EuCAPT

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Majorana neutrino oscillations in a magnetic field and CP violation

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Based on:

[1] A. Popov, A. Studenikin, “Manifestations of nonzero Majorana CP-violating phases in oscillations of supernova neutrinos”, Phys. Rev. D 103 (2021) 11, 115027;
[2] A. Popov, A. Studenikin, "Neutrino eigenstates and flavour, spin and spin-flavour oscillations in a constant magnetic field", Eur. Phys. J. C 79 (2019) 2, 144

Outline:

- Dirac and Majorana neutrinos
- CP-violation in neutrino oscillations
- Oscillations of Majorana neutrino in a magnetic field
- CP-violation in Majorana neutrino oscillations and implications in astrophysics

Majorana theory of fermions

Dirac field: left- and right-handed components are *independent*

$$\Psi_D = \Psi_L + \Psi_R$$

Majorana field: left- and right-handed components are *not independent*

$$\Psi_R = \Psi_L^c$$

$$\Psi_M = \Psi_L + \Psi_L^c$$

Thus, in Majorana case Ψ_L describes particles and Ψ_R describes antiparticles.

For a Majorana field the neutrality condition is satisfied:

$$\boxed{\Psi_M^c = \Psi_M}$$

Within SM neutrinos are the only particles that can potentially be Majorana fermions

Majorana neutrino mixing

Majorana neutrinos mixing is described by the following three relations:

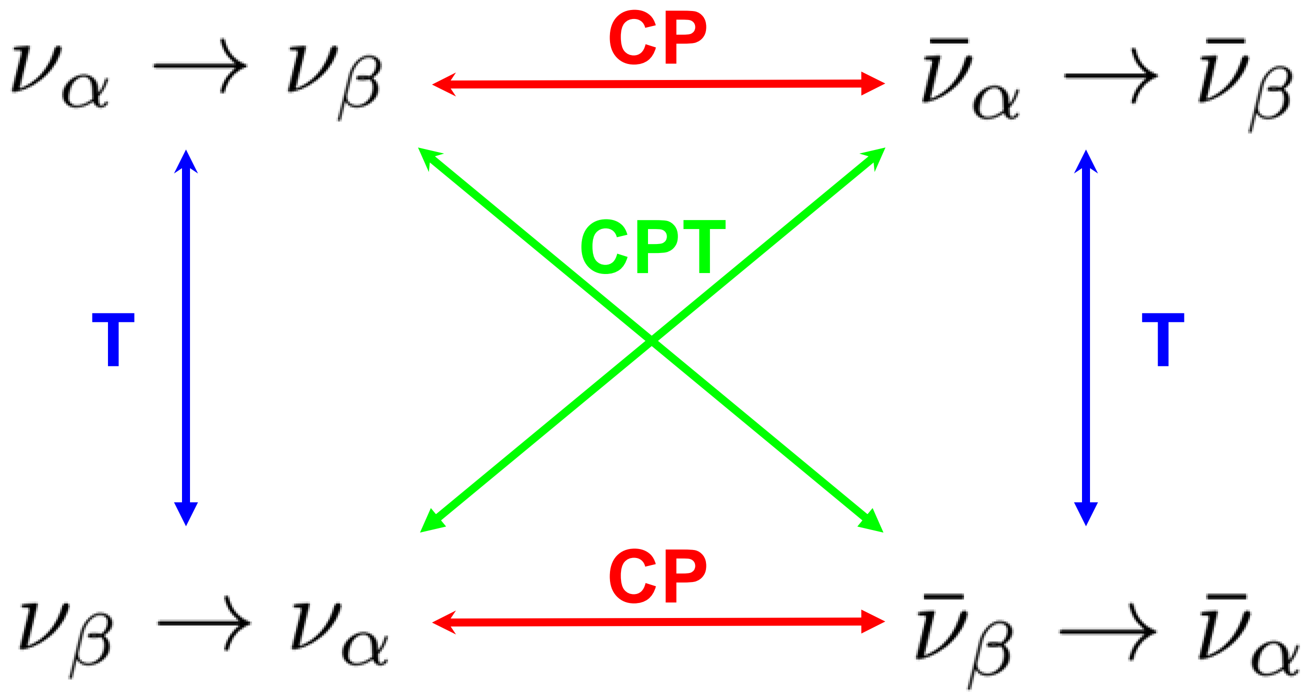
$$\nu_\alpha^L = \sum_i U_{\alpha i} \nu_i^L,$$

$$\nu_\alpha^R = (\nu_\alpha^L)^c = \sum_i U_{\alpha i}^* (\nu_i^L)^c,$$

$$\boxed{\nu_\alpha = \nu_\alpha^L + \nu_\alpha^R = \sum_i U_{\alpha i} \nu_i^L + \sum_i U_{\alpha i}^* (\nu_i^L)^c}$$

In contrast, in Dirac case the mixing matrix for right-handed neutrinos can be chosen arbitrarily.

