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Basics of CMB and structure formation

How neutrinos influence CMB and matter power spectrum

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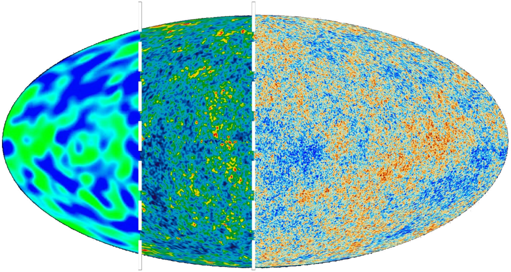
C Cosmic Microwave Background

Based on:

- Lesgourgues+, Neutrino Cosmology
- Planck Collaboration, 2018



Photon decoupling

Photons in equilibrium have $f_\gamma(q) = [\exp(q/T) - 1]^{-1}$

T fluid/photon temperature, q photon momentum

while electrons (e) are free, γ scatter and cannot move freely

when e and protons (p) form H atoms, γs can break atomic bound

H binding energy: $B_{\mathrm{H}} = m_e + m_p - m_{\mathrm{H}} \simeq 13.6$ eV

γs start to move freely when they cannot break H bound anymore

Notice: this depends on photon momentum distribution!

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generic Saha equation: $$\frac{n_c n_d}{n_a n_b} = \frac{\int d^3q e^{-E_c/T} \int d^3q e^{-E_d/T}}{\int d^3q e^{-E_a/T} \int d^3q e^{-E_b/T}}$$

(chemical equilibrium condition)

n_i number densities, E_i energies, T fluid temperature, q momenta

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Saha equation applied to $e + p \leftrightarrow \gamma + H$:

$$\boxed{\frac{n_p n_e}{n_{\rm H}} = \left(\frac{m_e T}{2\pi}\right)^{3/2} \exp\left(-\frac{B_{\rm H}}{T}\right)}$$

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define $X_e \equiv \frac{n_e}{n_e + n_{\mathrm{H}}}$, use $Y_p \equiv \frac{m_{\mathrm{He}} n_{\mathrm{He}}}{m_{\mathrm{N}} n_{\mathrm{B}}} \sim 0.25$, $\eta_{\mathrm{B}} \equiv \frac{n_{\mathrm{B}} - n_{\bar{\mathrm{B}}}}{n_\gamma} \sim 6 \times 10^{-10}$

$$\boxed{\frac{X_e^2}{1 - X_e} = \frac{1}{\eta_{\mathrm{B}}(1 - Y_p)} \left(\frac{m_e}{T}\right)^{3/2} \frac{\sqrt{\pi}}{2^{5/2}\zeta(3)} \exp\left(-\frac{B_{\mathrm{H}}}{T}\right)}$$

Y_p ^{4}He mass fraction, η_{B} baryon-to-photon ratio, $\zeta(3) \simeq 1.202\ldots$

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Y_p ^{4}He mass fraction, η_B baryon-to-photon ratio, $\zeta(3) \simeq 1.202\ldots$

For $T \simeq B_\mathrm{H}$, X_e is close to 1: too many high-E γs break H!

Fraction of free electrons decreases rapidly at $T \simeq 0.3$ eV

At that point (last scattering) photons start to move freely!

Cosmology with perturbations

[see also: Ma&Bertschinger, 1995]

Beyond homogeneous and isotropic universe: add perturbations!

$$\text{metric:}\quad g_{\mu\nu} \quad = \quad \bar{g}_{\mu\nu} \; + \; \delta g_{\mu\nu}$$

extend FLRW: $$ds^2 = a^2(\eta)[-(1+2\psi(\eta,\vec{x}))d\eta^2 + (1-2\phi(\eta,\vec{x}))d\vec{x}^2]$$

Newtonian gauge: ψ (Newtonian potential), ϕ metric perturbations

only scalar, no vector/tensor perturbations!

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stress-energy tensor: $T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$

4 scalars define the T perturbations:

$\delta = \delta\rho/\bar{\rho}$ density contrast

δP pressure perturbations

θ related to bulk velocity divergence

σ anisotropic stress

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Einstein equations (Fourier space):

$$k^2\phi + 3\frac{a'}{a}\left(\phi' + \frac{a'}{a}\psi\right) = -4\pi G a^2 \sum_i \delta\rho_i \quad \text{and} \quad k^2(\phi-\psi) = 12\pi G a^2 \textstyle\sum_i(\bar{\rho}_i + \bar{p}_i)\sigma_i$$

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Perturbed photon distribution:

$$f_\gamma(\eta,\vec{x},\vec{p}) = \left[\exp\left(\frac{y}{a(\eta)\,\bar{T}(\eta)\{1+\Theta_\gamma(\eta,\vec{x},\hat{n})\}}\right) - 1\right]^{-1}$$

$$\Theta'_\gamma + \hat{n}\cdot\vec{\nabla}\Theta_\gamma - \phi' + \hat{n}\cdot\vec{\nabla}\psi = a n_e \sigma_T(\Theta_{\gamma 0} - \Theta_\gamma + \hat{n}\cdot\vec{v}_{\rm B})$$

Cosmic Microwave Background (CMB)

Predicted in 1948 [Alpher, Herman]: blackbody background radiation at $T \simeq 5$ K

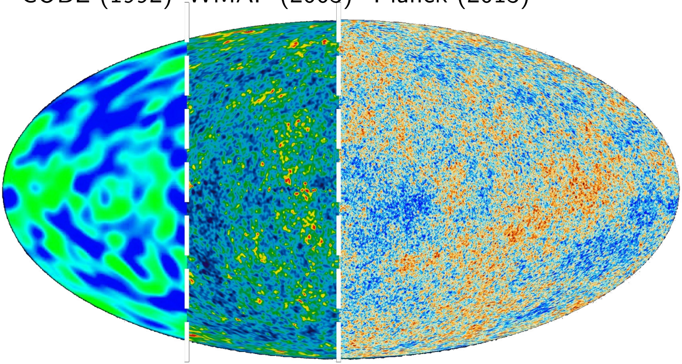
Discovery (accidental): [Penzias, Wilson 1964] ⟶ Nobel prize 1978

perfect black body spectrum at $T_{\rm CMB} = 2.72548 \pm 0.00057$ K [Fixsen, 2009]

Anisotropies at the level of 10^{-5}: very high precision measurements are needed.

