A complete EoS code for neutron star evolution and cooling processes

Including temperature and proton fraction dependence

Raquel Nicolás del Álamo, Martin Veselský, Vlasios Petousis

Ústav Technické a Experimentální Fyziky

České Vysoké Učení Technické v Praze

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Introduction to Neutron Stars

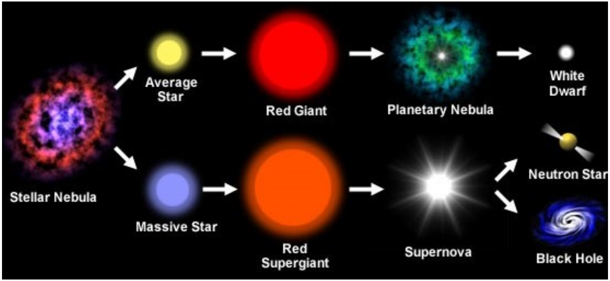

Figure 1: Life cycle of stars.

Source: NASA

Properties:

1. Mass $\sim 1.5\ M_\odot$
2. Radius ~ 12 km
3. Central density, $\mathrm{n}_c \sim 5 - 10\ \mathrm{n}_{sat}$

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Inner structure of NS

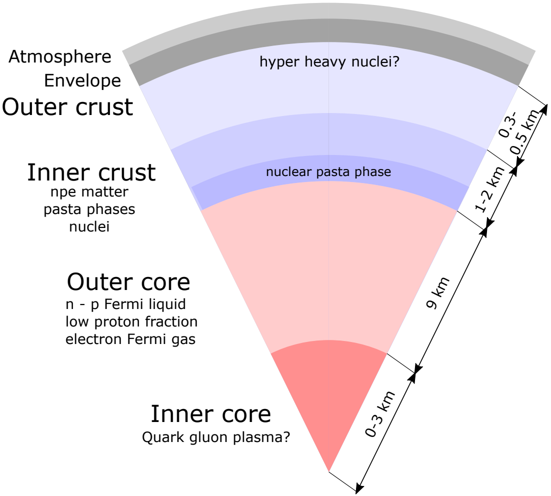

Figure 2: Inner structure of a NS.

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Cooling of neutron stars

URCA cooling

Neutron stars cool down mainly by neutrino emission.
Neutrinos are produced via URCA processes:

$$n \rightarrow p + e^{-} + \bar{\nu}_e$$

$$p + e^{-} \rightarrow n + \nu_e$$

G. Gamow and M. Schoenberg
Neutrino Theory of Stellar Collapse
Phys. Rev. 59, 539 – Published 1 April 1941

The interaction of neutrino with nuclei and "pasta phases" (coherent scattering) could affect the cooling processes, slowing down the cooling of the stars.

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Motivation

Neutron Stars

- Neutron stars are natural nuclear physics laboratories.
- A wide range of the energy spectrum could be analysed: from low energy (plasma) in the outer layers to high energy (quarks, condensed pions and hyperons) in the core.

EoS code development

- Simulations by Martin Veselský and Vlasios Petousis show the **fusion and stability of hyper-heavy nuclei in the crust**.
- The nuclei and the "pasta phases" (predicted by a number of published papers) could have an important contribution to the neutrino opacity, influencing the cooling rates.
- Neutrino opacity and phase transitions have a close relation with the EoS.

Motivation: Simulation results

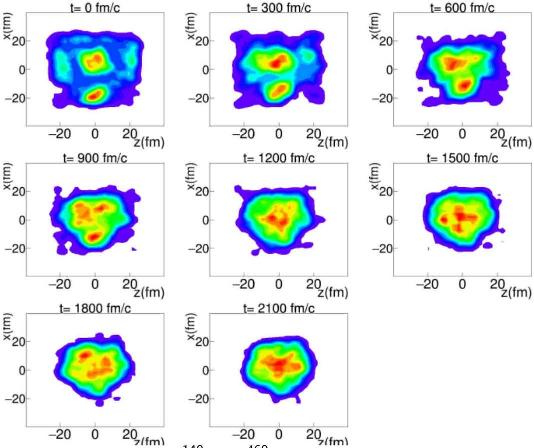

Figure 3: Evolution of the system Ni^{140} + U^{460} in the nucleon bath with Y_p=0.1 (10%) calculated using CoMD code at beam energy 0.1A MeV and Coulomb interaction cutoff 2 fm. Incomprenssibility is K_0= 254 MeV and intermediate density dependence of symmetry energy was used. The initial distance is 25 fm. The nucleons of the box are captured by the compound nucleus.

