CNB essentials 2: Basics of neutrino cosmology, neutrino decoupling

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ASTROPARTICLES
Astroparticles and High Energy Physics Group

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VNIVERSITAT DE VALÈNCIA

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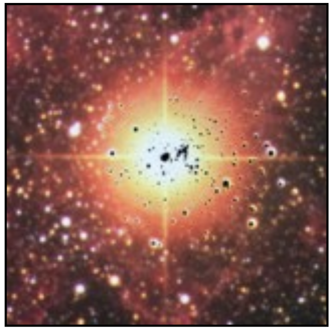
Where do neutrinos come from?

✓ Nuclear reactors

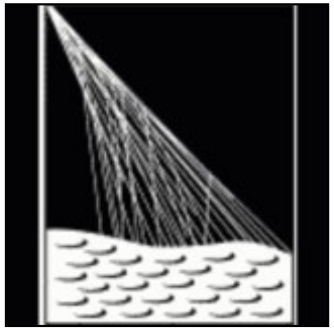
Sun ✓

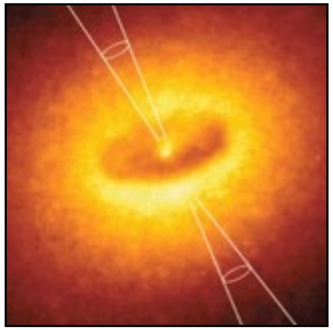
✓ Particle accelerators



Supernovae
SN 1987A ✓

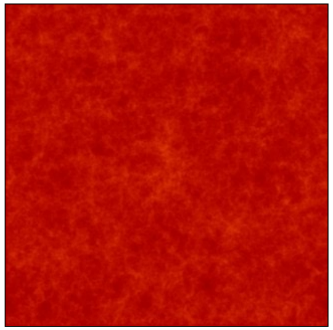
✓ Earth Atmosphere
(Cosmic rays)





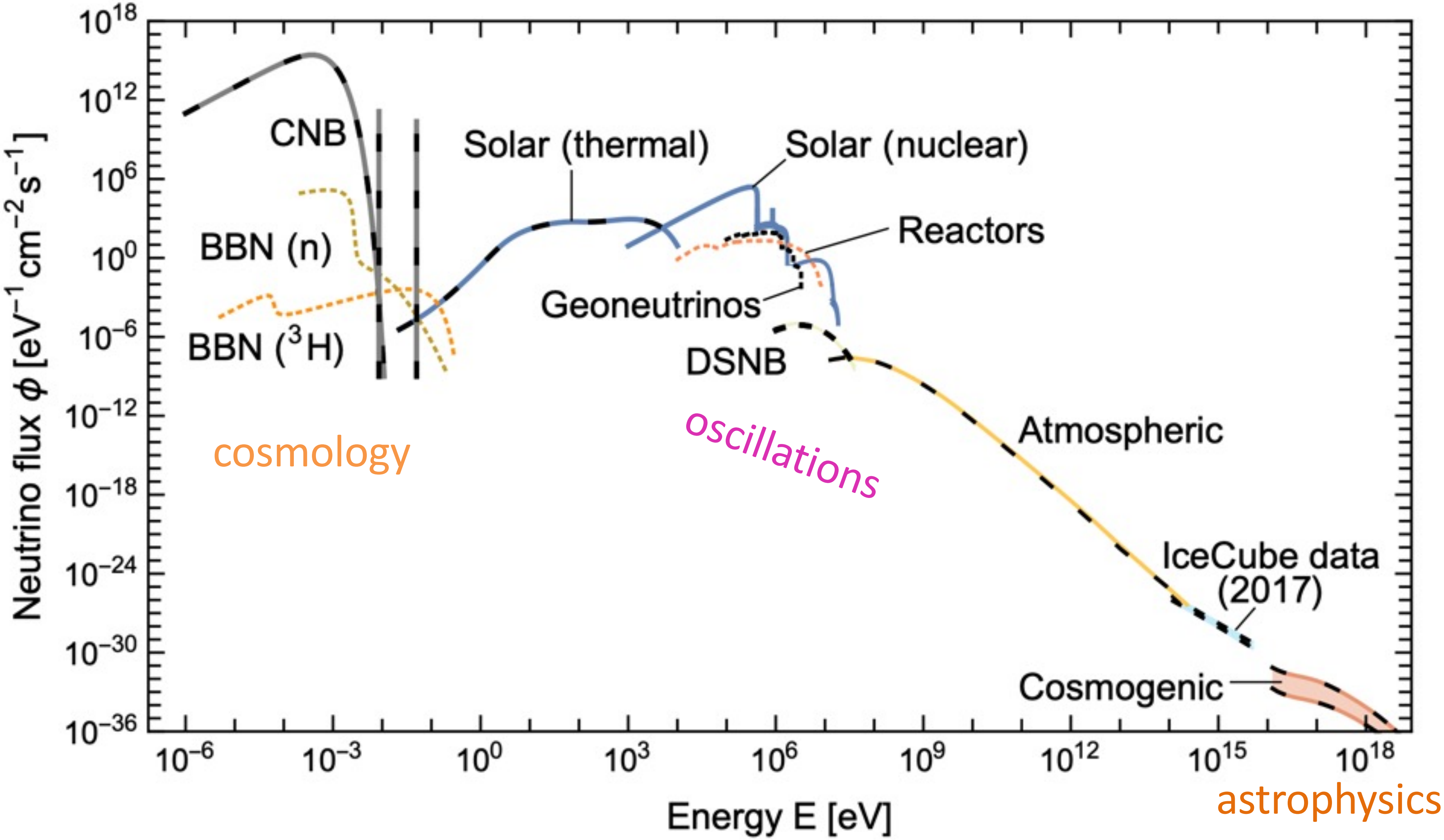
Accelerators in
astrophysical sources ?✓

✓ Earth interior
(Natural Radioactivity)

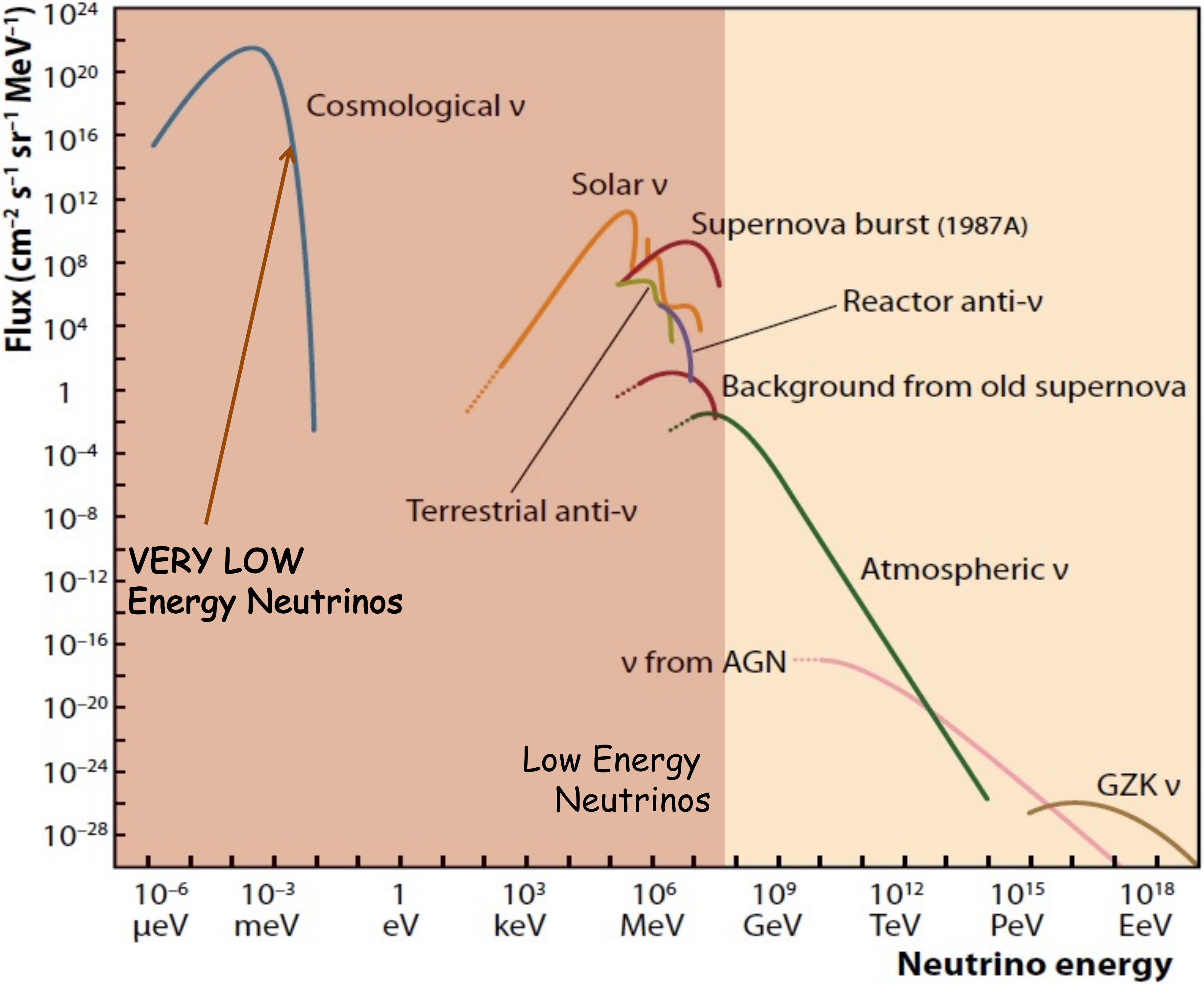


Early Universe
(today 336 ν/cm^3)
Indirect evidence

Grand Unified Neutrino Spectrum at Earth



Vitagliano, Tamborra, and Raffelt, Rev Mod Phys **92** (2020) 45006



Flux (cm⁻² s⁻¹ sr⁻¹ MeV⁻¹)
Cosmological ν
Solar ν
Supernova burst (1987A)
Reactor anti-ν
Background from old supernova
Terrestrial anti-ν
Atmospheric ν
ν from AGN
GZK ν
VERY LOW
Energy Neutrinos
Low Energy
Neutrinos
10⁻⁶ μeV
10⁻³ meV
1 eV
10³ keV
10⁶ MeV
10⁹ GeV
10¹² TeV
10¹⁵ PeV
10¹⁸ EeV
Neutrino energy