Improvement of the CMB experiments in 20 years:

COBE (1992) WMAP (2003) Planck (2013)



Power spectrum

Simplest assumption: only Gaussian fluctuations in the Early Universe

linear theory preserves gaussianity

all Gaussian fluctuations can be described by two-point correlation function

$$\langle A(\eta, \vec{k}) A^*(\eta, \vec{k}') \rangle$$

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stochastic gaussian field $\rightarrow$ uncorrelated wavevectors

$\rightarrow$ Fourier transform equal $\delta^{(3)}(\vec{k} - \vec{k}')$ times power spectrum P_A

Also defined as: $\mathcal{P}_A(k) = \frac{k^3}{2\pi^2} P_A(k)$

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Curvature perturbations: $\mathcal{R} = \psi - \frac{1}{3}\frac{\delta\rho_{\rm tot}}{\bar{\rho}_{\rm tot}+\bar{P}_{\rm tot}}$

Inflation predicts $\mathcal{P}_{\mathcal{R}}(k) = A_s (k/k_0)^{n_s-1}$ as initial spectrum

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Expression for the power spectrum of photon temperature perturbations:

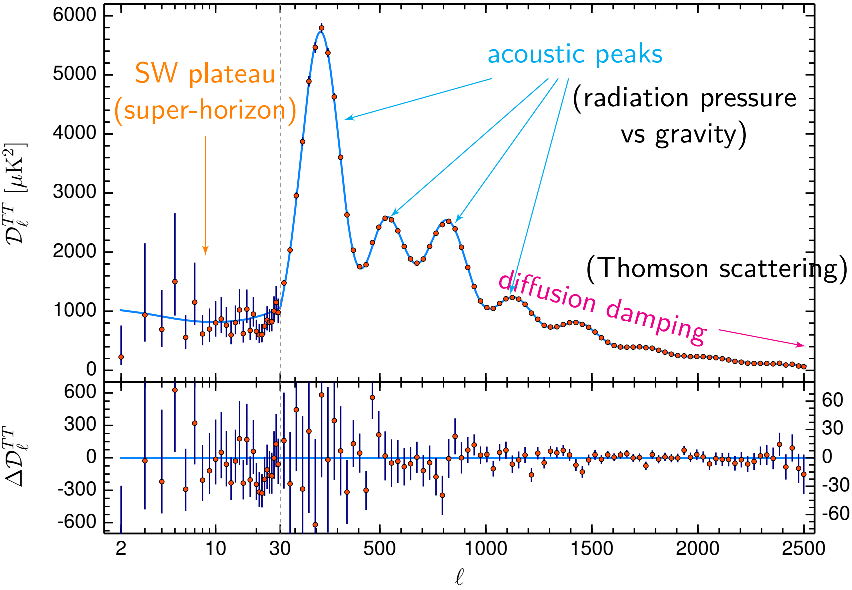
$$\boxed{\langle\Theta_{\gamma l}(\eta,\vec{k})\Theta^*_{\gamma l}(\eta,\vec{k}')\rangle=\frac{2\pi^2}{k^3}\mathcal{P}_\mathcal{R}(k)[\Theta_{\gamma l}(\eta,k)]^2\delta^{(3)}(\vec{k}-\vec{k}')}$$

$\Theta_{\gamma l}(\eta,k)\equiv[\Theta_{\gamma l}(\eta,\vec{k})/\mathcal{R}(\eta_{\rm in},\vec{k})]$ transfer function

Planck DR3 results - Temperature

[Planck Collaboration, 2018]

Planck legacy temperature auto-correlation power spectrum:



Cosmological parameters

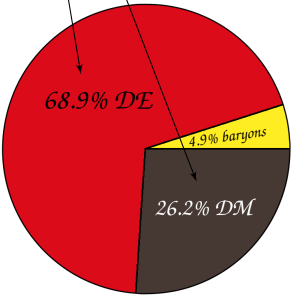
General Relativity + Homogeneity isotropy

$\downarrow$

Cosmological evolution

$\updownarrow$

ΛCDM model



[Planck Collaboration, 2018]

ΛCDM model described by 6 base parameters:

- $\omega_b = \Omega_b h^2$ baryon density today;
- $\omega_c = \Omega_c h^2$ CDM density today;
- τ optical depth to reionization;
- θ angular scale of acoustic peaks;
- n_s tilt and
- A_s amplitude of the power spectrum of initial curvature perturbations.

Other quantities can be studied:

- H_0 Hubble parameter today;
- σ_8 mean matter fluctuations at small scales;
- $\ldots$