Simulation provided by Martin Veselský.

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Cold EoS of neutron stars

Energy

$$E = \frac{g_v^2}{2m_v^2}\rho^2 + \frac{g_s^2}{2m_s^2}(m_N - m_N^*)^2 + \frac{\kappa}{6g_s^3}(m_N - m_N^*)^3 + \frac{\lambda}{24g_s^4}(m_N - m_N^*)^4 + \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\sqrt{k^2 + (m_N^*)^2}$$

Pressure

$$P = \frac{g_v^2}{2m_v^2}\rho^2 + \frac{g_s^2}{2m_s^2}(m_N - m_N^*)^2 + \frac{\kappa}{6g_s^3}(m_N - m_N^*)^3 + \frac{\lambda}{24g_s^4}(m_N - m_N^*)^4 + \frac{\gamma}{(2\pi)^3}\int_0^{k_F} d^3k\sqrt{k^2 + (m_N^*)^2}$$

Serot, Brian D. and Walecka, John Dirk
Recent Progress in Quantum Hadrodynamics
International Journal of Modern Physics E, Volume 06, 1997

EoS of neutron stars

Equation expansion around SNM, $Y_P = 0.5$.

Energy

$$E = E_{T=0,\,SNM} + E_{leptons} + E_{sym}\ (1 - Y_P)^2 + E_T$$

$$E_{leptons} = 3\,K\,Y_P^{\frac{4}{3}}\,\rho^{\frac{1}{3}}$$

$$E_{sym} = \eta\,E_{kin} + (S_0 - \eta\,E_{kin}\,(T=0))\left(\frac{\rho}{\rho_0}\right)^{\gamma} \qquad (\eta = 1)$$

$$E_{kin} = 12.5\left(\frac{\rho}{\rho_0}\right)^{\frac{2}{3}}\mathrm{MeV}$$

$$E_T = \frac{4\sigma f_S T^4}{c\rho} + \left\{\frac{2}{3k_B T} + \left[a\left(\rho, Y_p = 0.5, m^*_{SM}\right) + a\left(\rho, Y_P, m_e\right)\right]^{-1} T^{-2}\right\}^{-1}$$

Carolyn A. Raithel, Feryal Ozel, Dimitrios Psaltis
Finite-temperature extension for cold neutron star equations of state
The Astrophysical Journal, Volume 875, 2019

EoS of neutron stars

Equation expansion around SNM, $Y_P = 0.5$.

Pressure

$$P = P_{T=0,\,SNM} + P_{leptons} + P_{sym}\ (1 - Y_P)^2 + P_T$$

$$P = \rho^2 \frac{\partial E}{\partial \rho}$$

$$P_{leptons} = 3K\ Y_P^{\frac{4}{3}}\ \rho^{\frac{4}{3}}$$

$$P_{sym} = \left\{ \frac{2}{3}\eta\, E_{kin} + \left[S_0 - \eta\, E_{kin}\,(T = 0)\right]\gamma \left(\frac{\rho}{\rho_0}\right)^{\gamma} \right\} \rho$$

$$P_T = \frac{4\sigma f_S T^4}{3c} + \left\{ \frac{1}{\rho k_B T} - \left[\frac{\partial a(\rho, Y_P = 0.5, m^*_{SM})}{\partial \rho} + \frac{\partial a(\rho, Y_P, m_e)}{\partial \rho} \right] \rho^{-2} T^{-2} \right\}$$