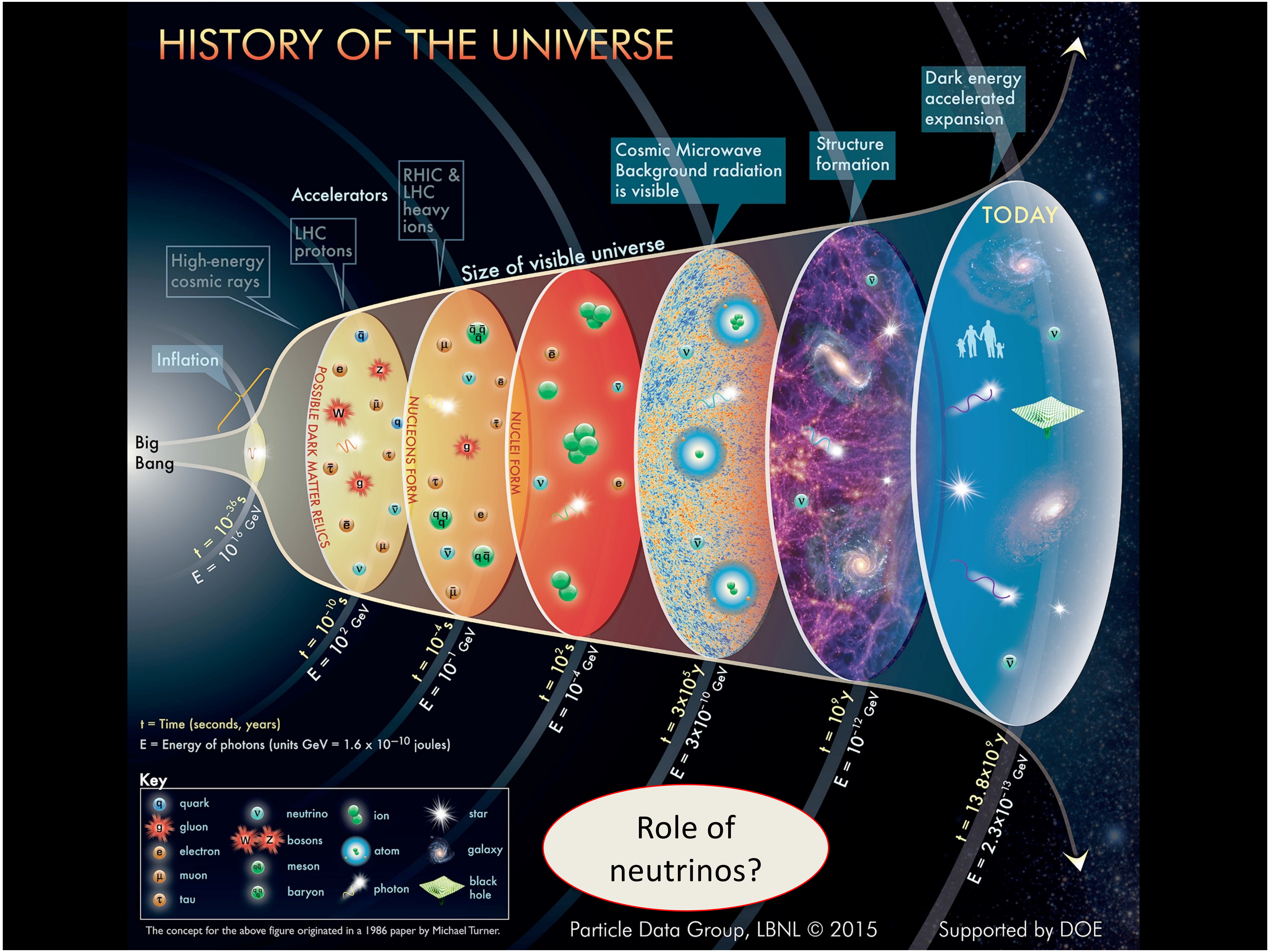
Outline

Introduction: neutrinos and the history of the Universe

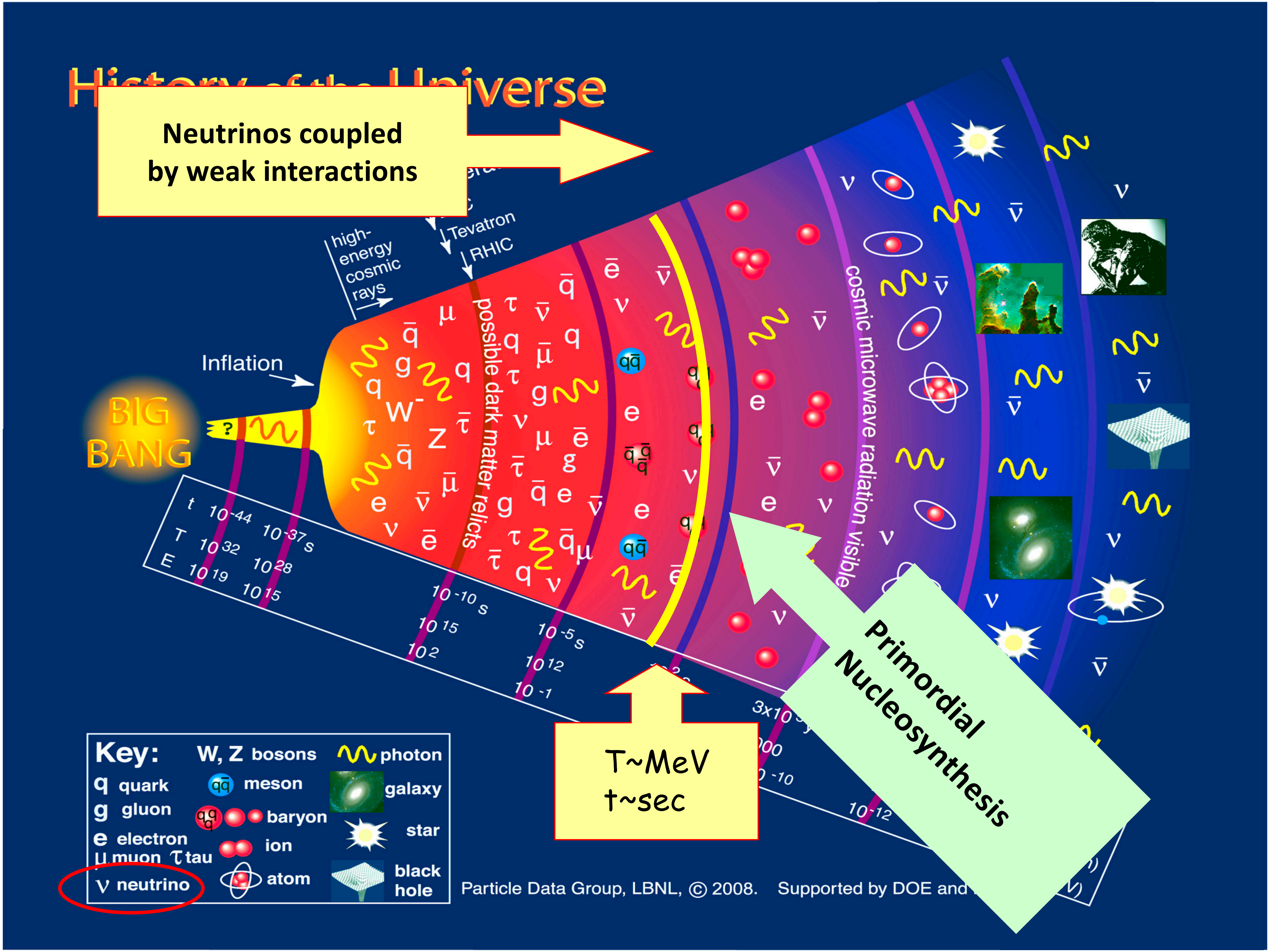
Production and decoupling of relic neutrinos

The radiation content of the Universe (N_{eff})

Introduction: neutrinos and the history of the Universe



HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Cosmic Microwave Background radiation is visible
Structure formation
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Size of visible universe
TODAY
Inflation
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
t = 10^-36 s
E = 10^16 GeV
t = 10^-10 s
E = 10^2 GeV
t = 10^-4 s
E = 10^-1 GeV
t = 10^2 s
E = 10^-4 GeV
t = 3x10^5 y
E = 3x10^-10 GeV
t = 10^9 y
E = 10^-12 GeV
t = 13.8x10^9 y
E = 2.3x10^-13 GeV
t = Time (seconds, years)
E = Energy of photons (units GeV = 1.6 x 10^-10 joules)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
The concept for the above figure originated in a 1986 paper by Michael Turner.
Role of neutrinos?
Particle Data Group, LBNL © 2015
Supported by DOE