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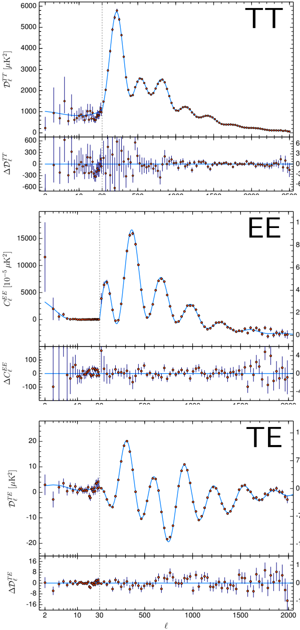
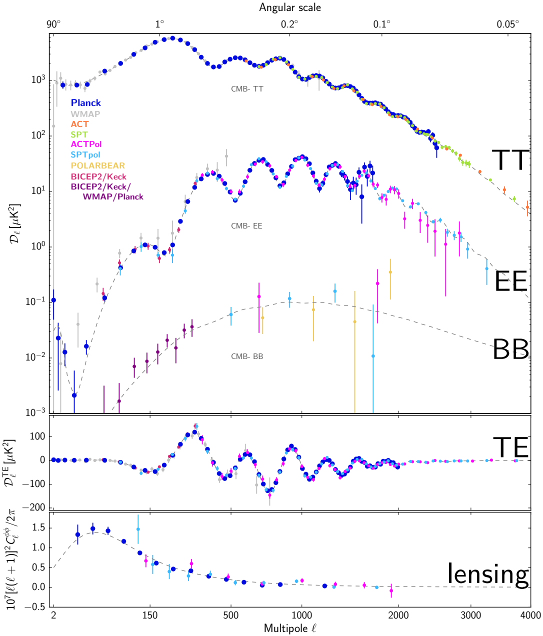
CMB spectra as of 2018

[Planck Collaboration, 2018]

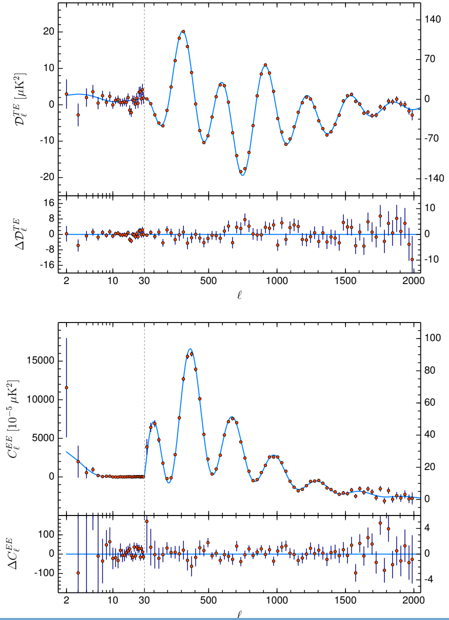




Planck DR3 results - Polarization

[Planck Collaboration, 2018]

- TE cross-correlation and EE auto-correlation measured with high precision;
- ΛCDM explains very well the data;
- Note: in the plots, the red curve is the prediction based on the TT only best-fit for ΛCDM model → very good consistency between temperature and polarization spectra.



M Matter power spectrum

Based on:

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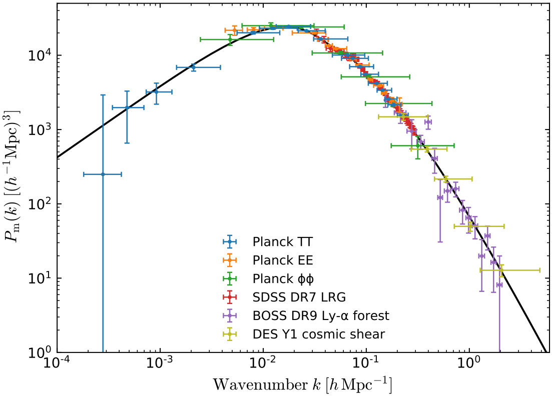

Matter perturbations

What about evolution of matter density perturbations?

$$\langle \delta(\eta, \vec{k}) \delta^*(\eta, \vec{k}') \rangle = \delta^{(3)}(\vec{k} - \vec{k}') P(\eta, k)$$

goal: determine matter power spectrum

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fluctuations with wavelengths k smaller or larger
than the casual horizon behave differently!

large scales
small k $\boxed{\text{superhorizon}}$

grow with expansion of the
universe (no gravity effect)

$\boxed{\text{sub-horizon}}$ small scales
large k

growth from gravitational collapse

balance between expansion
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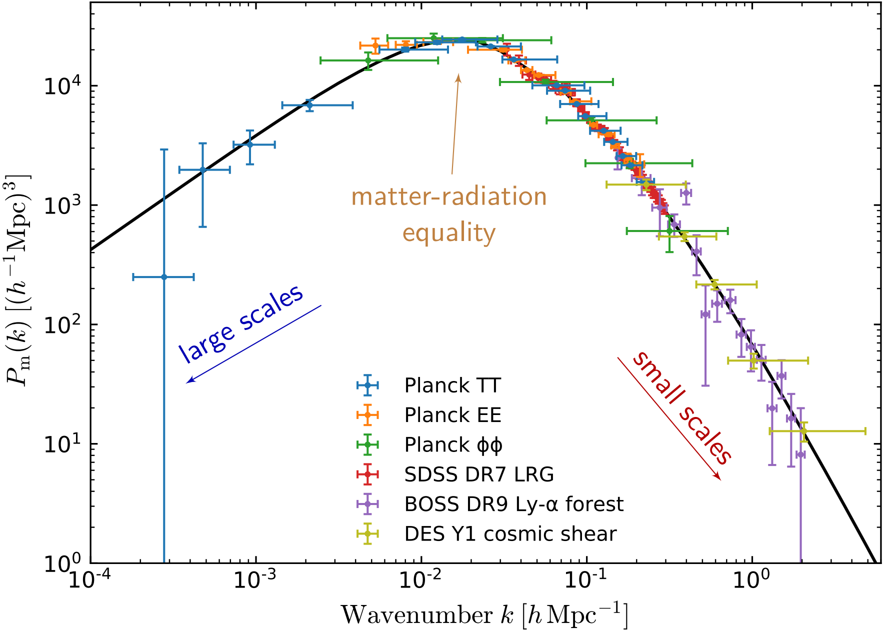
approximated $P(a, k)$ with negligible baryon fraction:

$$P(a,k) = \left(\frac{a}{a_0} \frac{a_M \, \delta_C(a,k)}{a \, \delta_C(a_M,k)} \right)^2 \frac{k \mathcal{P}_{\mathcal{R}}(k)}{(\Omega_m a_0^2 H_0^2)^2} \times \begin{cases} \frac{8\pi^2}{25} & (a_0 H_0 < k < k_{\rm eq}) \\ \frac{k_{\rm eq}^4}{2k^4} \left(\alpha + \beta \ln \left(\frac{k}{k_{\rm eq}} \right) \right)^2 & (k > k_{\rm eq}) \end{cases}$$