Carolyn A. Raithel, Feryal Ozel, Dimitrios Psaltis
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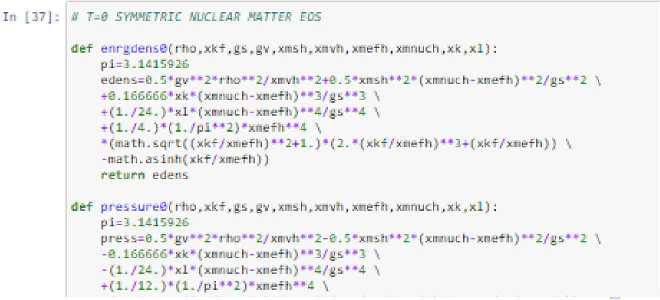
The code

1. Code language: Python.
2. Includes thermal and proton fraction dependence.
3. Can reproduce the whole NS structure in one single code.
4. Although default set of parameters is NL3, other sets of parameters could also be used.
5. Data output from the calculations can be input into Tolman-Oppenheimer-Volkoff (TOV) equations to compare with experimental data (NICER, etc.).



In [37]:

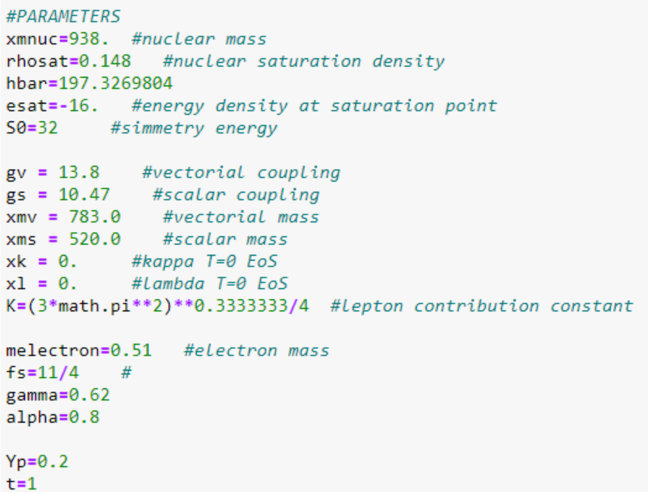
```
# T=0 SYMMETRIC NUCLEAR MATTER EOS

def enrgdens0(rho,xkf,gs,gv,xmsh,xmvh,xmefh,xmnuch,xk,xl):
    pi=3.1415926
    edens=0.5*gv**2*rho**2/xmvh**2+0.5*xmsh**2*(xmnuch-xmefh)**2/gs**2 \
    +0.166666*xk*(xmnuch-xmefh)**3/gs**3 \
    +(1./24.)*xl*(xmnuch-xmefh)**4/gs**4 \
    +(1./4.)*(1./pi**2)*xmefh**4 \
    *(math.sqrt((xkf/xmefh)**2+1.)*(2.*(xkf/xmefh)**3+(xkf/xmefh)) \
    -math.asinh(xkf/xmefh))
    return edens

def pressure0(rho,xkf,gs,gv,xmsh,xmvh,xmefh,xmnuch,xk,xl):
    pi=3.1415926
    press=0.5*gv**2*rho**2/xmvh**2-0.5*xmsh**2*(xmnuch-xmefh)**2/gs**2 \
    -0.166666*xk*(xmnuch-xmefh)**3/gs**3 \
    -(1./24.)*xl*(xmnuch-xmefh)**4/gs**4 \
    +(1./12.)*(1./pi**2)*xmefh**4 \
```

Set of parameters used

Default set: NL3



```
#PARAMETERS
xmnuc=938.  #nuclear mass
rhosat=0.148   #nuclear saturation density
hbar=197.3269804
esat=-16.   #energy density at saturation point
S0=32     #simmetry energy

gv = 13.8    #vectorial coupling
gs = 10.47    #scalar coupling
xmv = 783.0    #vectorial mass
xms = 520.0    #scalar mass
xk = 0.     #kappa T=0 EoS
xl = 0.     #lambda T=0 EoS
K=(3*math.pi**2)**0.3333333/4  #lepton contribution constant

melectron=0.51   #electron mass
fs=11/4    #
gamma=0.62
alpha=0.8

Yp=0.2
t=1
```