Neutrinos coupled
by weak interactions
T~MeV
t~sec
Primordial
Nucleosynthesis
BIG
BANG
Inflation
high-energy cosmic rays
Tevatron
RHIC
possible dark matter relicts
cosmic microwave radiation visible
t 10^-44 10^-37 s
T 10^32 10^28
E 10^19 10^15
10^-10 s
10^15
10^2
10^-5 s
10^12
10^-1
Key:
q quark
g gluon
e electron
μ muon τ tau
ν neutrino
W, Z bosons
meson
baryon
ion
atom
photon
galaxy
star
black hole
Particle Data Group, LBNL, © 2008.
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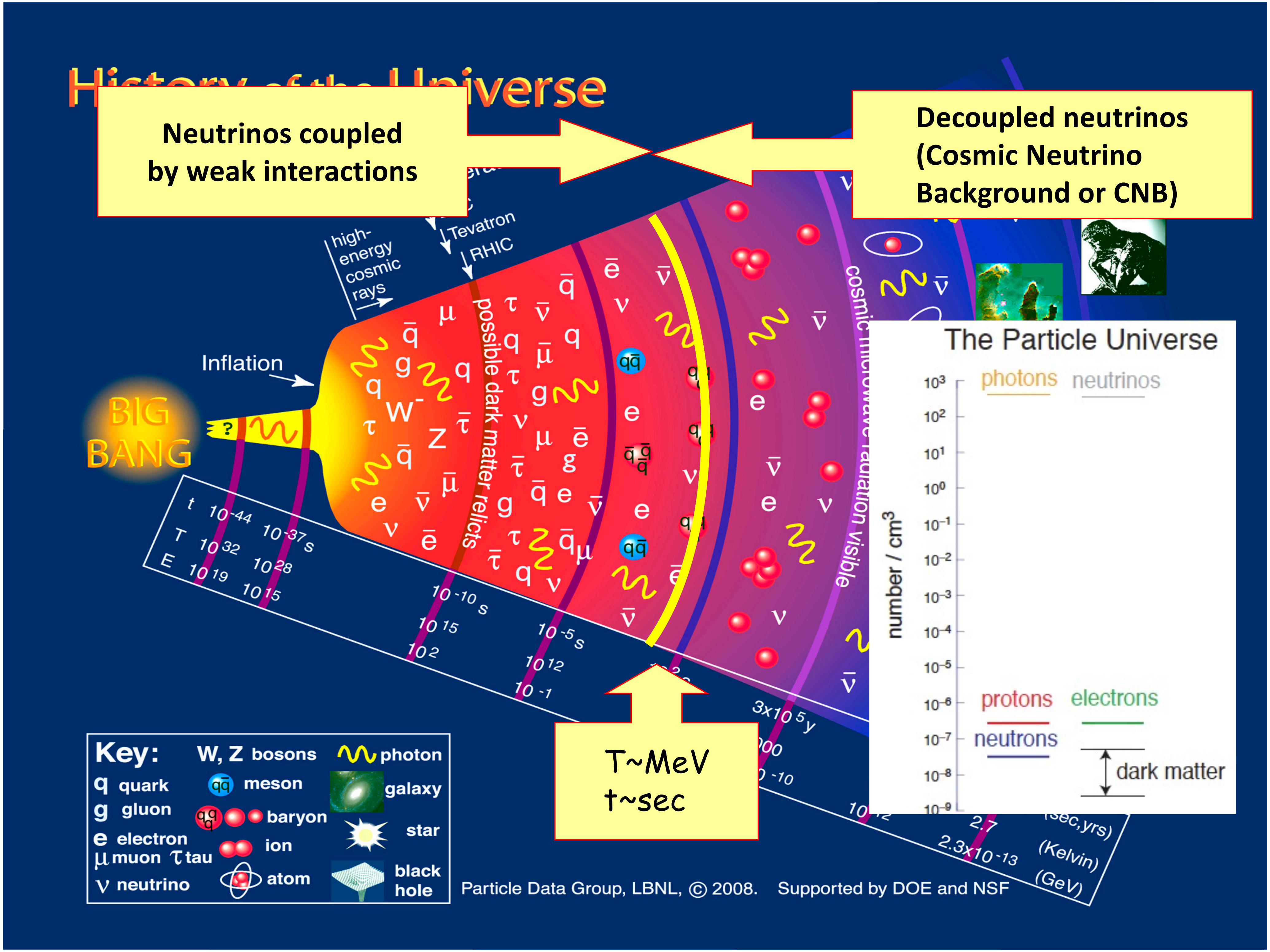
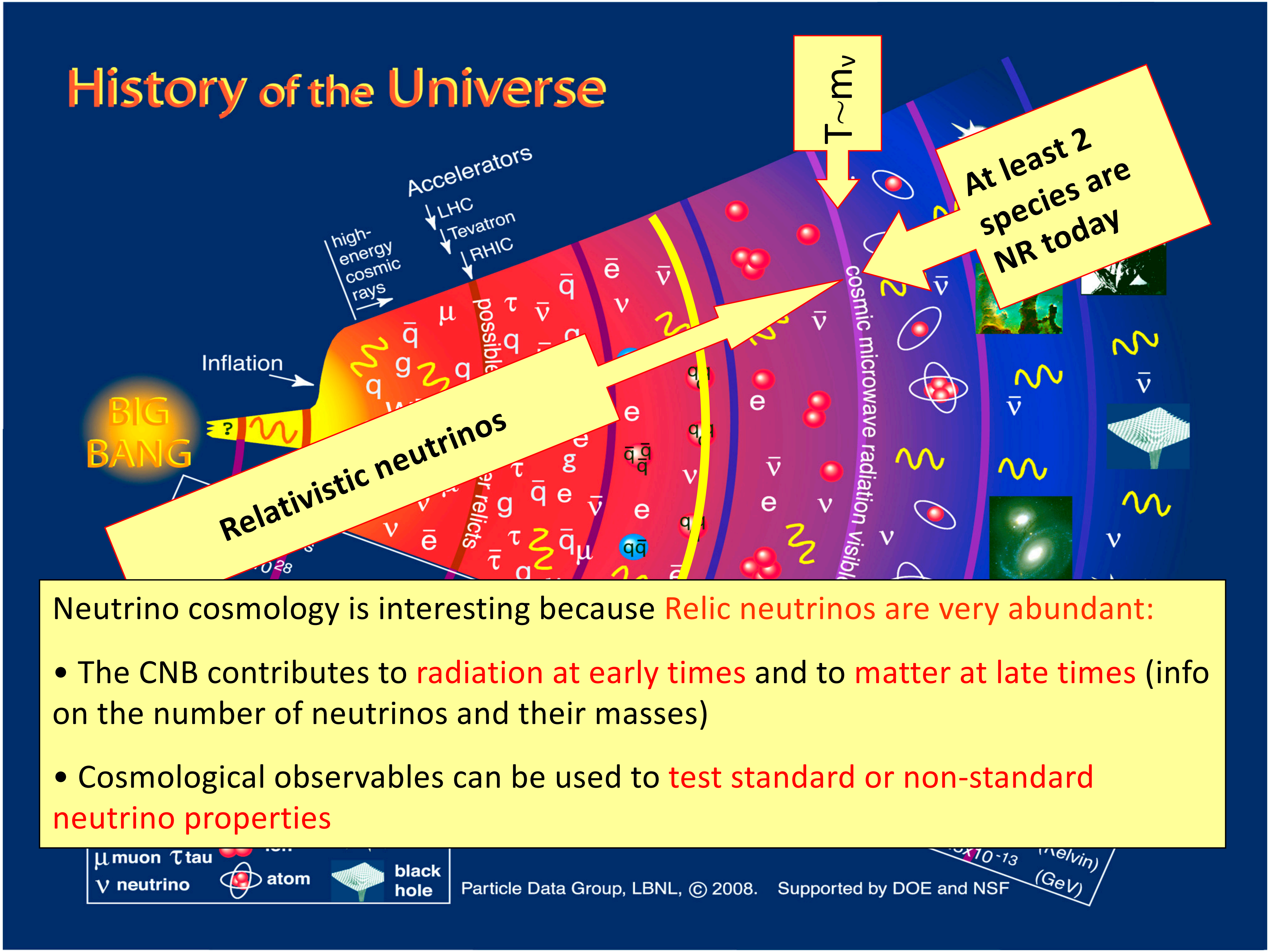
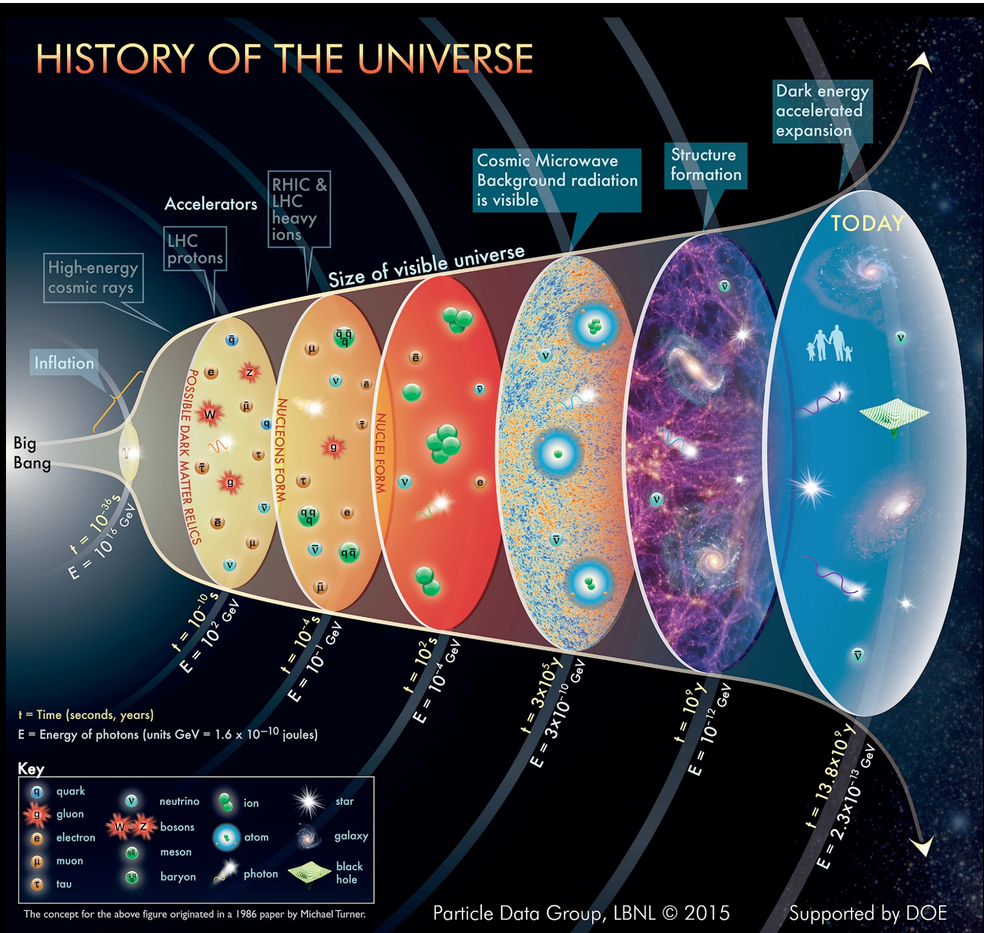

History of the Universe
Neutrinos coupled by weak interactions
Decoupled neutrinos (Cosmic Neutrino Background or CNB)
T~MeV
t~sec
BIG BANG
Inflation
high-energy cosmic rays
Tevatron
RHIC
possible dark matter relicts
cosmic microwave radiation visible
t 10^-44 10^-37 s
T 10^32 10^28
E 10^19 10^15
10^-10 s
10^15
10^2
10^-5 s
10^12
10^-1
3x10^5 y
2.7
2.3x10^-13
(sec,yrs)
(Kelvin)
(GeV)
The Particle Universe
photons
neutrinos
protons
electrons
neutrons
dark matter
number / cm^3
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
star
e electron
ion
μ muon τ tau
atom
black hole
ν neutrino
Particle Data Group, LBNL, © 2008. Supported by DOE and NSF



Neutrino cosmology is interesting because Relic neutrinos are very abundant:

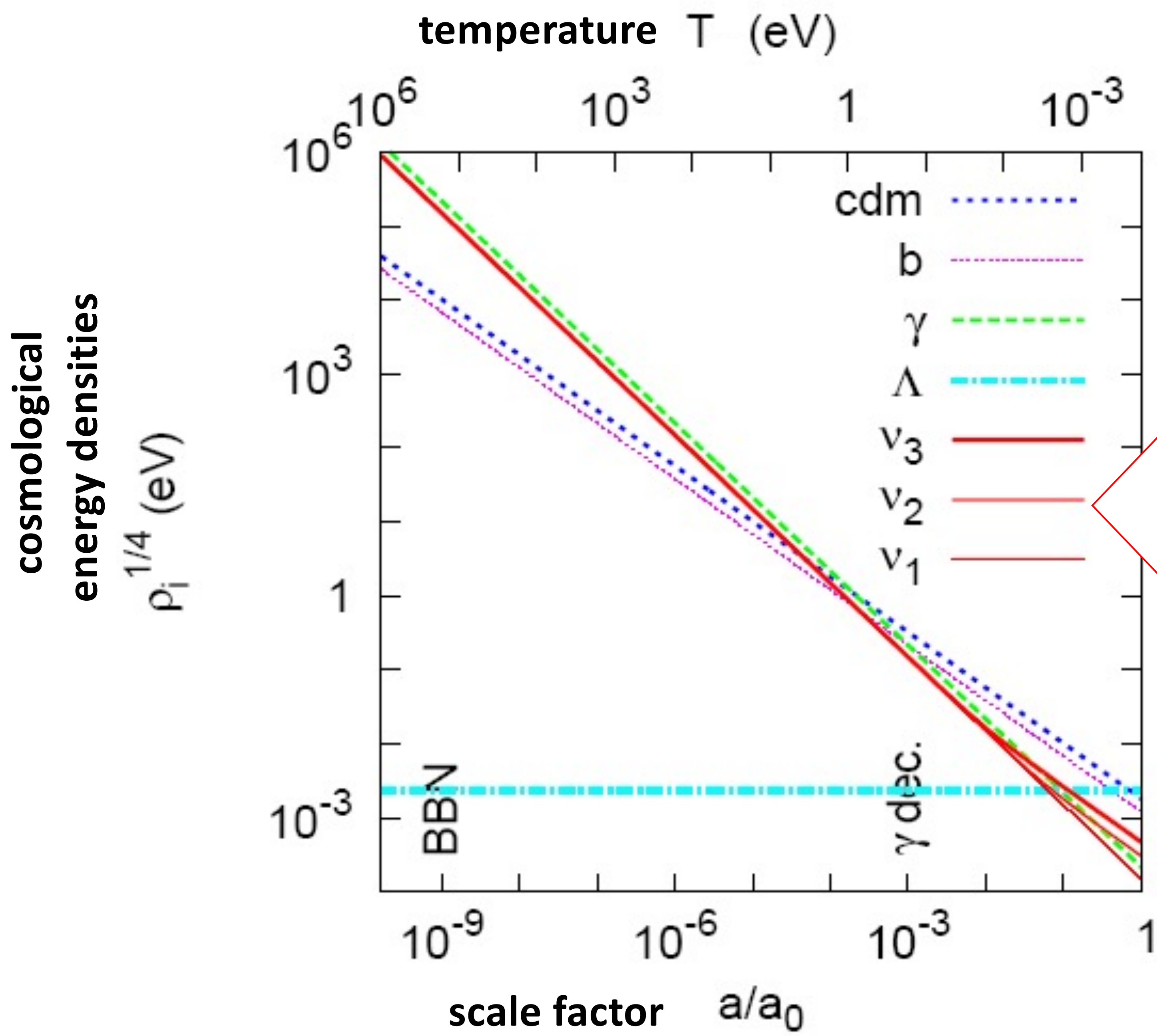
- The CNB contributes to radiation at early times and to matter at late times (info on the number of neutrinos and their masses)
- Cosmological observables can be used to test standard or non-standard neutrino properties