(Linear) matter power spectrum

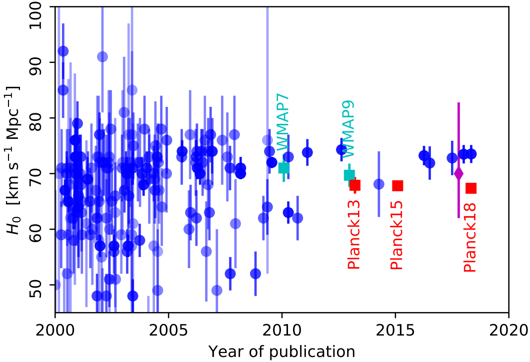
[Planck Collaboration, 2018]



O Other observables

Based on:

- Planck Collaboration, 2018



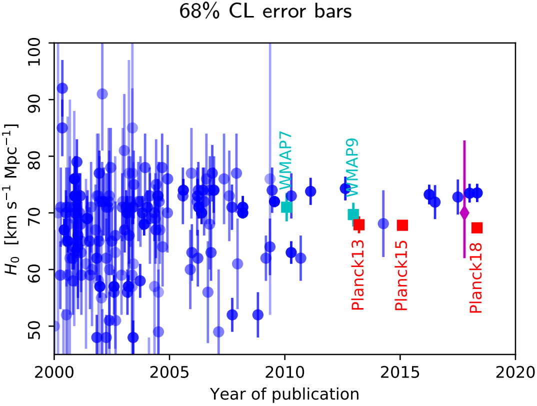
Tension I: the Hubble parameter H_0

[Planck Collaboration, 2018]

$$\boxed{\begin{aligned} v &= H_0 d, \\ \text{with } H_0 &= H(z=0) \end{aligned}}$$

Local measurements:
$H(z=0)$,
local and independent on evolution (model independent, but **systematics?**)

CMB measurements
(probe $z \simeq 1100$):
H_0 from the cosmological evolution (model dependent, well controlled systematics)



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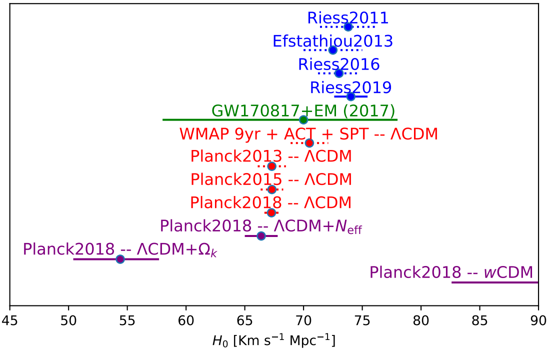
68% CL error bars



Using HST Cepheids:
[Efstathiou 2013] $H_0 = 72.5 \pm 2.5\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$
[Riess+, 2019] $H_0 = 74.03 \pm 1.42\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

GW: [Abbott+, 2017] $H_0 = 70^{+12}_{-8}\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

(ΛCDM model - CMB data only)
[Planck 2013]: $H_0 = 67.3 \pm 1.2\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$
[Planck 2018]: $H_0 = 67.27 \pm 0.60\,\text{Km s}^{-1}\ \text{Mpc}^{-1}$

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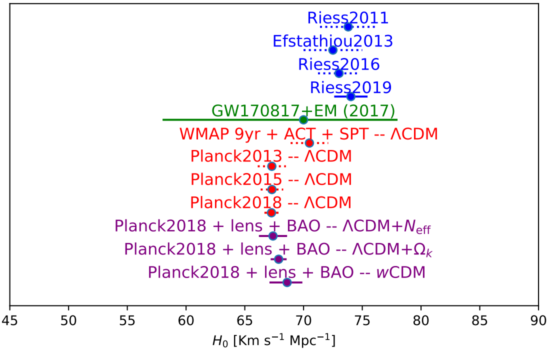

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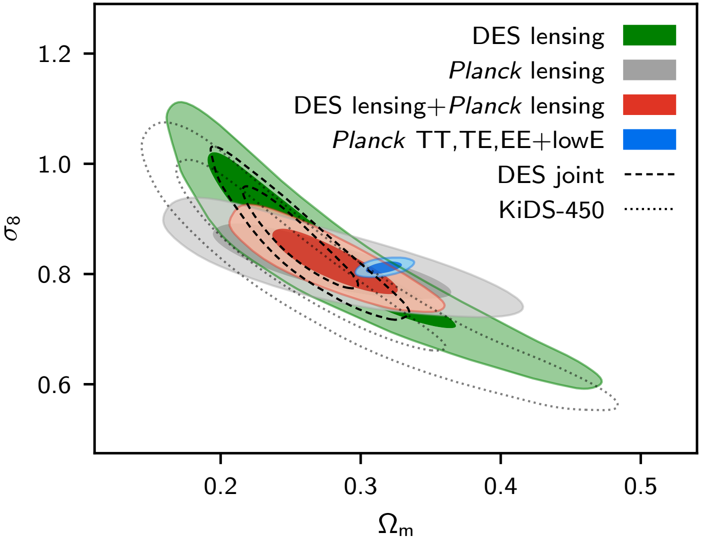
Tension II (?): the matter distribution at small scales

Assuming ΛCDM model:

σ_8: rms fluctuation in total matter (baryons + CDM + neutrinos) in $8h^{-1}$ Mpc spheres, today;

Ω_m: total matter density today divided by the critical density



[Planck collaboration, 2018]

N Neutrinos in cosmology

Impact of neutrinos on CMB and $P_m(k)$?