CP-violation in neutrino oscillations



For a review of leptonic CP-violation see
G.C. Branco, R.Gonzalez Felipe, F.R. Joaquim, "Leptonic CP Violation", Rev.Mod.Phys. 84 (2012) 515-565

$$U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e^{i\alpha_1} & 0 & 0 \\ 0 & e^{i\alpha_2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$c_{ik} = \cos\theta_{ik}$$
$$s_{ik} = \sin\theta_{ik}$$

- δ is the Dirac CP-violating phase

  The Dirac CP-violation phase can be measured in neutrino oscillations:

$$\Delta P_{\alpha\beta} = P(\nu_\alpha \to \nu_\beta) - P(\bar{\nu}_\alpha \to \bar{\nu}_\beta) = 4\sum_{i>j} \mathrm{Im}(U^*_{\beta i} U_{\alpha i} U_{\beta j} U^*_{\alpha j}) \sin\left(\frac{\Delta m^2_{ij} L}{2E}\right)$$

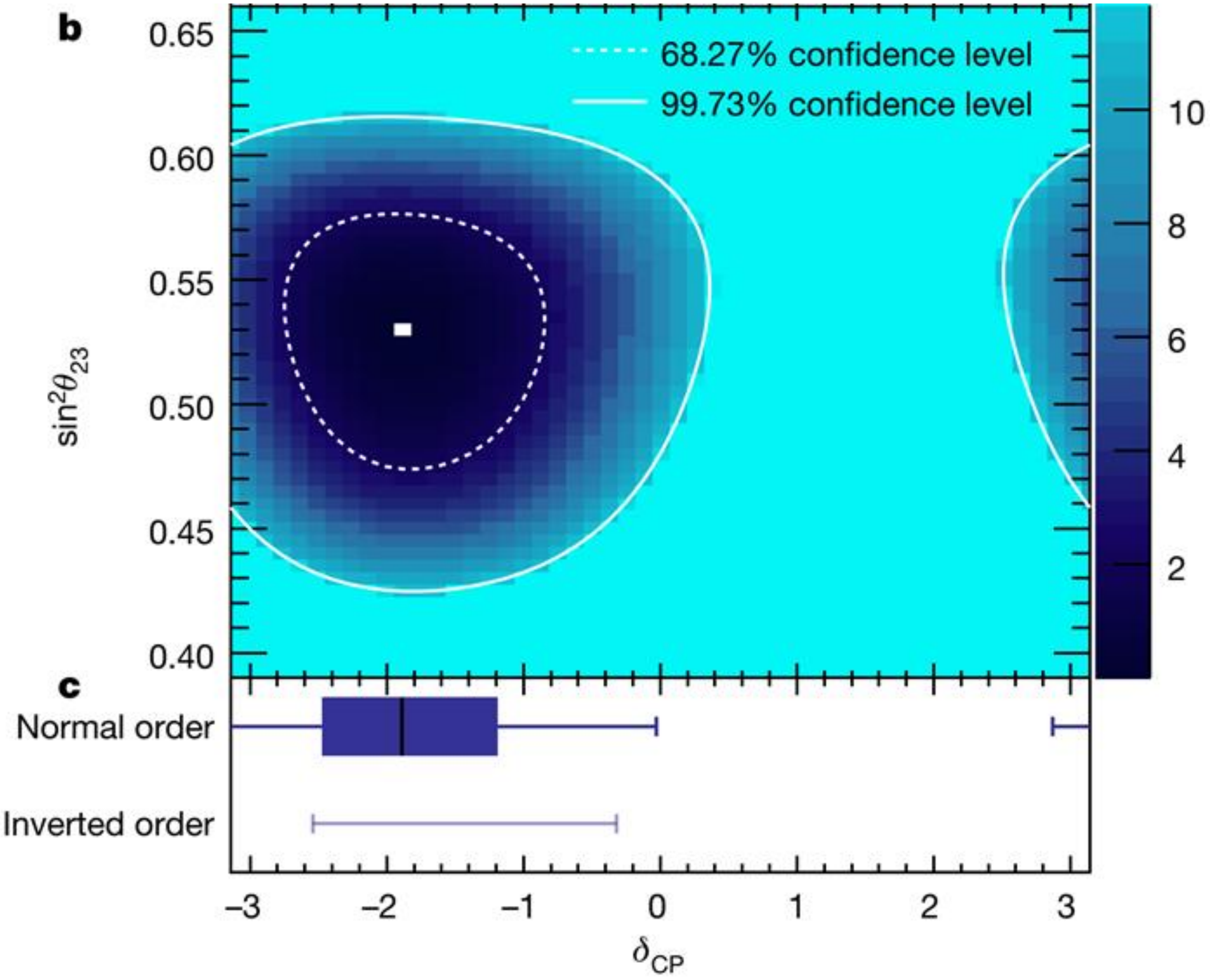
Depends on the Dirac CP-violating phase δ