energy density: $\rho(a) = a^{-3(1+w)}$

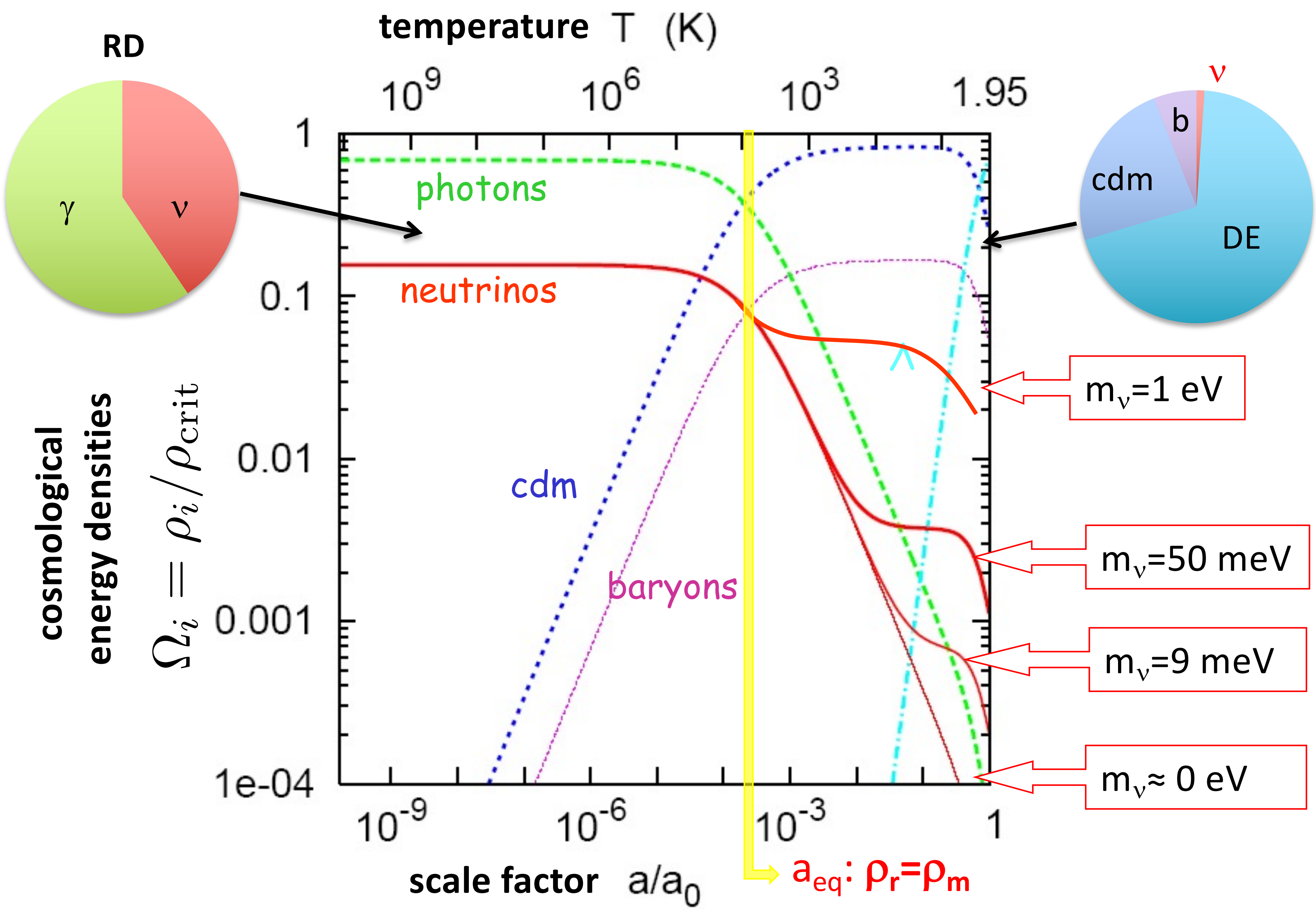
$$\rho_R \sim a^{-4} \quad , \qquad w = 1/3 \qquad \text{(Radiation)}$$
$$\rho_M \sim a^{-3} \quad , \qquad w = 0 \qquad \text{(Matter)}$$
$$\rho_\Lambda \sim \text{const.} \quad , \qquad w = -1 \qquad \text{(Cosmological constant)}$$

Evolution of the background densities: 1 MeV → now

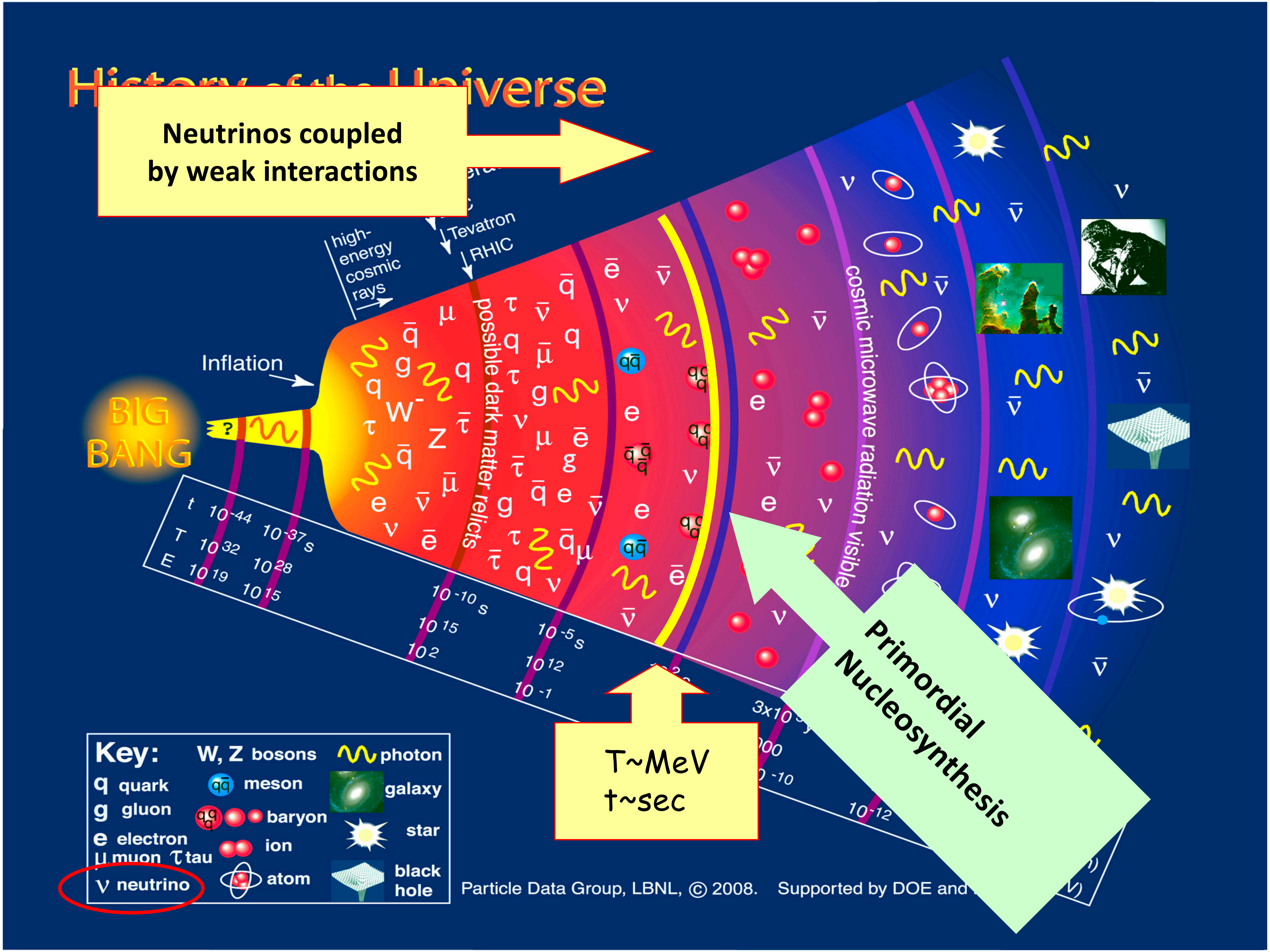


Three neutrino species with different masses

Evolution of the background densities: 1 MeV → now



Production and decoupling of relic neutrinos



Neutrinos coupled by weak interactions
T~MeV
t~sec
Primordial Nucleosynthesis
BIG BANG
Inflation
high-energy cosmic rays
Tevatron
RHIC
possible dark matter relicts
cosmic microwave radiation visible
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
star
e electron
ion
μ muon τ tau
ν neutrino
atom
black hole
Particle Data Group, LBNL, © 2008. Supported by DOE and

Equilibrium thermodynamics

Particles in equilibrium when T are high and interactions effective

T~1/a(t)

Distribution function of particle momenta in equilibrium

$$f_i^{eq}(p,T) = \left[\exp\left(\frac{E_i - \mu_i}{T}\right) \mp 1\right]^{-1}$$

Thermodynamical variables

VARIABLE	RELATIVISTIC		NON REL.
	BOSE	FERMI	
n	$\frac{\zeta(3)}{\pi^2} g T^3$	$\frac{3}{4}\frac{\zeta(3)}{\pi^2} g T^3$	$g\left(\frac{mT}{2\pi}\right)^{3/2} e^{-m/T}$
ρ	$\frac{\pi^2}{30} g T^4$	$\frac{7}{8}\frac{\pi^2}{30} g T^4$	mn
p	$\frac{\rho}{3}$		$nT \ll \rho$
$\langle E \rangle$	$2{,}701 T$	$3{,}151 T$	$m + \frac{3}{2} T$

$$n = g_i \int \frac{d^2\vec{p}}{(2\pi)^3} f_i(p,T) \qquad \rho = g_i \int \frac{d^2\vec{p}}{(2\pi)^3} E_i \; f_i(p,T)$$

$$p = g_i \int \frac{d^2\vec{p}}{(2\pi)^3} \frac{p^2}{3E_i} f_i(p,T) \qquad \langle E \rangle = \rho/n$$