Based on:

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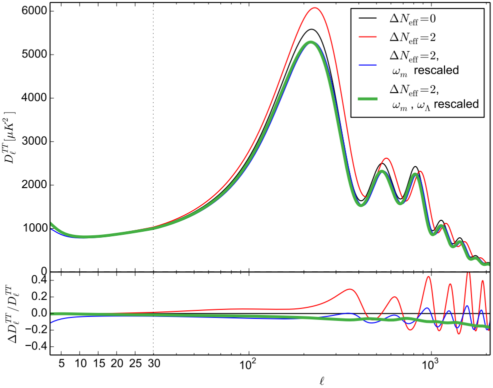
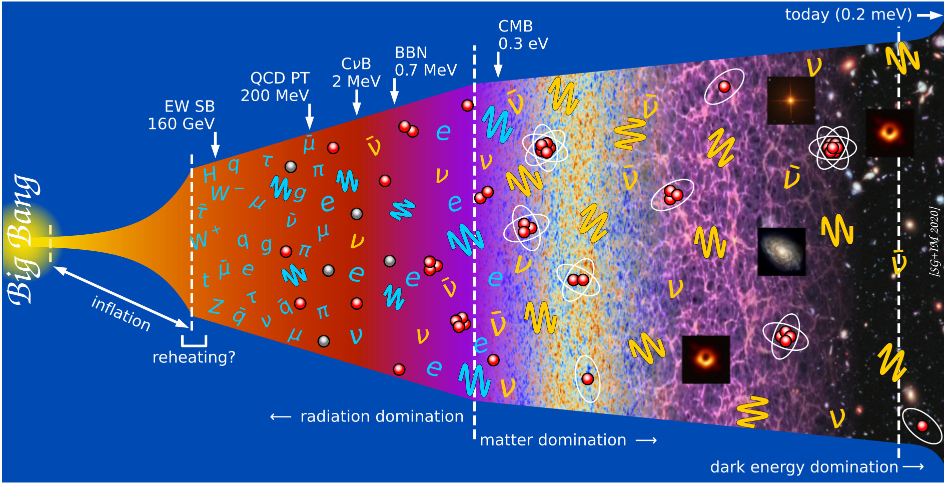


History of the universe



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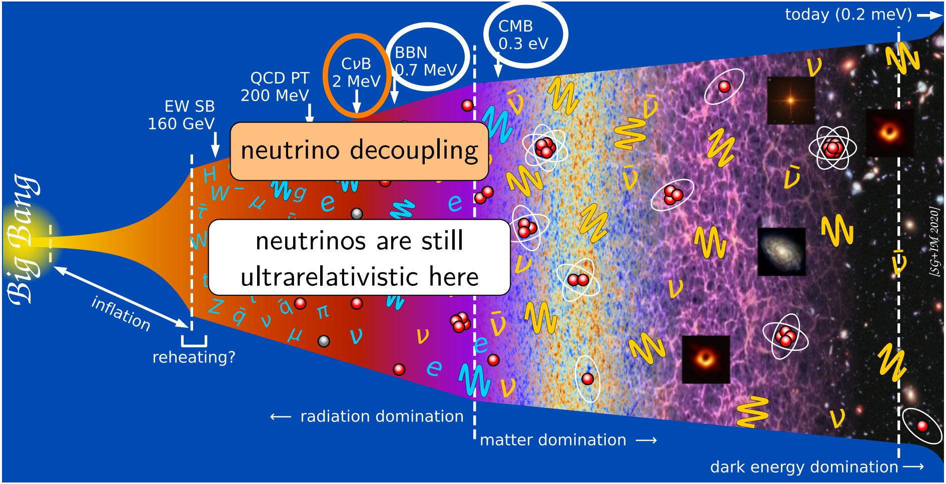

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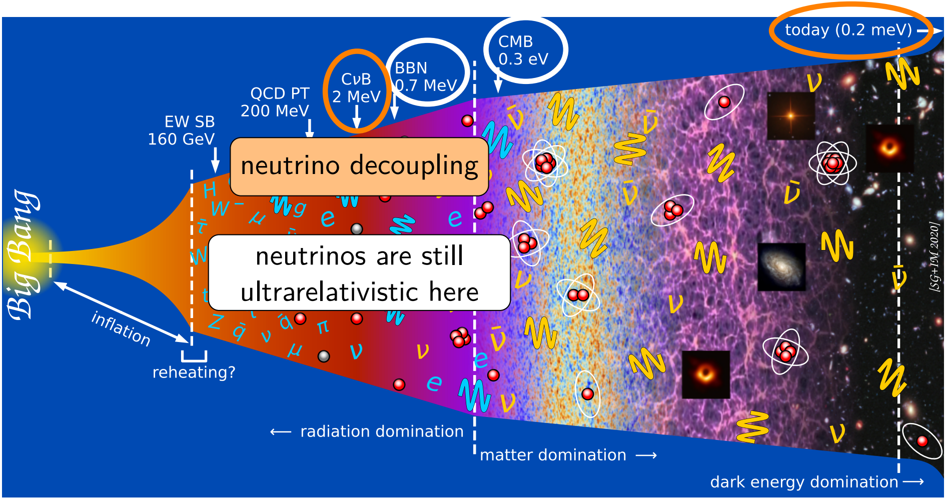


∃ at least 2 mass eigenstates with $m_i \gtrsim 8$ meV $\left(=\sqrt{\Delta m^2_{\rm sol}}\right) > \langle E_\nu \rangle$

→ many relic neutrinos are non-relativistic today!

