau_{S_1}\tau_{S_2}}{\tau_{S2} - \tau_{S1}} = \tau_L$.

(3)The lepton family numbers are always within the range $[0, 1]$.

The dependence on Majorana phase:

$m_1 \approx m_2$=**0.01 < q=0.02**$< m_3 = 0.03$

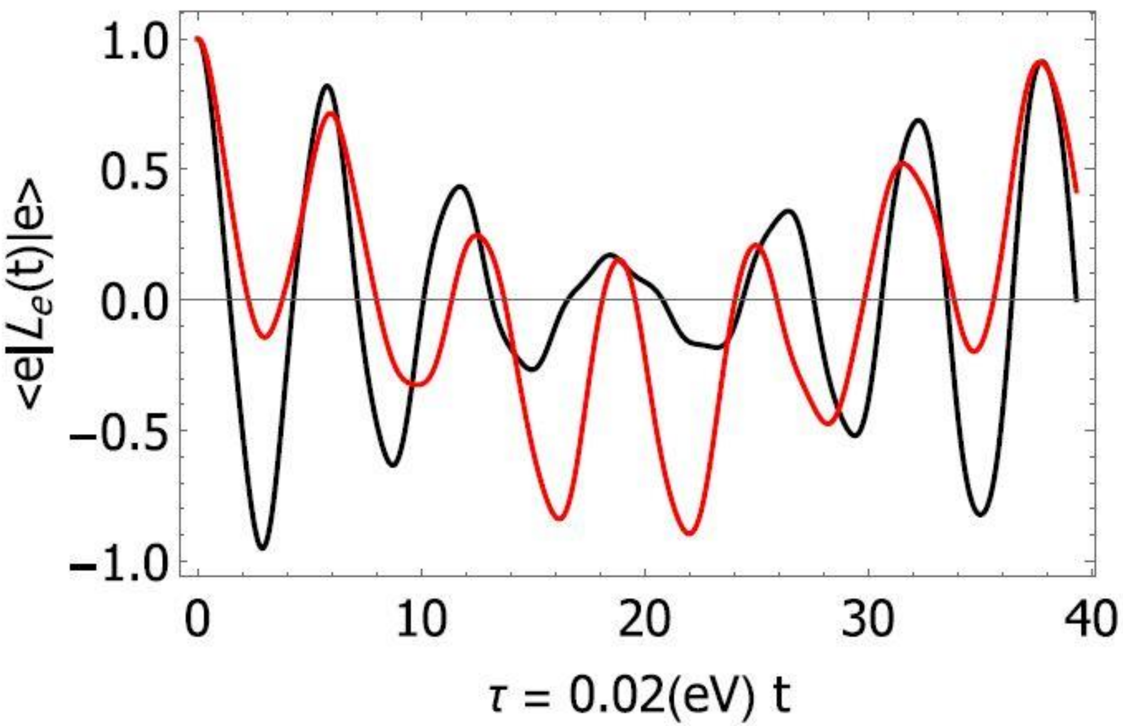
Figure3 $\langle e|L_e(t)|e\rangle$ electron number.