Neutrinos in Equilibrium

$$1\ \mathrm{MeV} \lesssim \mathrm{T} \lesssim \mathrm{m}_\mu$$

$$T_\nu = T_{e^\pm} = T_\gamma$$

$$\nu_\alpha \nu_\beta \leftrightarrow \nu_\alpha \nu_\beta$$

$$\nu_\alpha \bar{\nu}_\beta \leftrightarrow \nu_\alpha \bar{\nu}_\beta$$

$$\nu_\alpha e^\pm \leftrightarrow \nu_\alpha e^\pm$$

$$\nu_\alpha \bar{\nu}_\alpha \leftrightarrow e^+ e^-$$

$$\mathcal{L}_{\mathrm{SM}} = -2\sqrt{2} G_F \left\{ (\bar{\nu}_e \gamma^\mu L \nu_e)(\bar{e}\gamma_\mu L e) + \sum_{P,\alpha} g_P (\bar{\nu}_\alpha \gamma^\mu L \nu_\alpha)(\bar{e}\gamma_\mu P e) \right\}$$

$$P = L, R = (1 \mp \gamma_5)/2$$

$$g_L = -\tfrac{1}{2} + \sin^2\theta_W \text{ and } g_R = \sin^2\theta_W$$

Cosmological energy densities: radiation

Energy density of **relativistic particles** with $f_i(p)$

$$\rho_i = g \int \frac{d^3p}{(2\pi)^3} \frac{p}{e^{p/T_i} \pm 1}$$

$$\rho_i = 3P_i = \begin{cases} \frac{\pi^2}{30} g\, T_i^4 \;, & \text{boson} \\ \frac{7}{8}\frac{\pi^2}{30} g\, T_i^4 \;, & \text{fermion} \end{cases}$$

$$\rho_\gamma = \frac{\pi^2}{15} T_\gamma^4$$

$$\rho_\nu = 3 \times \frac{7}{8} \times \frac{\pi^2}{15} T_\nu^4$$

Neutrino decoupling

As the Universe expands, particle densities are diluted and temperatures fall. Weak interactions become ineffective to keep neutrinos in good thermal contact with the e.m. plasma

Rough, but quite accurate estimate of the decoupling temperature

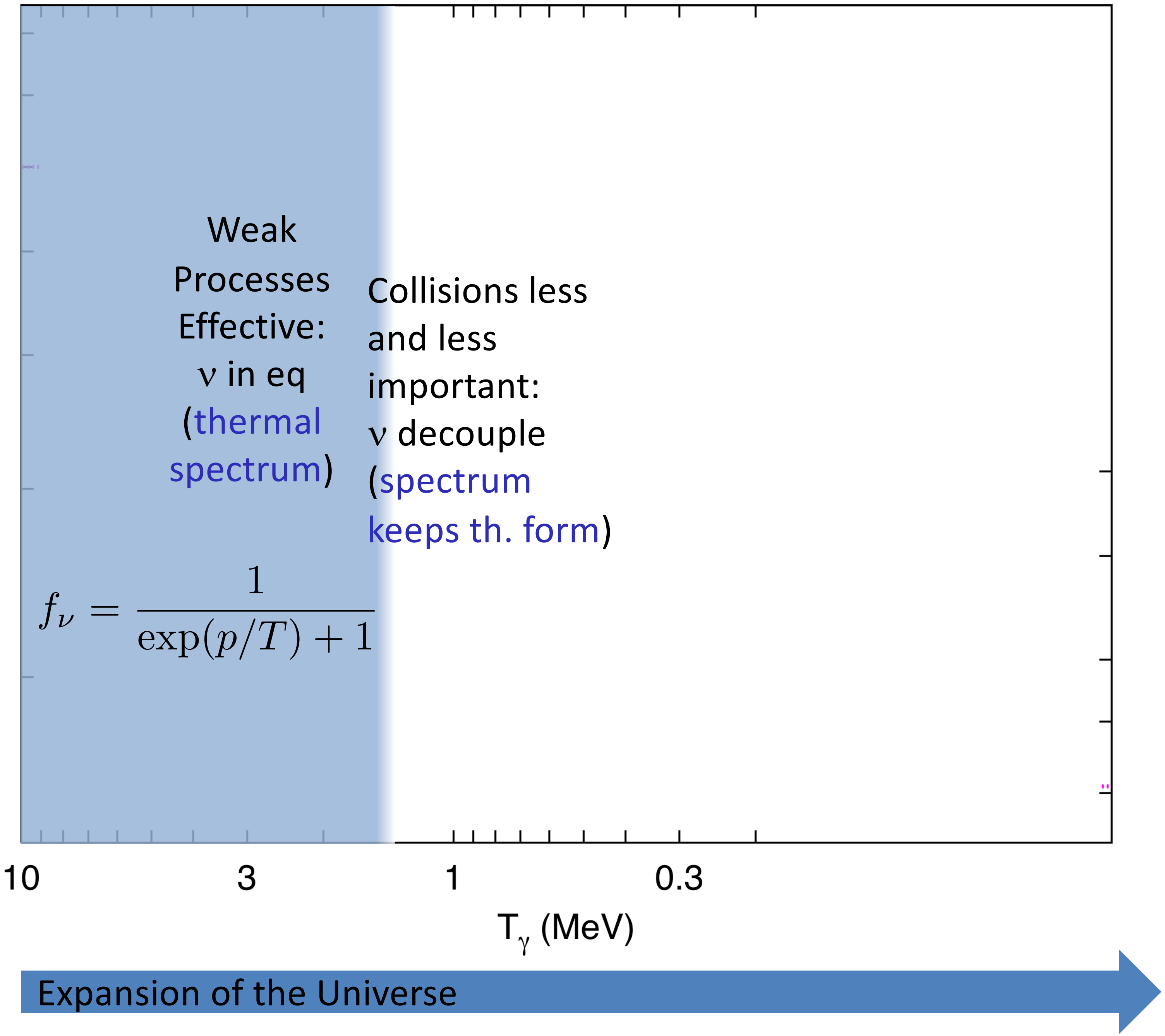
Rate of weak processes ~ Hubble expansion rate

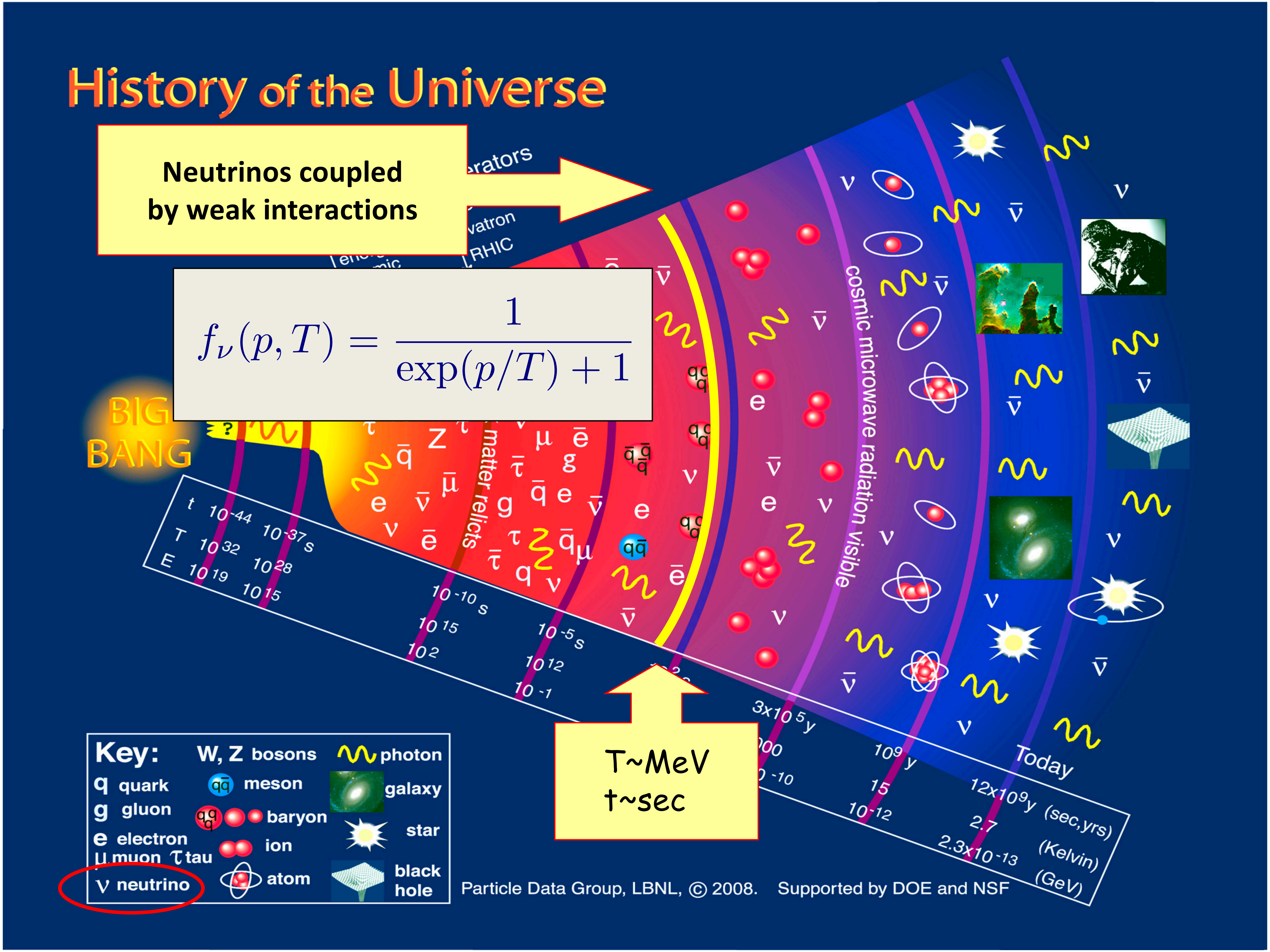
$$\Gamma_W \approx \sigma_W |v| n, \;\; H^2 = \frac{8\pi \rho_{\rm rad}}{3M_P^2} \;\; \rightarrow \;\; G_F^2 T^5 \approx \sqrt{\frac{8\pi \rho_{\rm rad}}{3M_P^2}} \;\; \rightarrow \;\; T_{\rm dec}(\nu) \approx 1 \text{ MeV}$$

Since ν_e have both CC and NC interactions with $e^{\pm}$

$$T_{\rm dec}(\nu_e) \simeq 2 \text{ MeV} \quad T_{\rm dec}(\nu_{\mu,\tau}) \simeq 3 \text{ MeV}$$