Additional Radiation in the Early Universe

$$\rho_r = [1 + 0.2271 N_{\rm eff}]\, \rho_\gamma$$

$$H^2 = 8\pi G \rho_T / 3$$

$N_{\rm eff}$ controls the expansion rate H in the early Universe, during radiation dominated phase

influence on

Big Bang Nucleosynthesis: production of light nuclei

abundances today

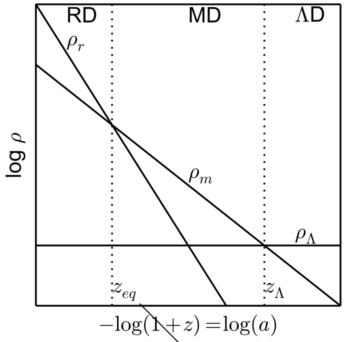
matter-radiation equality

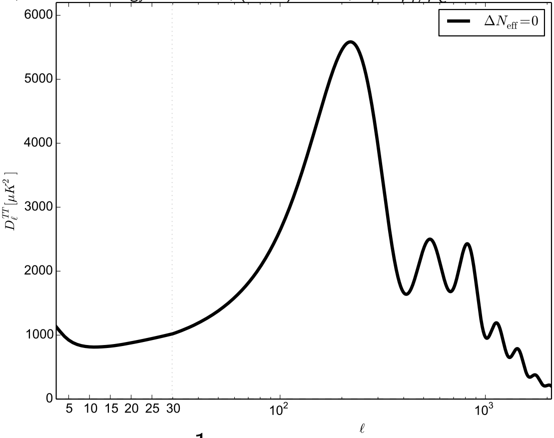
expansion rate at CMB decoupling

Additional Radiation: Effects on the CMB

Starting configuration:

RD: Radiation Dominated, MD: Matter Dominated, ΛD: Dark Energy Dominated; $(1+z) = a^{-1}$; $\omega_i = \rho_i/\rho_C$



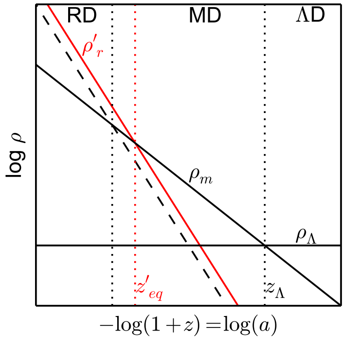


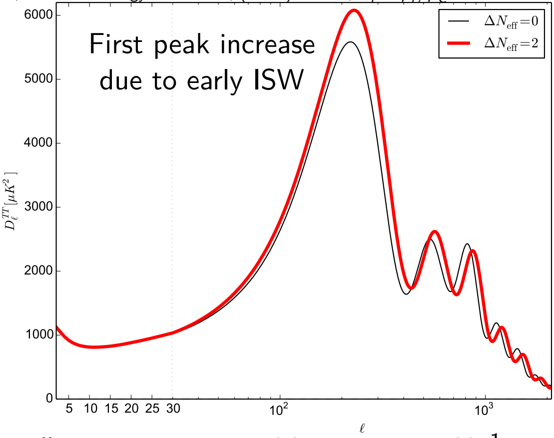
$$1 + z_{\rm eq} = \frac{\omega_m}{\omega_r} = \frac{\omega_m}{\omega_\gamma}\frac{1}{1 + 0.2271\,N_{\rm eff}}$$

Additional Radiation: Effects on the CMB

If we increase $N_{\rm eff}$, all the other parameters fixed:

RD: Radiation Dominated, MD: Matter Dominated, ΛD: Dark Energy Dominated; $(1+z) = a^{-1}$; $\omega_i = \rho_i/\rho_C$





At z_{CMB}: higher $H \propto \rho_r \Rightarrow$ smaller comoving sound horizon $r_s \propto H^{-1}$

$\Rightarrow$ decrease of the angular scale of the acoustic peaks $\theta_s = r_s/D_A$

$\Rightarrow$ shift of the peaks at higher ℓ

Additional Radiation: Effects on the CMB

If we increase $N_{\rm eff}$, plus ω_m to fix $z_{\rm eq}$:

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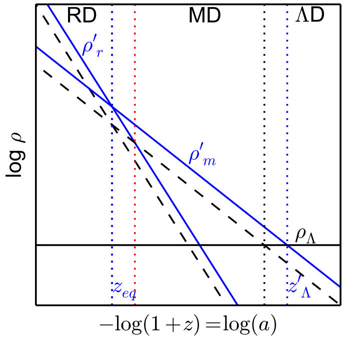
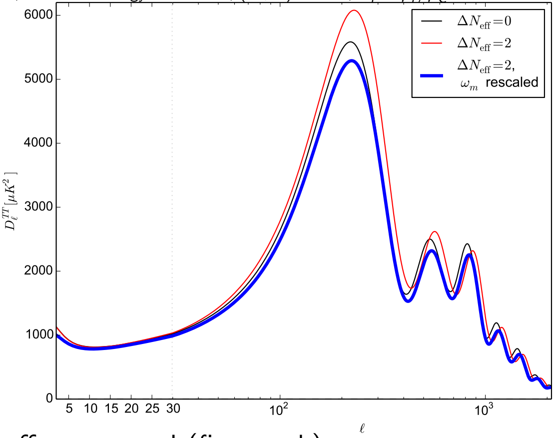


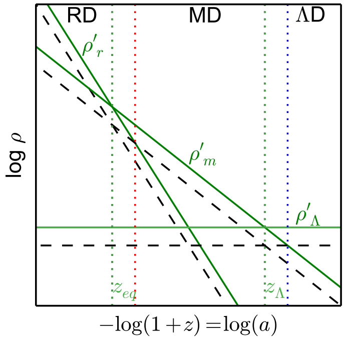


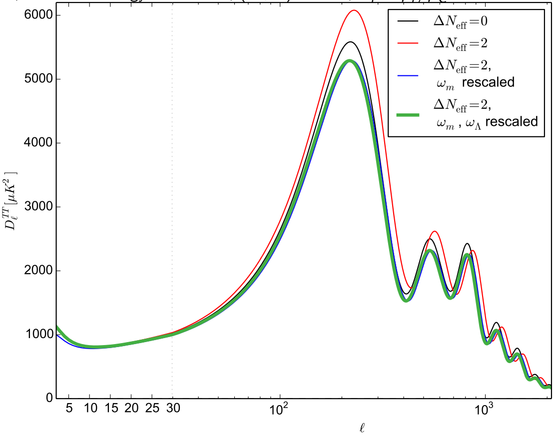
- Contribution from early ISW effect restored (first peak)
- different slope of the Sachs-Wolfe plateau, peak positions, envelope of high-ℓ peaks $\Rightarrow$ due to later z_Λ