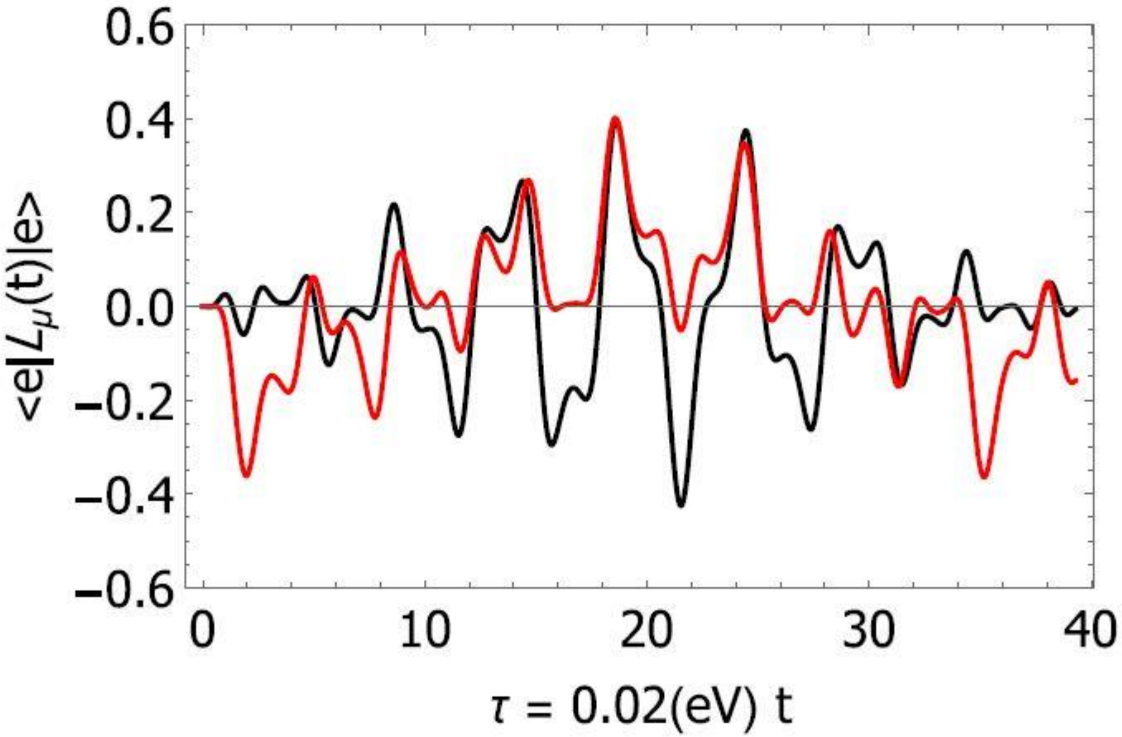
The sharp decrease of the electron number for the largest $|m_{ee}|$.(black curve) $|m_{ee}|$ is the smallest for (red curve).

$\nu_e \to \bar{\nu}_e$

Further decrease of the momentum to q=0.0002(eV). Both electron and muon numbers alternate their sign.