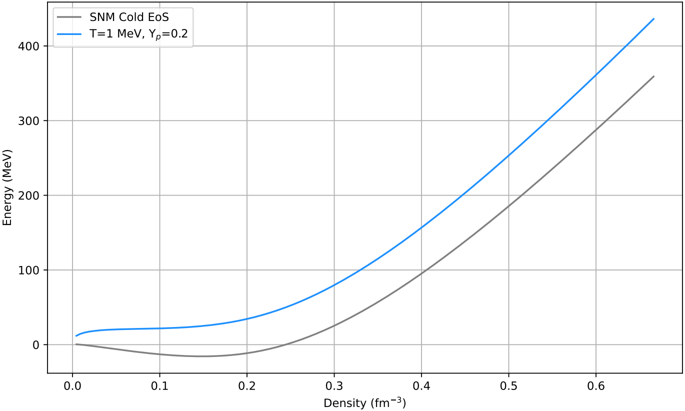
Set of parameters used

Code parameter	Parameter	Default value (NL3)	Units
xmnuc	Nuclear mass	938	MeV
rhosat	Nuclear saturation density	0.148	fm^{-3}
hbar	$\hbar$	197	MeV fm
gv	Vectorial coupling	13.8	1
gs	Scalar coupling	10.47	1
xmv	Vectorial mass	783	MeV
xms	Scalar mass	520	MeV
xk	κ	0	1
xl	λ	0	1
K	Lepton energy associated constant	$\sqrt[3]{3\pi^2}/4$	1
melectron	Electron mass	0.51	MeV
fs	Number of ultra-relativistic species that contribute to thermal pressure	11/4	1
gamma	γ (EoS)	0.62	1
alpha	α (EoS)	0.8	1
Yp	Proton fraction	User defined	1
t	Temperature	User defined	MeV

Table 1: Code parameters and their default values. All of the calculations are made in units of fm.

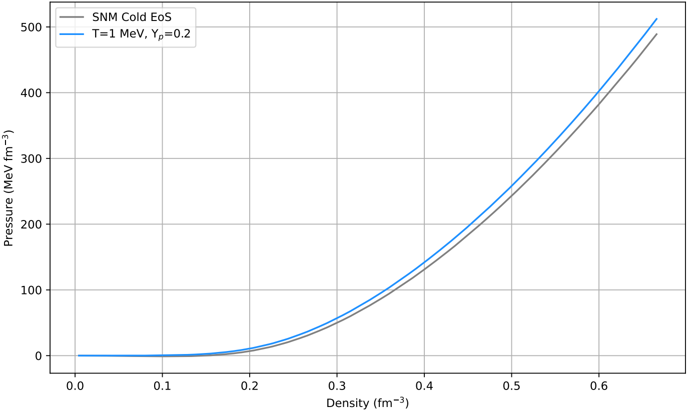
Output

Energy per nucleon (E/A)



Output

Pressure



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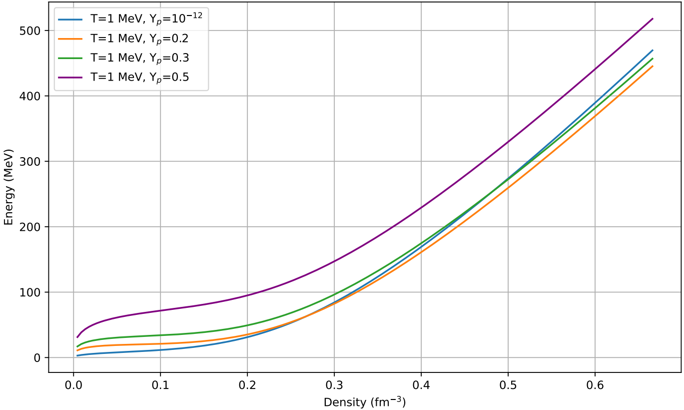
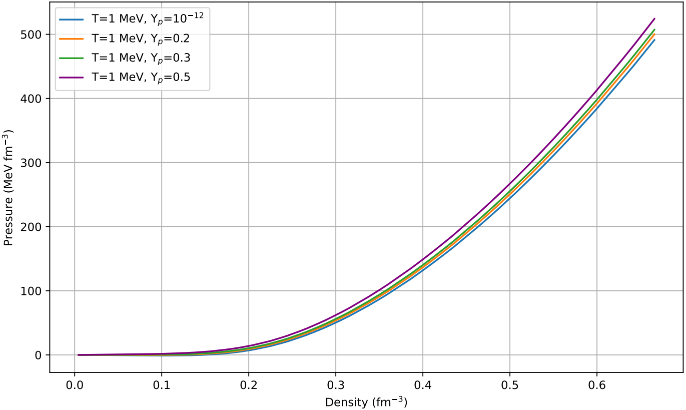
Results