Additional Radiation: Effects on the CMB

If we increase $N_{\rm eff}$, plus ω_m, ω_Λ to fix $z_{\rm eq}$, z_Λ:

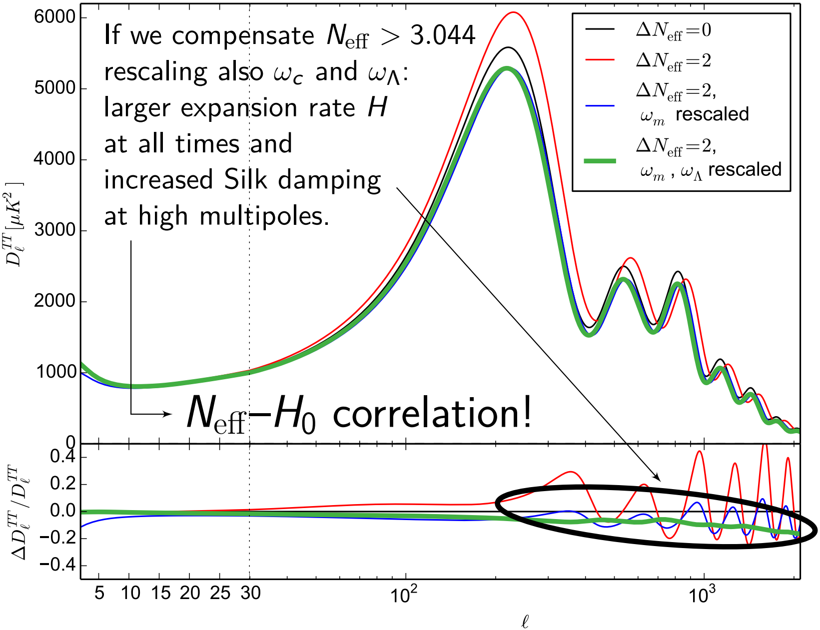
RD: Radiation Dominated, MD: Matter Dominated, ΛD: Dark Energy Dominated; $(1+z) = a^{-1}$; $\omega_i = \rho_i/\rho_C$





- peak positions recovered;
- slope of the Sachs-Wolfe plateau recovered;
- peak amplitude not recovered!

Additional Radiation: Effects on the CMB



Neutrino masses from CMB

ω_i energy density of species i,
$i \in$ (*r*adiation, *m*atter, *b*aryons, *c*old dark matter, ν)
$z_{\rm eq}$ matter-radiation equality redshift

$$1 + z_{\rm eq} = (\omega_b + \omega_c)/\omega_r$$

independent of m_ν

$$\omega_m^0 = \omega_b^0 + \omega_c^0 + \omega_\nu^0 \text{ today}$$

mass of species relativistic at recombination affects late time evolution only

small effects on the SW plateau (cosmic variance, degeneracies...)

Effects on the early ISW effect

$$\frac{\Delta C_\ell}{C_\ell} \simeq -\left(\frac{\sum m_\nu}{0.1\text{ eV}}\right)\%$$

effects on the position of peaks

$$\theta_s = r_s(\eta_{LS})/D_A(\eta_{LS})$$

$$D_A = \int_0^{z_{rec}} \frac{dz}{H(z)}$$

(this effect can be compensated reducing H_0)

correlation m_ν–H_0

[Lesgourgues+, Neutrino Cosmology]

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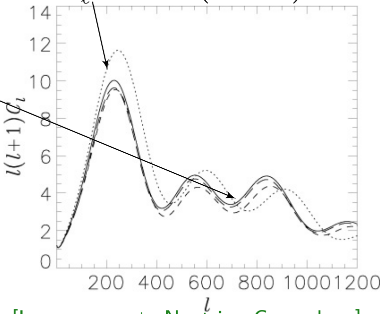


[Lesgourgues+, Neutrino Cosmology]

Free-streaming - I

Non-cold relics $\Longrightarrow$ damping in the perturbations due to free-streaming

Growth equation: $\ddot{\delta} + \underbrace{\boxed{2H\dot{\delta}}}_{\text{Hubble drag}} + \underbrace{\boxed{c_s^2 k^2 \frac{\delta}{a^2}}}_{\text{pressure}} = \underbrace{\boxed{4\pi G_N \rho\, \delta}}_{\text{gravity}}$

Jeans scale: pressure=gravity

$$k_J \equiv \sqrt{\frac{4\pi G_N \rho}{c_s^2 (1+z)^2}}$$

$k \quad < \quad k_J$

growth of density perturbations

$k \quad > \quad k_J$

no growth can occur

neutrino free-streaming scale

$$\boxed{k_{fs}(z) \equiv \sqrt{\frac{3}{2}\frac{H(z)}{(1+z)\sigma_{v,\nu}(z)}} \simeq 0.7 \left(\frac{m_\nu}{1\ \text{eV}}\right) \sqrt{\frac{\Omega_M}{1+z}}\, h/\text{Mpc}}$$

ρ energy density of a given fluid
$\delta = \delta\rho/\rho$ perturbation (single fluid)
c_s sound speed of the fluid

$\sigma_{v,\nu}(z)$ ν velocity dispersion
$H = H(z)$ Hubble factor at redshift z
h reduced Hubble factor today