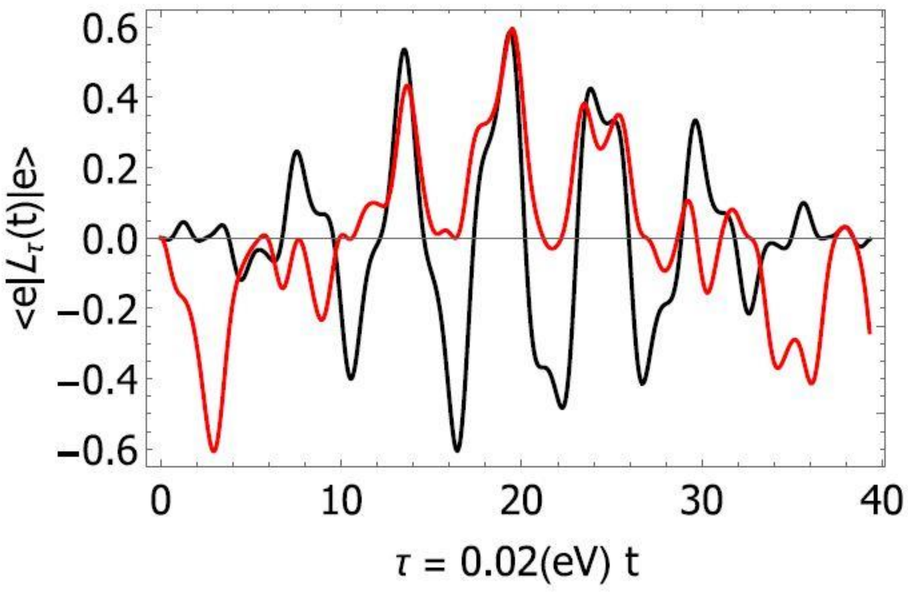
Electron number for normal hierarchy



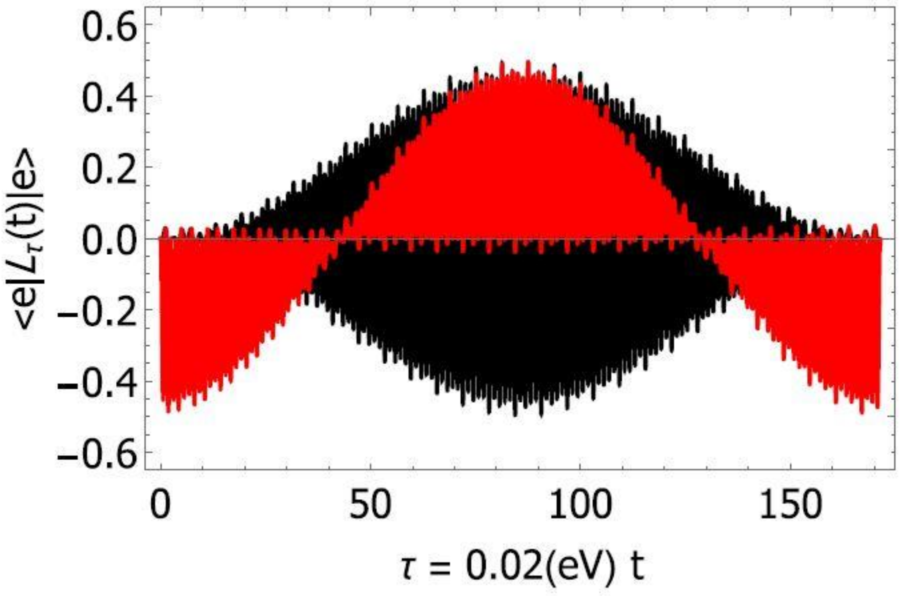
Muon number for normal hierarchy

The shape of the envelopes depends on the choice of Majorana phase.

Dependence on normal or inverted



Tauon number for normal hierarchy



Tauon number for inverted hierarchy

$$\frac{(m_2 + m_1)_{Inverted}}{(m_2 + m_1)_{Normal}} = \frac{(m_2 - m_1)_{Normal}}{(m_2 - m_1)_{Inverted}} = 5$$

Comparison with Dirac Neutrino case

Insight into neutrino mass phenomenology exploring the non-relativistic regime in quantum field theory .
: 2106.02783

$$\mathcal{L}^D = \overline{\nu_{L\alpha}} i\gamma^\mu \partial_\mu \nu_{L\alpha} + \overline{\nu_{R\alpha}} i\gamma^\mu \partial_\mu \nu_{R\alpha} - \theta(t)\left(\overline{\nu_{R\alpha}} m_{\alpha\beta} \nu_{L\beta} + \text{h.c.}\right)$$

Two kinds of lepton family numbers

$$L^L_\alpha \equiv \int d^3x \overline{\nu_{L\alpha}} \gamma^0 \nu_{L\alpha},$$

Can be measured by charged current process