Proton fraction dependence



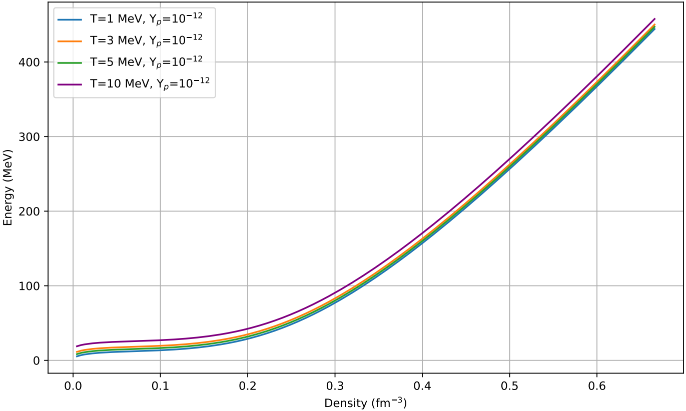
Results

Proton fraction dependence



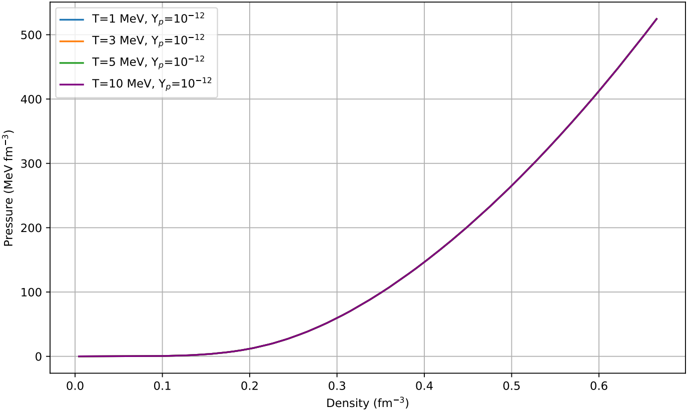
Results

Temperature dependence



Results

Temperature dependence



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About the code

Important remarks

- The code excludes the rotation of the star, which leads to deformability.
- Hyperon and quark matter contributions in the inner core are also excluded.
- Behaviour of matter at the stars core is still under debate, predictions of the EoS are **extrapolations** of the yet known data.

Further work

Tolman-Oppenheimer-Volkoff equations

The EoS output data can be used together with the TOV equations to predict M-R distributions. These results can be compared with experimental data to see if theoretical predictions match the observations.

$$\frac{dP(r)}{dr} = -\frac{G\varepsilon(r)m(r)}{c^2r^2}\left(1+\frac{P(r)}{\varepsilon(r)}\right)\left(1+\frac{4\pi P(r)r^3}{m(r)}\right)\left(1-\frac{2Gm(r)}{r}\right)^{-1}$$

$$\frac{dm(r)}{dr} = \frac{4\pi r^2\varepsilon(r)}{c^2}$$

J. R. Oppenheimer and G. M. Volkoff
On Massive Neutron Cores
Phys. Rev. 55, 374 – Published 15 February 1939

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Questions

THANK YOU

for your attention