- α_1, α_2 are the Majorana CP-violating phases

  The experiments on neutrinoless double beta decay are potentially sensitive to the magnitude of $\alpha_1 - \alpha_2$

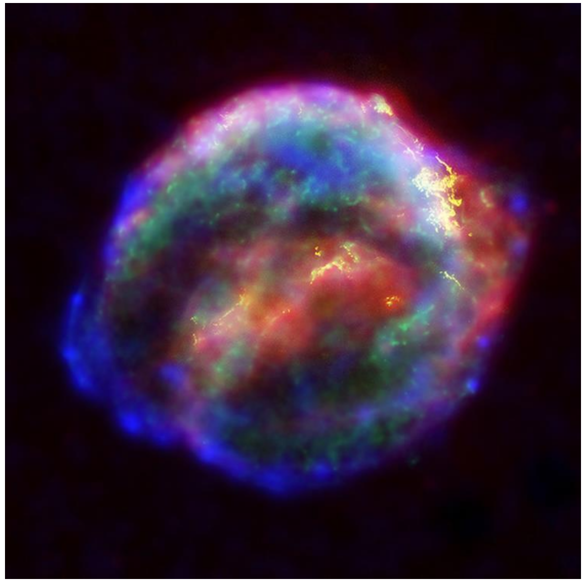
Recent results on the Dirac CP-violating phase

K. Abe et al. [T2K], Nature 580 (2020) no.7803, 339-344



Neutrino oscillations in astrophysics

- Magnetic fields $10^{12}\mathrm{Gauss}$ and more
- Baryon number density $10^{30}\mathrm{cm}^{-3}$ and higher
- Neutrino energies ~10 MeV (for supernova neutrinos)
- Possibly, supernova neutrinos will be detected by the future neutrino experiments, such as JUNO and Hyper-Kamiokande



For a review on topic of supernova neutrinos see
A.Mirizzi, I.Tamborra, H.T.Janka, N.Saviano, K.Scholberg, R.Bollig, L.Hudepohl and S.Chakraborty, Riv. Nuovo Cim. 39 (2016) no.1-2, 1

Majorana neutrino interaction with a magnetic field

$$\boxed{\mathcal{L}_{mag} = -\sum_{i,k} \mu_{ik} \left[\overline{(\nu_i^L)^c} \boldsymbol{\Sigma B} \nu_k^L + \overline{\nu_i^L} \boldsymbol{\Sigma B} (\nu_k^L)^c \right]}$$

Magnetic field can induce neutrino-antineutrino oscillations $\boxed{\nu \to \bar{\nu}}$

Magnetic moments matrix for Majorana neutrinos can be parameterized as follows:

$$\mu_{ij} = \begin{pmatrix} 0 & i|\mu_{12}| & i|\mu_{13}| \\ -i|\mu_{12}| & 0 & i|\mu_{23}| \\ -i|\mu_{13}| & -i|\mu_{23}| & 0 \end{pmatrix}$$

$$\boxed{\mu_\nu^{exp} < 2.8 \times 10^{-11} \mu_B}$$

See *C.Giunti, A.Studenikin, "Neutrino electromagnetic interactions: a window to new physics", Rev.Mod.Phys. 87 (2015) 531* for a review

Majorana neutrino interaction with matter

$$\mathcal{L}_{mat} = -\sum_{\alpha} V_{\alpha}^{(f)} \overline{\nu_{\alpha}} \gamma_0 (1 + \gamma_5) \nu_{\alpha}$$

$$V^{(f)} = \mathrm{diag}\left(\frac{G_F n_e}{\sqrt{2}} - \frac{G_F n_n}{2\sqrt{2}}, -\frac{G_F n_n}{2\sqrt{2}}, -\frac{G_F n_n}{2\sqrt{2}} \right)$$ is the Wolfenstein potential

n_e and n_n are electron and neutron number densities respectively

For Majorana fermions the following relation holds: $\boxed{\bar{\Psi}\gamma_{\mu}\Psi = 0}$

$$\boxed{\mathcal{L}_{mat}^{M} = -\sum_{\alpha} V_{\alpha}^{(f)} \left[\overline{\nu_{\alpha}^{L}} \gamma_0 \nu_{\alpha}^{L} - \overline{(\nu_{\alpha}^{L})^c} \gamma_0 (\nu_{\alpha}^{L})^c \right]}$$