$$L^R_\alpha \equiv \int d^3x \overline{\nu_{R\alpha}} \gamma^0 \nu_{R\alpha},$$

Can not be measured by the SM charged current process. If SU(2)R is also gauged, it can be measured by that interaction.

Results of expectation values with the left-handed state

$$|\sigma_L(\mathbf{q})\rangle = \frac{a^\dagger_{L\sigma}(\mathbf{q})|0\rangle}{\sqrt{\langle 0|a_{L\sigma}(\mathbf{q})a^\dagger_{L\sigma}(\mathbf{q})|0\rangle}},$$

Dirac left-handed

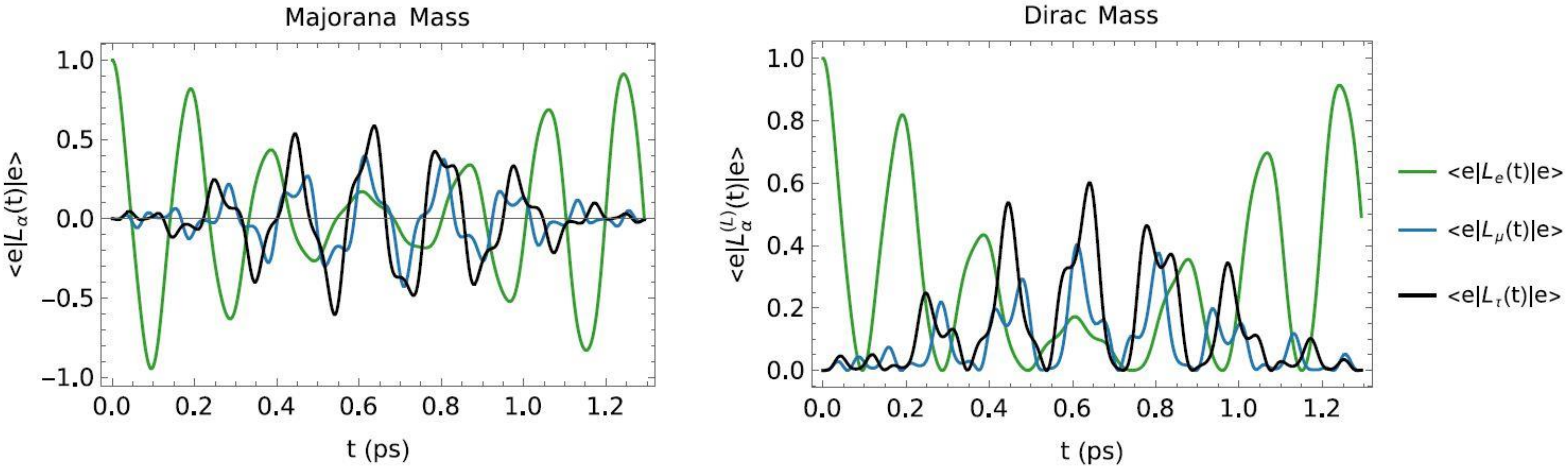
$$\langle\sigma_L(\mathbf{q})|L^L_\alpha(t)|\sigma_L(\mathbf{q})\rangle = \sum_{i,j}^{3}\Big\{\mathrm{Re}(V^*_{\alpha i}V_{\sigma i}V_{\alpha j}V^*_{\sigma j})\left(\cos(E_i(\mathbf{q})t)\cos(E_j(\mathbf{q})t) + \frac{\mathbf{q}^2}{E_i(\mathbf{q})E_j(\mathbf{q})}\sin(E_i(\mathbf{q})t)\sin(E_j(\mathbf{q})t)\right)$$
$$- \mathrm{Im}(V^*_{\alpha i}V_{\sigma i}V_{\alpha j}V^*_{\sigma j})\left(\frac{|\mathbf{q}|}{E_i(\mathbf{q})}\sin(E_i(\mathbf{q})t)\cos(E_j(\mathbf{q})t) - \frac{|\mathbf{q}|}{E_j(\mathbf{q})}\cos(E_i(\mathbf{q})t)\sin(E_j(\mathbf{q})t)\right)\Big\}$$