Neutrino decoupling

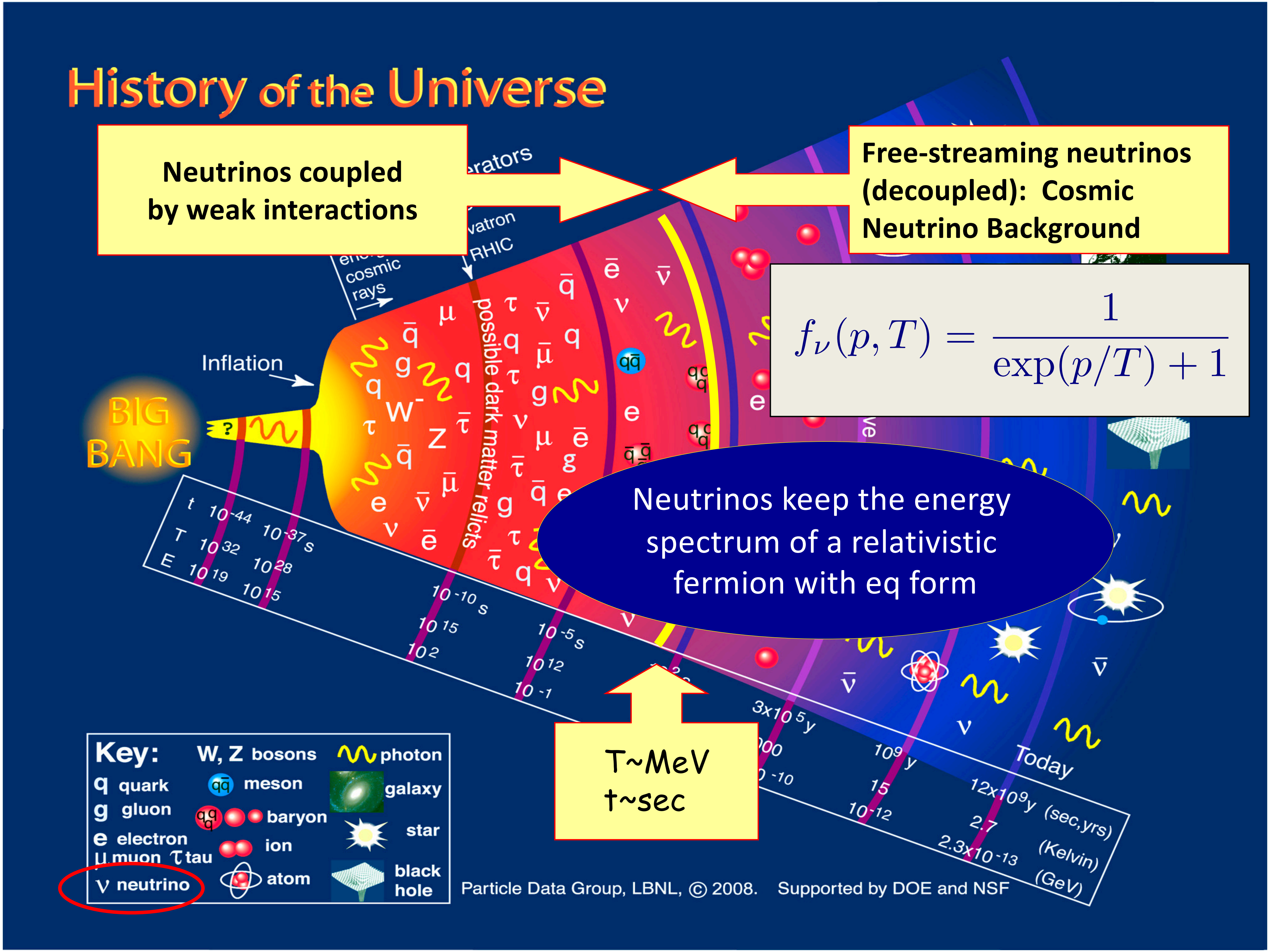




Neutrinos coupled
by weak interactions

$$f_\nu(p,T) = \frac{1}{\exp(p/T)+1}$$

T~MeV
t~sec

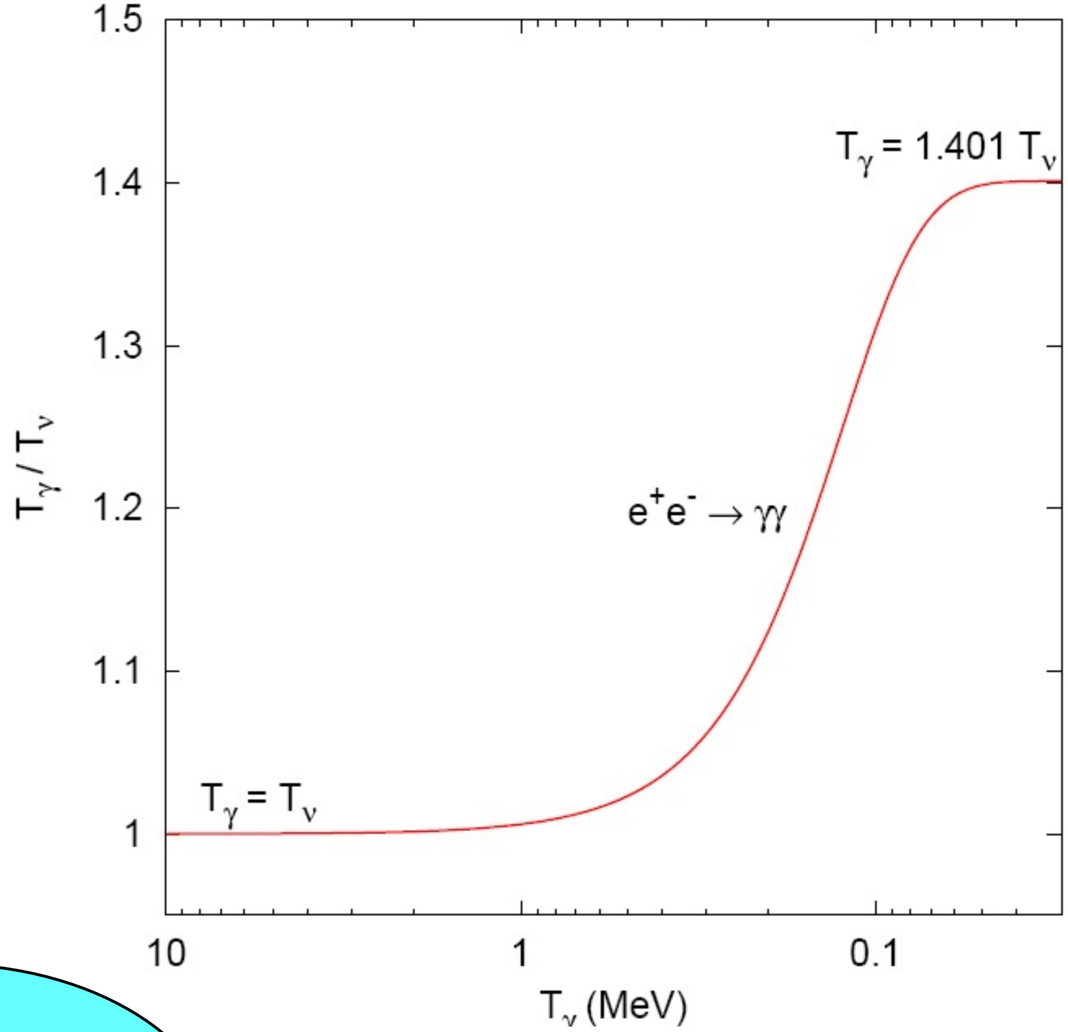

History of the Universe
Neutrinos coupled by weak interactions
Free-streaming neutrinos (decoupled): Cosmic Neutrino Background
$$f_\nu(p,T) = \frac{1}{\exp(p/T)+1}$$
Neutrinos keep the energy spectrum of a relativistic fermion with eq form
T~MeV
t~sec
BIG BANG
Inflation
cosmic rays
RHIC
possible dark matter relicts
t 10^-44 10^-37 s
T 10^32 10^28
E 10^19 10^15
10^-10 s
10^15
10^2
10^-5 s
10^12
10^-1
3x10^5 y
10^9 y
15
10^-12
Today
12x10^9 y (sec,yrs)
2.7 (Kelvin)
2.3x10^-13 (GeV)
Key:
W, Z bosons
photon
q quark
meson
galaxy
g gluon
baryon
e electron
star
ion
μ muon τ tau
ν neutrino
atom
black hole
Particle Data Group, LBNL, © 2008. Supported by DOE and NSF

Neutrino and photon (CMB) temperatures

At T~m_e, electron-positron pairs annihilate

$$e^+e^- \to \gamma\gamma$$

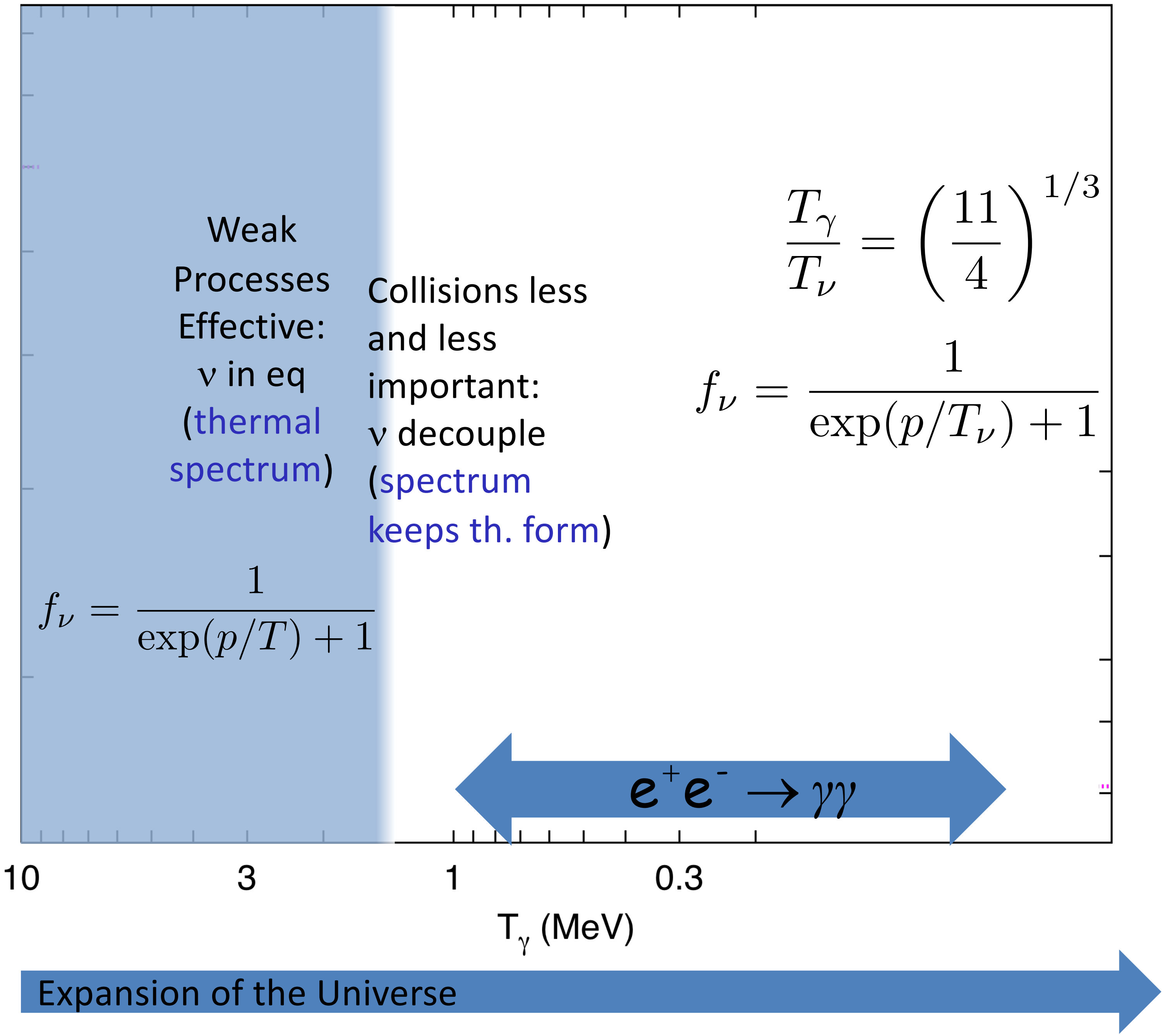
heating photons but not the decoupled neutrinos



$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$

$$f_\nu(p,T) = \frac{1}{\exp(p/T_\nu) + 1}$$

Neutrino decoupling and $e^{\pm}$ annihilations



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$$e^+e^- \rightarrow \gamma\gamma$$