Majorana neutrino oscillations in a magnetic field

Neutrino propagation in a magnetic field and media is described by the following equation:

$$\boxed{(i\gamma^{\mu}\partial_{\mu} - m_i - V_{ii}^{(m)}\gamma^0\gamma_5)\nu_i(x) - \sum_{k\neq i}(\mu_{ik}\boldsymbol{\Sigma}\boldsymbol{B} + V_{ik}^{(m)}\gamma^0\gamma_5)\nu_k(x) = 0}$$

$$V^{(m)} = U^{\dagger}V^{(f)}U$$

The equation can be represented in a Hamiltonian form:

$$i\frac{\partial}{\partial t}\nu(x) = \begin{pmatrix} H_{11} & H_{12} & H_{13} \\ H_{21} & H_{22} & H_{23} \\ H_{31} & H_{32} & H_{33} \end{pmatrix}\nu(x) = H\nu(x) \qquad \nu = (\nu_e, \nu_{\mu}, \nu_{\tau})^T$$

$$H_{ik} = \delta_{ik}\gamma_0\boldsymbol{\gamma}\boldsymbol{p} + m_i\delta_{ik}\gamma_0 + \mu_{ik}\gamma_0\boldsymbol{\Sigma}\boldsymbol{B} + V_{ik}^{(m)}\gamma_5$$

Neutrino oscillations probabilities

$$P(\nu_\alpha^s \to \nu_\beta^{s'}) = \left| \langle \nu_\beta^{s'}(0) | \nu_\alpha^s(x) \rangle \right|^2 = \left| \sum_{i,k} \left(U_{\beta k}^{s'} \right)^* U_{\alpha i}^s \langle \nu_k^{s'}(0) | \nu_i^s(x) \rangle \right|^2$$

$\alpha, \beta = e, \mu, \tau$ $\qquad$ $s, s' = L, R$

Neutrino flavours

Neutrino helicities

(Note that in the Majorana case L and R stand for neutrinos and antineutrinos respectively)

Note that for the Majorana neutrinos case

$$\boxed{U^L = U, U^R = U^*}$$

Neutrino oscillations probabilities

We derive the following expression for the probabilities of neutrino oscillations:

$$\boxed{P(\nu_\alpha^s \to \nu_\beta^{s'}) = \delta_{\alpha\beta}\delta_{ss'} - 4\sum_{n>m} \mathrm{Re}(A_{\alpha\beta nm}^{ss'}) \sin^2\left(\frac{E_n - E_m}{2}\right) x + 2\underbrace{\sum_{n>m} \mathrm{Im}(A_{\alpha\beta nm}^{ss'}) \sin\left(E_n - E_m\right) x}_{\text{T-violating term}}}$$

The expression is valid for the ultrarelativistic case (E >> m)

T-violating term

(See *M.Jacobson, T.Ohlsson, "Extrinsic CPT Violation in Neutrino Oscillations in Matter", Phys. Rev. D 69 (2004), 013003* for a review of extrinsic CPT-violating effects in neutrino oscillation)

$$A_{\alpha\beta nm}^{ss'} = \sum_{i,j,k,l} \left(U_{\beta k}^{s'}\right)^* U_{\alpha i}^{s} U_{\beta l}^{s'} \left(U_{\alpha j}^{s}\right)^* \left(C_{nki}^{ss'}\right)^* C_{mlj}^{ss'}$$

$$H = \sum_n E_n \left|n\right\rangle \left\langle n\right| , H\left|n\right\rangle = E_n \left|n\right\rangle$$