$$\langle\sigma(\mathbf{p})\,|L^M_\alpha(t)|\,\sigma(\mathbf{p})\rangle = \sum_{i,j}^{3}\Big\{\mathrm{Re}(V^*_{\alpha i}V_{\sigma i}V_{\alpha j}V^*_{\sigma j})\left(\cos(E_i(\mathbf{q})t)\cos(E_j(\mathbf{q})t) + \frac{\mathbf{q}^2}{E_i(\mathbf{q})E_j(\mathbf{q})}\sin(E_i(\mathbf{q})t)\sin(E_j(\mathbf{q})t)\right)$$
$$- \mathrm{Im}(V^*_{\alpha i}V_{\sigma i}V_{\alpha j}V^*_{\sigma j})\left(\frac{|\mathbf{q}|}{E_i(\mathbf{q})}\sin(E_i(\mathbf{q})t)\cos(E_j(\mathbf{q})t) - \frac{|\mathbf{q}|}{E_j(\mathbf{q})}\cos(E_i(\mathbf{q})t)\sin(E_j(\mathbf{q})t)\right)\Big\}$$

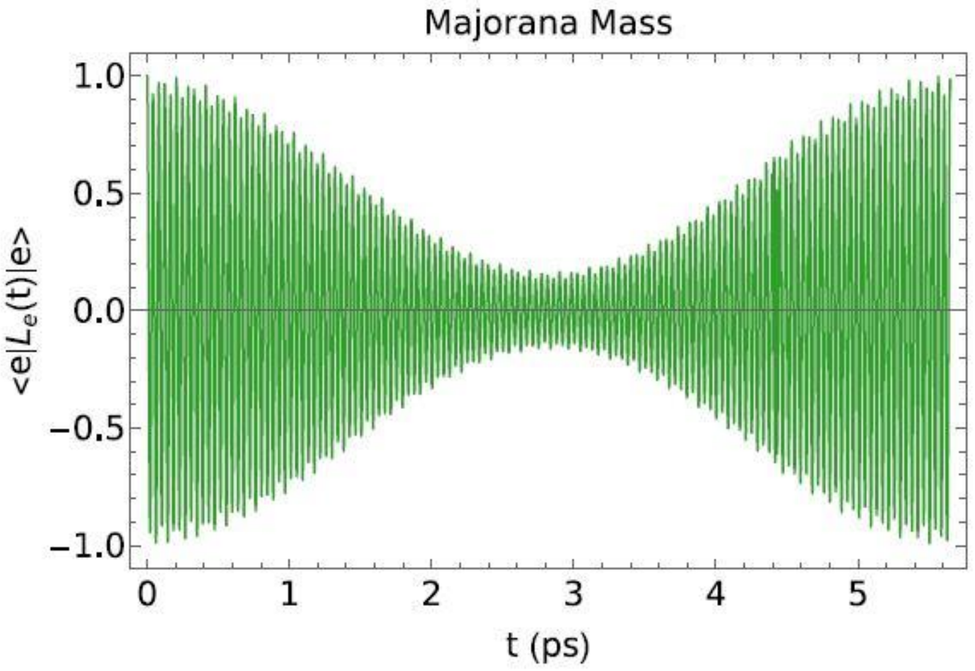
Majorana-Dirac Left-handed difference

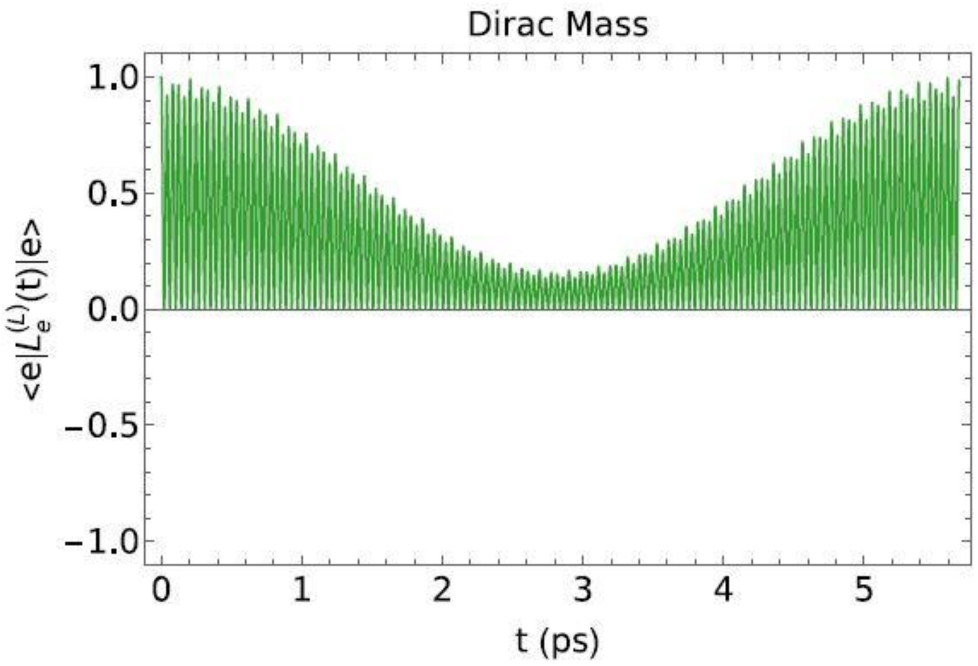
$$\boxed{-\sum_{i,j}^{3}\left\{\mathrm{Re}(V^*_{\alpha i}V^*_{\sigma i}V_{\alpha j}V_{\sigma j})\frac{m_i}{E_i(\mathbf{q})}\frac{m_j}{E_j(\mathbf{q})}\sin(E_i(\mathbf{q})t)\sin(E_j(\mathbf{q})t)\right\},}$$

Numerical comparison:
Majorana vs Dirac left-handed lepton family number
q=0.0002 (eV)
The lightest neutrino mass=0.01(eV)
normal hierarchy case, Majorana phases are set to be zero



Numerical comparison:
Majorana vs Dirac left-handed electron family number
q=0.0002 (eV)
The lightest neutrino mass=0.01(eV)
Inverted hierarchy case, Majorana phases are set to be zero





The momentum variation of electron family number for fixed time.