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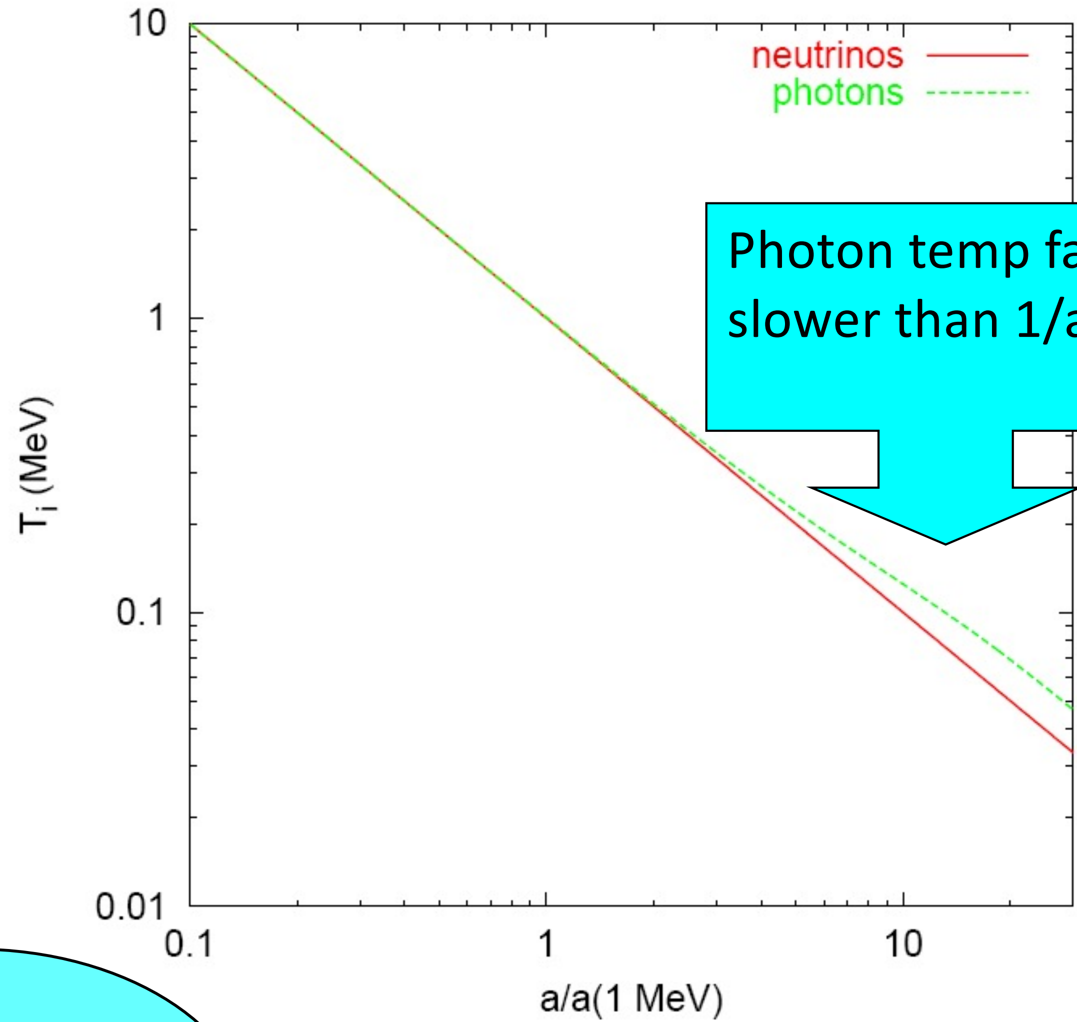


Photon temp falls slower than 1/a(t)

$$\frac{T_\gamma}{T_\nu} = \left(\frac{11}{4}\right)^{1/3}$$

$$f_\nu(p,T) = \frac{1}{\exp(p/T_\nu)+1}$$

The Cosmic Neutrino Background

Neutrinos decoupled at T~MeV, keeping a spectrum as that of a relativistic species

$$f_\nu(p,T) = \frac{1}{\exp(p/T_\nu)+1}$$

- Number density

$$n_\nu = \int \frac{d^3p}{(2\pi)^3} f_\nu(p,T_\nu) = \frac{3}{11} n_\gamma = \frac{6\zeta(3)}{11\pi^2} T_{\rm CMB}^3$$

- Energy density

$$\rho_{\nu_i} = \int \sqrt{p^2+m_{\nu_i}^2}\, \frac{d^3p}{(2\pi)^3} f_\nu(p,T_\nu) \rightarrow \begin{cases} \frac{7\pi^2}{120}\left(\frac{4}{11}\right)^{4/3} T_{\rm CMB}^4 & \text{Massless} \\ m_{\nu_i} n_\nu & \text{Massive } m_\nu \gg T \end{cases}$$

The Cosmic Neutrino Background

Neutrinos decoupled at T~MeV, keeping a spectrum as that of a relativistic species

$$f_\nu(p,T) = \frac{1}{\exp(p/T_\nu)+1}$$

- Number density

At present 112 $(\nu + \bar{\nu})$ cm^{-3} per flavour

- Energy density

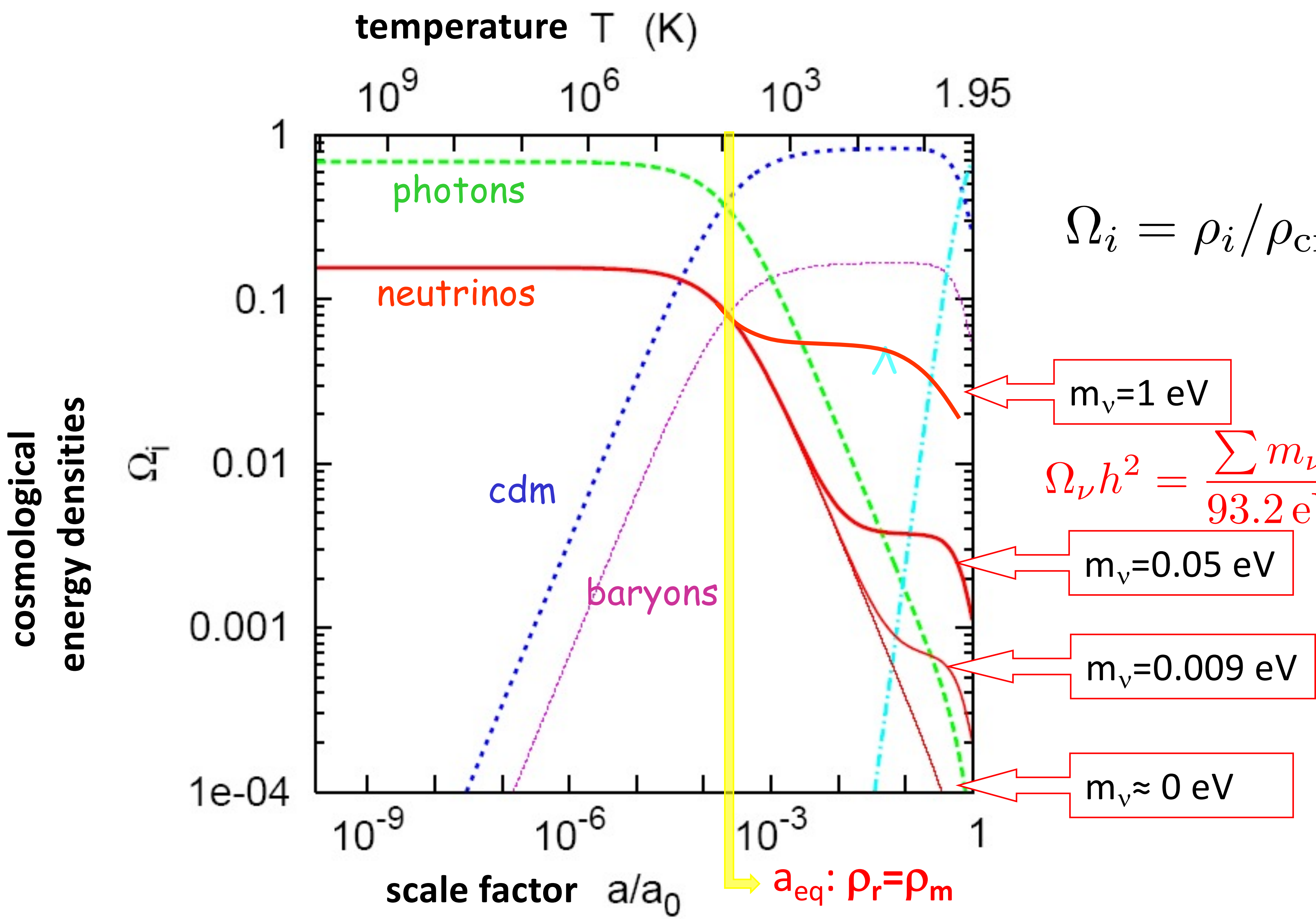
Contribution to the energy density of the Universe