$$C_{nki}^{ss'} = \langle \psi_k^{s'}(0) | P_n | \psi_i^{s}(0) \rangle$$

$$P_n = \left|n\right\rangle \left\langle n\right|$$

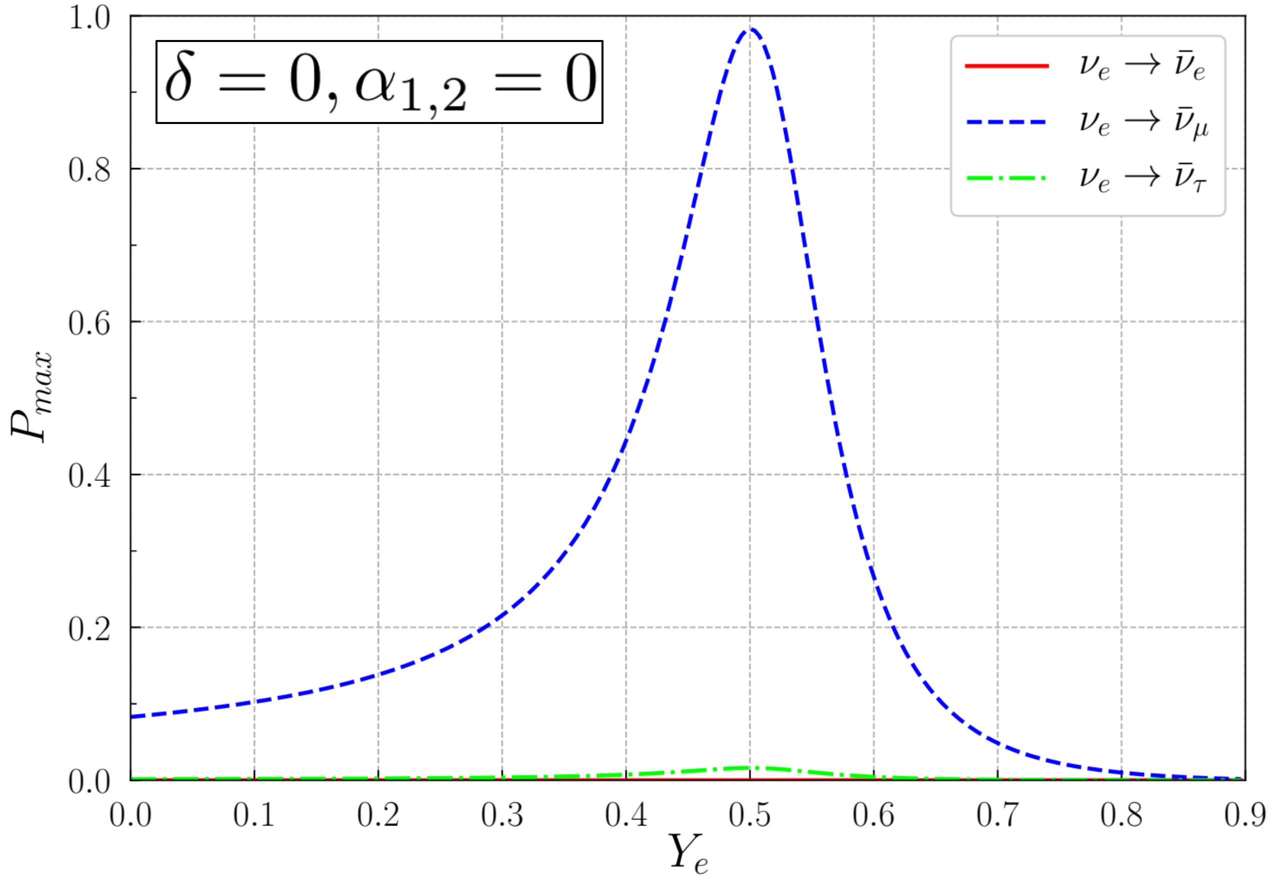
Initial neutrino states

Oscillations amplitudes:

$$P(\nu_\alpha^s \to \nu_\beta^{s'})_{max} = \left(\sum_n |\mathcal{I}_{n\alpha\beta}^{ss'}| \right)^2 ,$$

$$\mathcal{I}_{n\alpha\beta}^{ss'} = \sum_{i,k} (U_{\beta k}^{s'})^* U_{\alpha i}^{s} C_{nki}^{ss'}.$$