Free-streaming - II

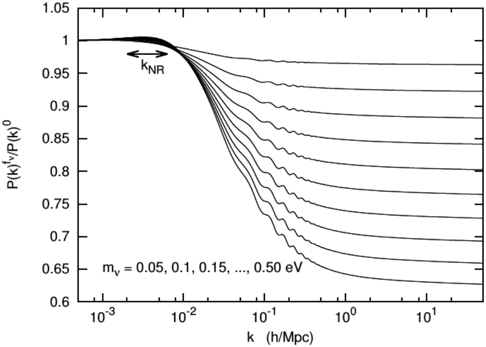
Damping occurs for all $k \gtrsim k_{\rm nr}$

k_{nr}: corresponding
to ν non-relativistic transition

Plot: $\dfrac{P_{m_\nu>0}(k)}{P_{m_\nu=0}(k)}$

- top to bottom: $m_\nu = 0.05$ eV
  to $m_\nu = 0.5$ eV
- $\dfrac{\Delta P}{P} \simeq -\dfrac{8\Omega_\nu}{\Omega_M} \simeq -\dfrac{\sum m_\nu}{0.01\ \mathrm{eV}}\%$

[Lesgourgues+, Neutrino Cosmology]
(fixed h, ω_m, ω_b, ω_Λ)

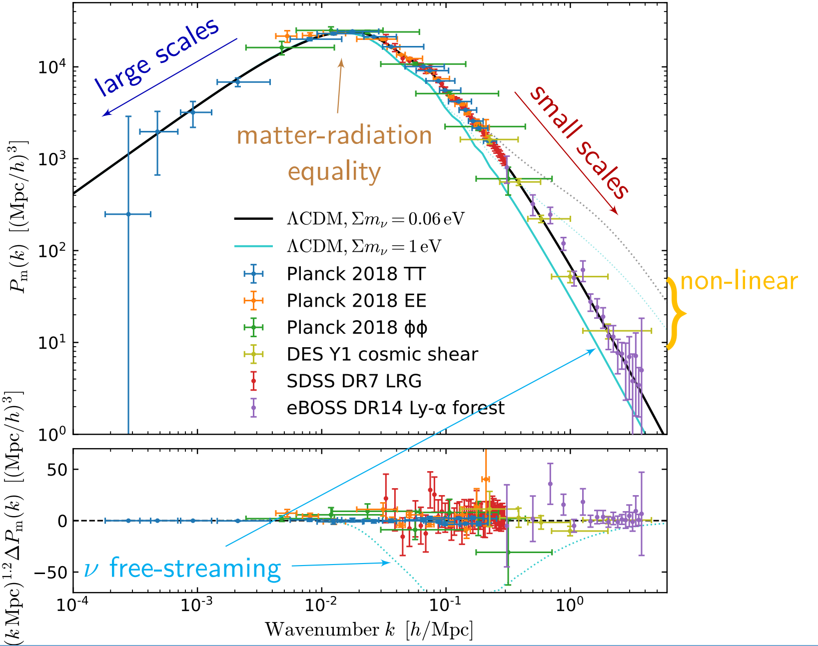


Expected constraints from future surveys:

- Planck CMB + DES: $\sigma(m_\nu) \simeq 0.04$–$0.06$ eV [Font-Ribera+, 2014]
- Planck CMB + Euclid: $\sigma(m_\nu) \simeq 0.03$ eV [Audren+, 2013]

(Linear) matter power spectrum with νs

[Chabanier+, 2019]

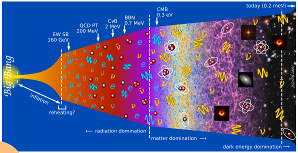


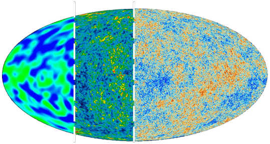
S Summary

What did we learn about CMB and matter $P(k)$?

T

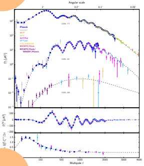
CMB decoupling, power spectra, ΛCDM

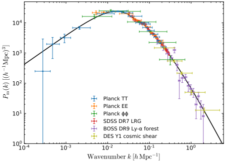


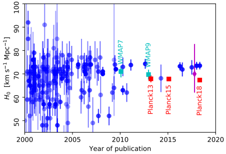


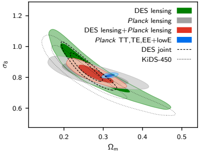
O

Observables: CMB and matter power spectrum



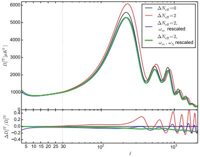


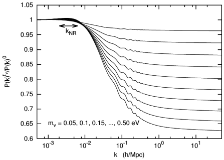


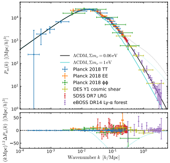


N

Neutrino effects







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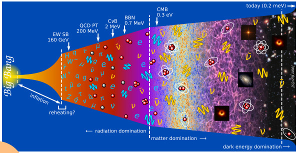
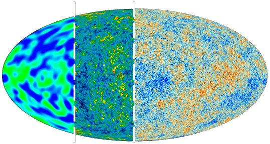
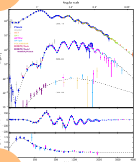
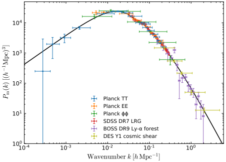


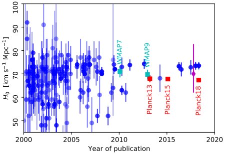


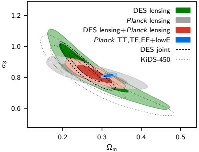
O

Observables: CMB and matter power spectrum









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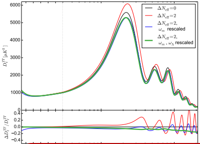

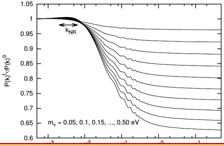

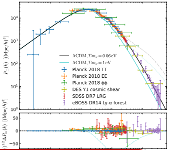

Thanks for your attention!