Normal

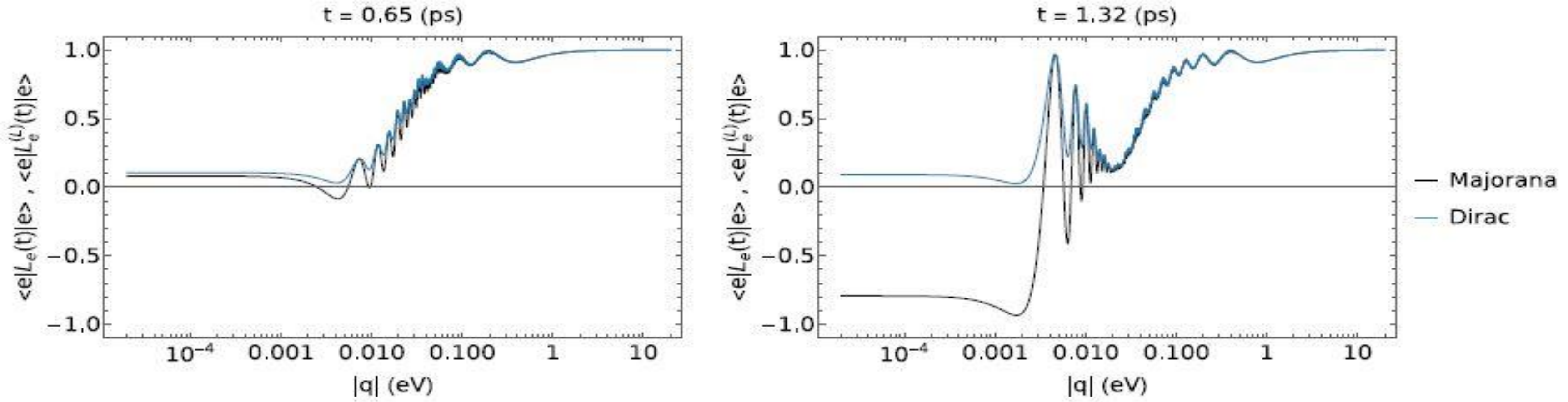


FIG. 3. The momentum variation of the expectation value for the electron neutrino operator at fixed time slices t. In both panels, the normal mass hierarchy is taken and the value of the lightest neutrino mass is 0.01 eV. The values of $t = 0.65$ ps and $t = 1.32$ ps are chosen to emphasize the changes to the steady state value in small momentum region. Both of the Majorana phases are set to zero.

Inverted

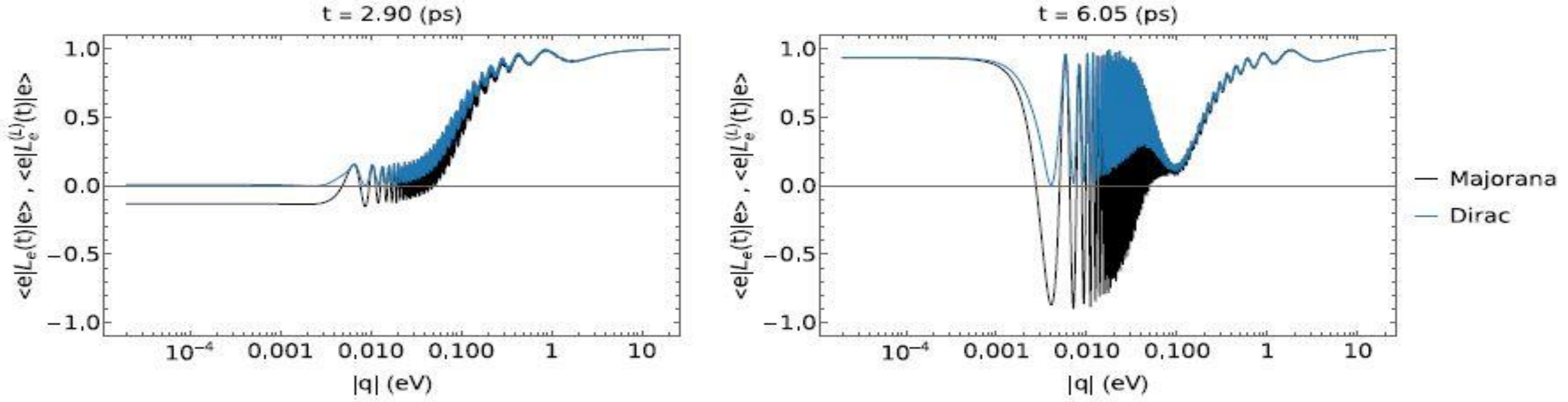


FIG. 4. The momentum variation of the expectation value for the electron neutrino operator at fixed time slices t. In both panels, the inverted mass hierarchy is taken and the lightest neutrino mass is 0.01 eV. The values of $t = 2.90$ ps and $t = 6.05$ ps are chosen to emphasize the changes to the steady state value in small momentum region. Both of the Majorana phases are set to zero.

Conclusion and OutLook

- **We investigated time evolution of lepton family numbers both Majorana neutrinos and Dirac neutrinos.**
- **The formula valid from relativistic regime to non-relativistic regime is obtained.**
- **In non-relativistic regime, lepton family numbers of Majorana neutrinos predict the distinguishable behavior through Majorana phase dependent terms.**

Outlook

1. Application to CNB. (from1 MeV to 2K).

2. Since Majorana neutrinos for the light-neutrinos are predicted by seesaw model, we may have stronger connection of CP violation of leptogenesis through Majorana phases.