Case of absense of CP-violating phases



$$|\mu_{12}| = |\mu_{13}| = |\mu_{23}| = 10^{-12}\mu_B$$

$$n_n = 10^{31}\ \mathrm{cm}^{-3}$$

$$B = 10^{12}\ \mathrm{G}$$

$$E = 10\ \mathrm{MeV}$$

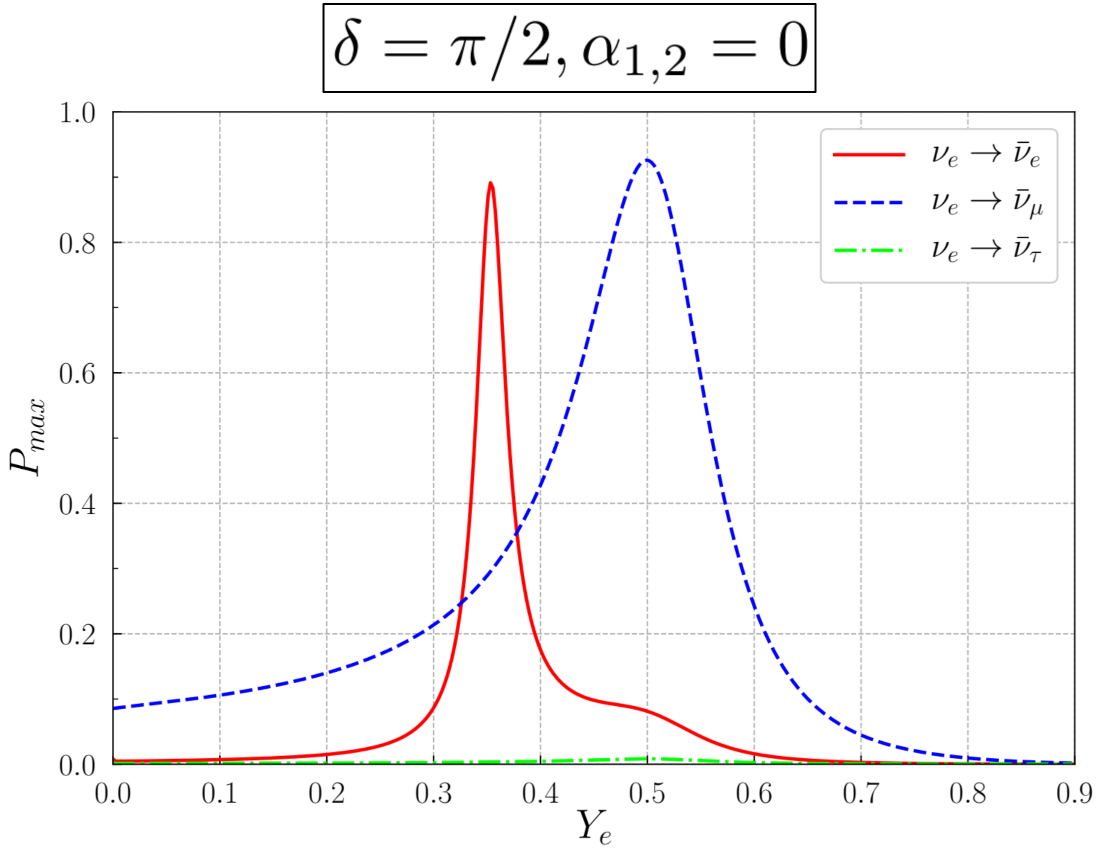
$$Y_e = n_e/n_B,\ n_B = n_n + n_p$$

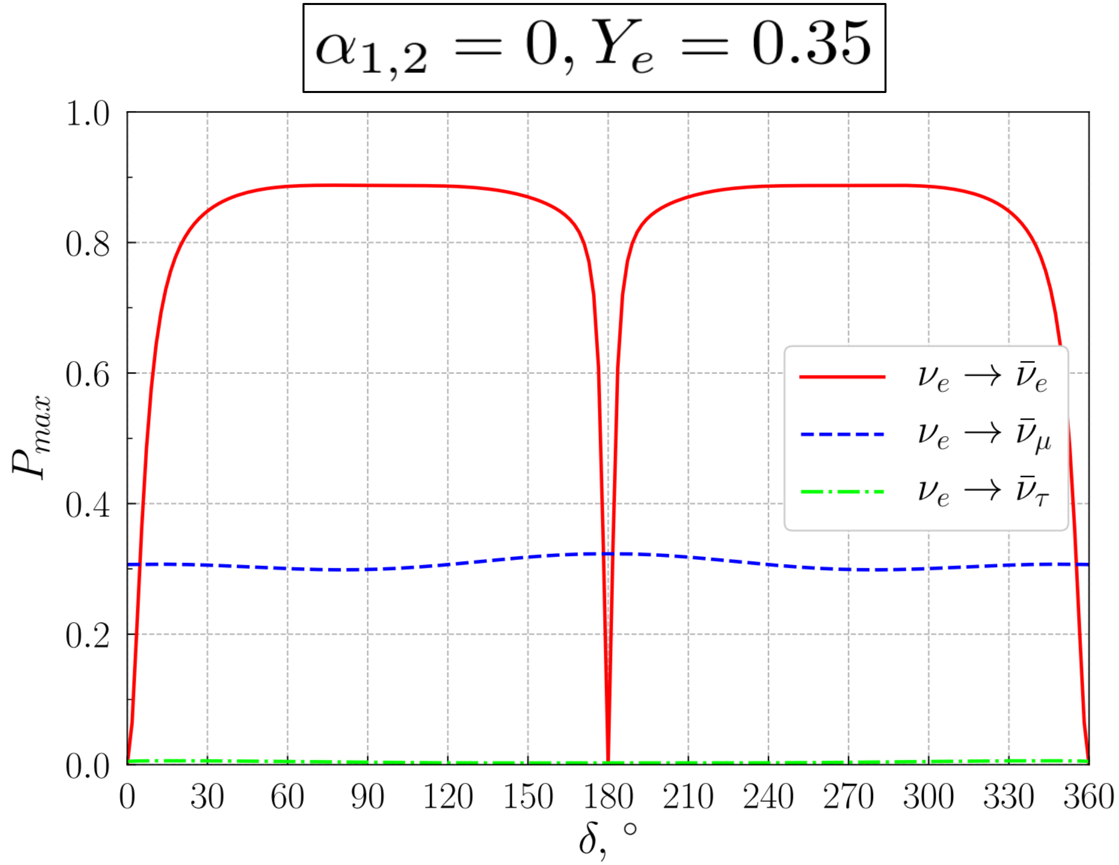
Electron fraction

The amplitudes of neutrino-antineutrino oscillations as functions of the electron fraction Y_e for the case of CP conservation.

See *E. Akhmedov, "Resonant amplification of neutrino spin rotation in matter and the solar-neutrino problem" Phys. Lett. B 213, 64 (1988)*

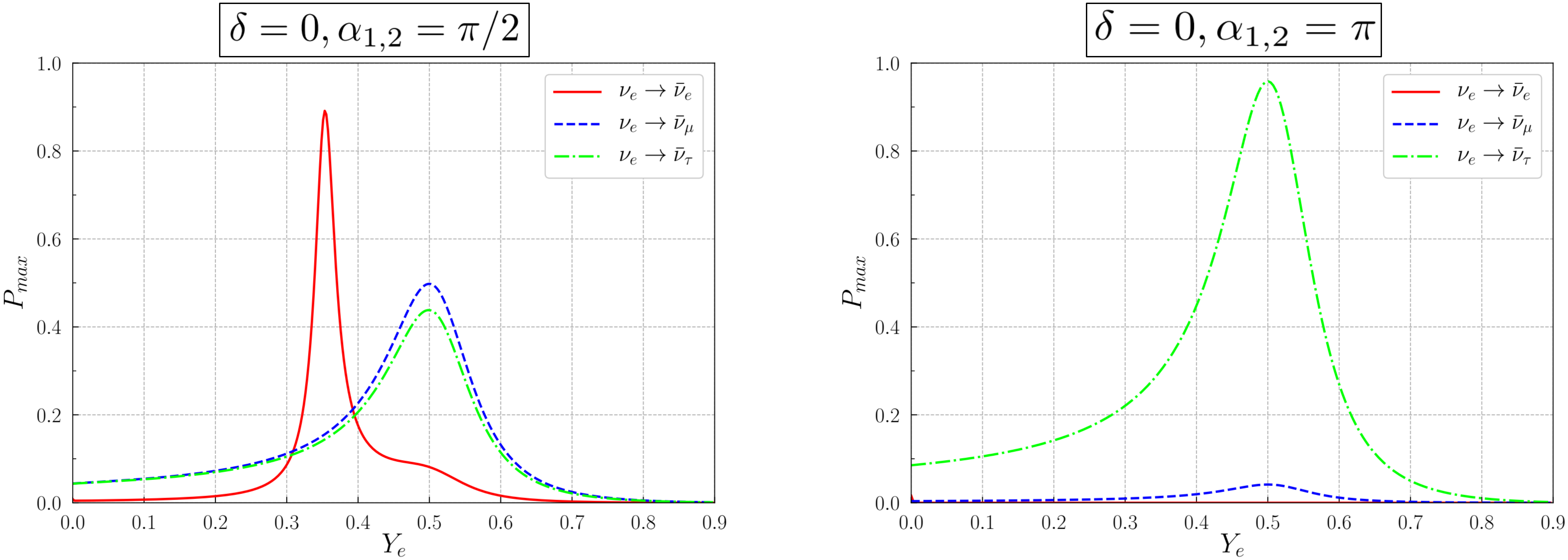
Nonzero Dirac CP-violating phase



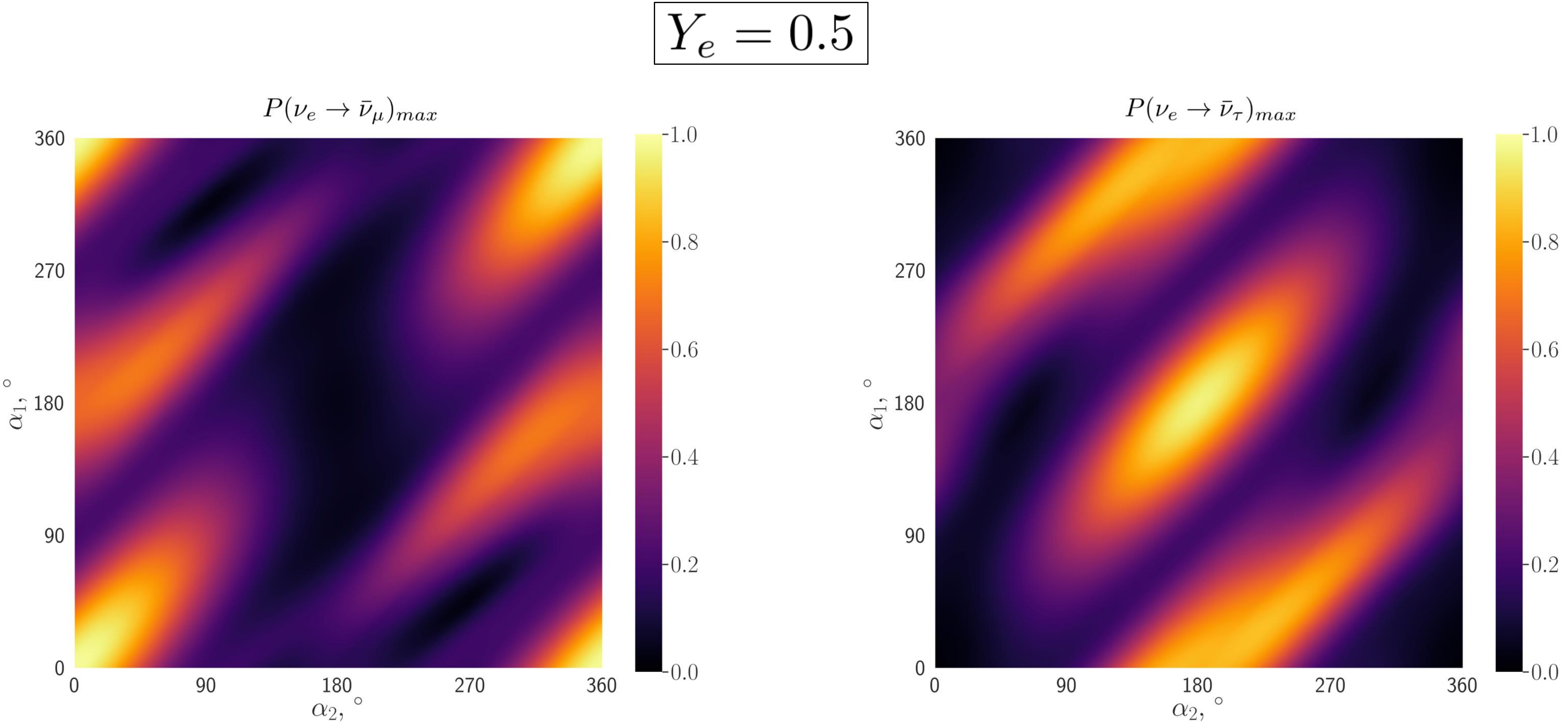


Left: the amplitudes of neutrino-antineutrino oscillations as functions of the electron fraction Y_e for the case of $\delta = \pi/2$, $\alpha_1 = 0$ and $\alpha_2 = 0$. Right: the amplitudes of neutrino-antineutrino oscillations as functions of the Dirac CP-violating phase δ for the case $Y_e = 0.35$, $\alpha_1 = 0$ and $\alpha_2 = 0$.

Nonzero Majorana CP-violating phases



The amplitudes of neutrino-antineutrino oscillations as functions of the electron fraction Y_e for the case of $\delta = 0$. Left: $\alpha_1 = \alpha_2 = \pi/2$. Right: $\alpha_1 = \alpha_2 = \pi$.



Left: The amplitude of $\nu_e \to \overline{\nu}_\mu$ oscillations for the case $Y_e = 0.5$ as a function of the Majorana CP-violating phases α_1 and α_2. Right: Same, but for the amplitude of $\nu_e \to \overline{\nu}_\tau$ oscillations.

Conclusion:

- We find new resonances in neutrino-antineutrino oscillations in a magnetic field in the case of nonzero Dirac and Majorana CP-violating phases

- The resonances appear at $Y_e = 0.35$ and $Y_e = 0.5$

- New resonances possibly can alter evolution of supernova neutrino fluxes and affect the flavour composition

- Thus, we conclude that astrophysical neutrino experiments potentially can be used to probe both the nature of neutrino mass and the presence of leptonic CP-violation