$$\Omega_\nu h^2 \simeq 1.7 \times 10^{-5}$$ Massless

$$\Omega_\nu h^2 = \frac{\sum_i m_{\nu_i}}{94.1 \text{ eV}}$$ Massive

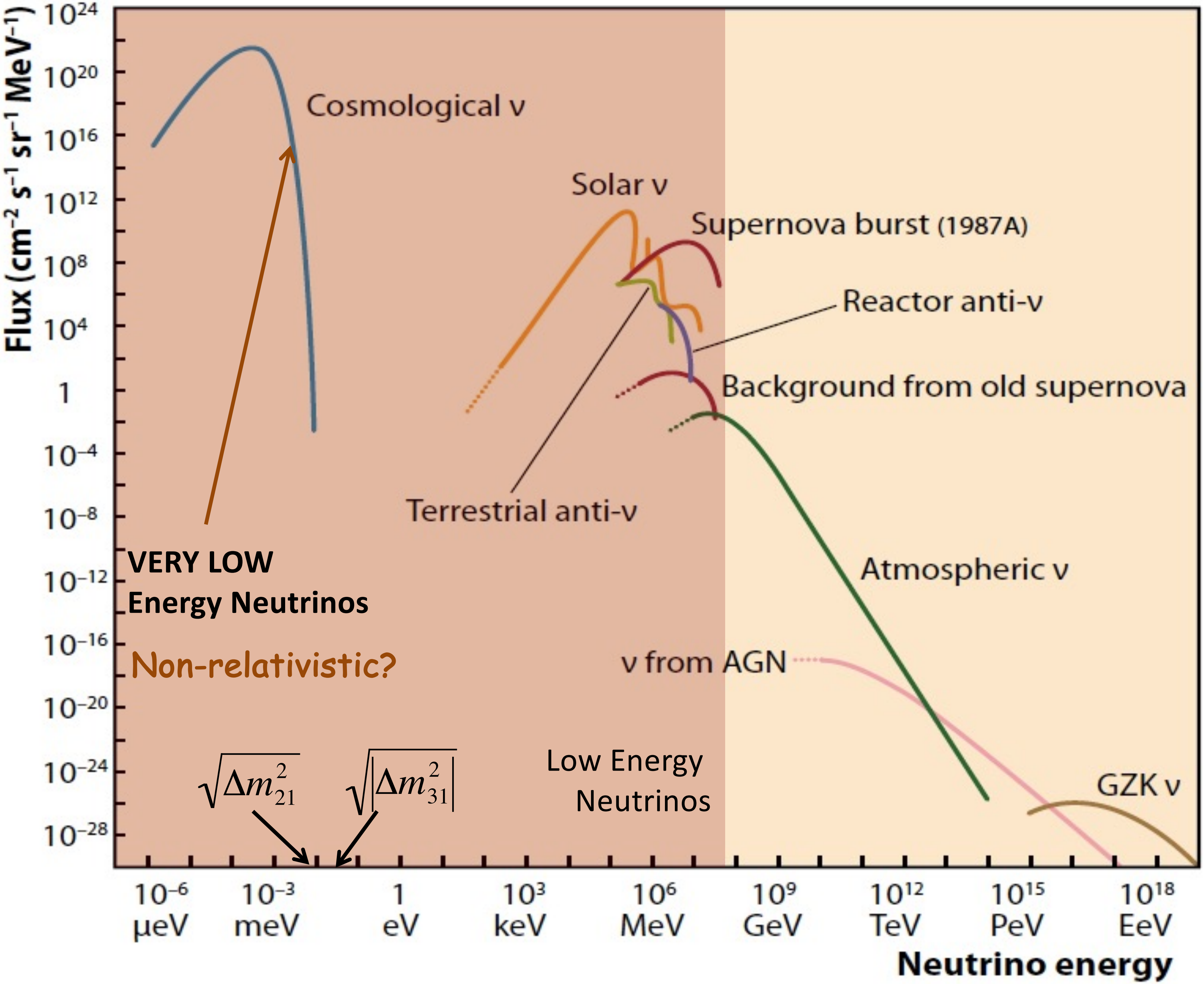
$m_\nu >> T$

Evolution of the background densities: 1 MeV → now



$$\Omega_i = \rho_i/\rho_{\rm crit}$$

$$\Omega_\nu h^2 = \frac{\sum m_{\nu_i}}{93.2\,{\rm eV}}$$



Flux (cm^{-2} s^{-1} sr^{-1} MeV^{-1})
10^{24}
10^{20}
10^{16}
10^{12}
10^{8}
10^{4}
1
10^{-4}
10^{-8}
10^{-12}
10^{-16}
10^{-20}
10^{-24}
10^{-28}
Cosmological ν
Solar ν
Supernova burst (1987A)
Reactor anti-ν
Background from old supernova
Terrestrial anti-ν
Atmospheric ν
ν from AGN
GZK ν
VERY LOW
Energy Neutrinos
Non-relativistic?
Low Energy
Neutrinos
$\sqrt{\Delta m^2_{21}}$
$\sqrt{|\Delta m^2_{31}|}$
10^{-6} μeV
10^{-3} meV
1 eV
10^{3} keV
10^{6} MeV
10^{9} GeV
10^{12} TeV
10^{15} PeV
10^{18} EeV
Neutrino energy

The radiation content of the Universe (N_{eff})

Relativistic particles in the universe

At $T<m_e$, the radiation content of the Universe is

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} \times 3\right]$$

Valid for standard neutrinos in the
instantaneous decoupling approximation

Relativistic particles in the universe

At $T<m_e$, the radiation content of the Universe is

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu + \rho_x = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]$$

effective number of relativistic neutrino species

(effective number of neutrinos)

N_{eff} is a way to measure the ratio $\dfrac{\rho_\nu + \rho_x}{\rho_\gamma}$

1960s-1970s : $N_{eff} = N_\nu$, **extra neutrinos** would enhance the cosmological expansion

>1980s: N_{eff} = **additional relativistic particles**

Number of light neutrino types (LEP data) $N_\nu = 2.984 \pm 0.008$

Relativistic particles in the universe

At $T<m_e$, the radiation content of the Universe is

$$\rho_{\rm rad} = \rho_\gamma + \rho_\nu + \rho_x = \rho_\gamma \left[1 + \frac{7}{8}\left(\frac{4}{11}\right)^{4/3} N_{\rm eff}\right]$$

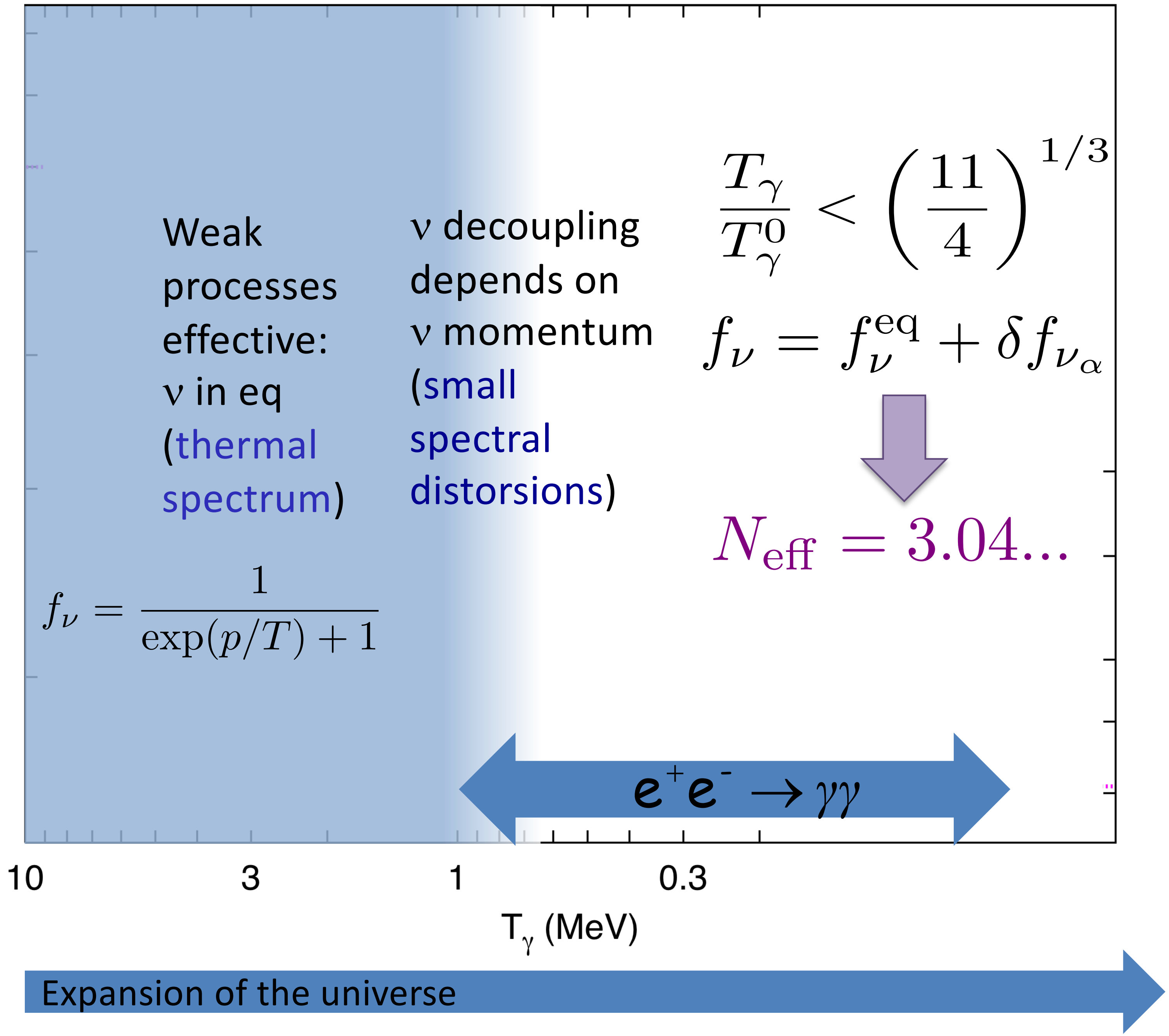
effective number of neutrinos

$N_{\rm eff} \neq 3$

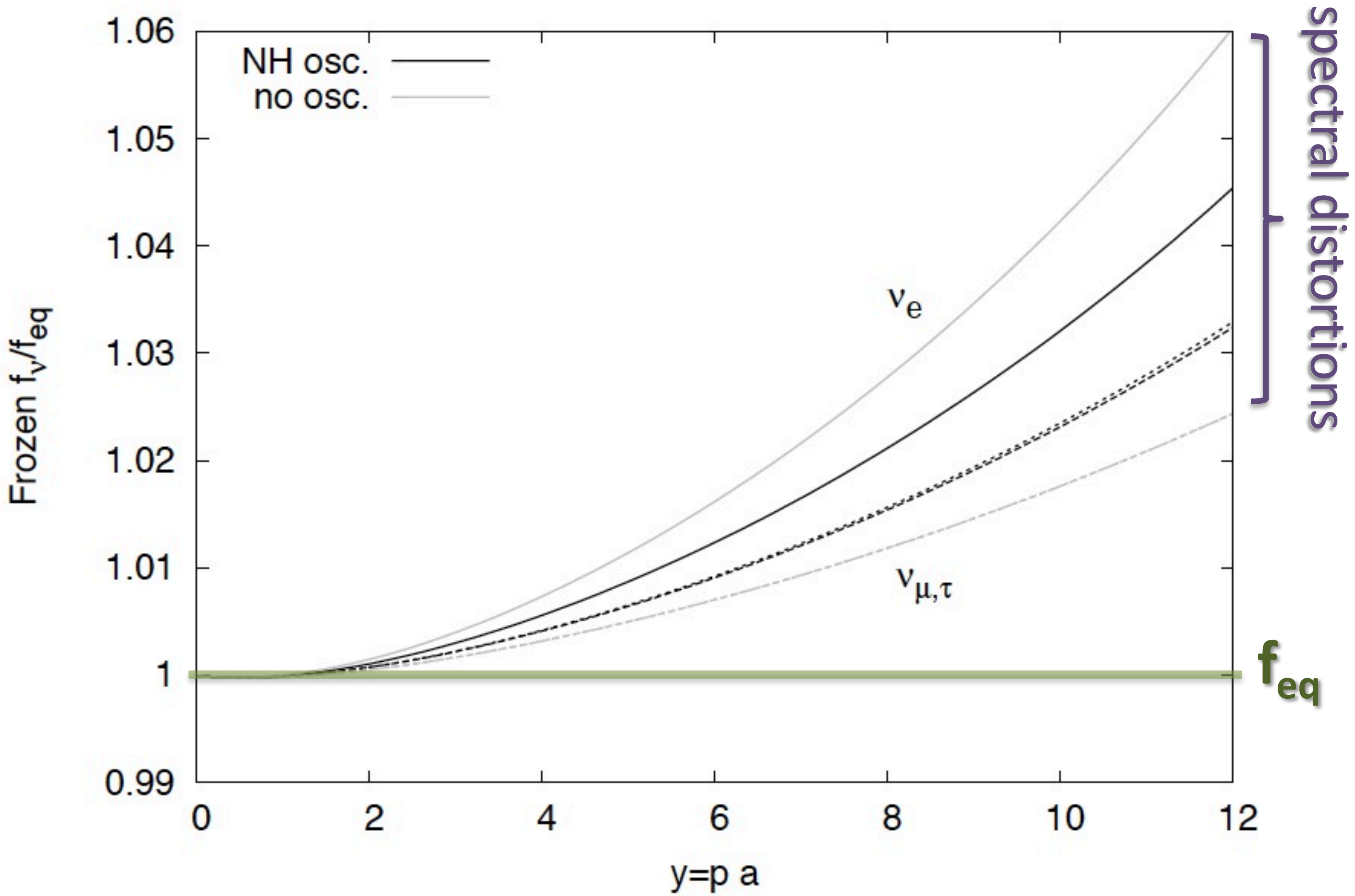
- **additional relativistic particles** (scalars, pseudoscalars, decay products of heavy particles,…)
- **non-standard neutrino physics** (primordial neutrino asymmetries, totally or partially thermalized light sterile neutrinos, non-standard interactions with electrons,…)

$N_{eff} \neq 3$ in the standard case

$N_{eff} > 3$: small neutrino heating

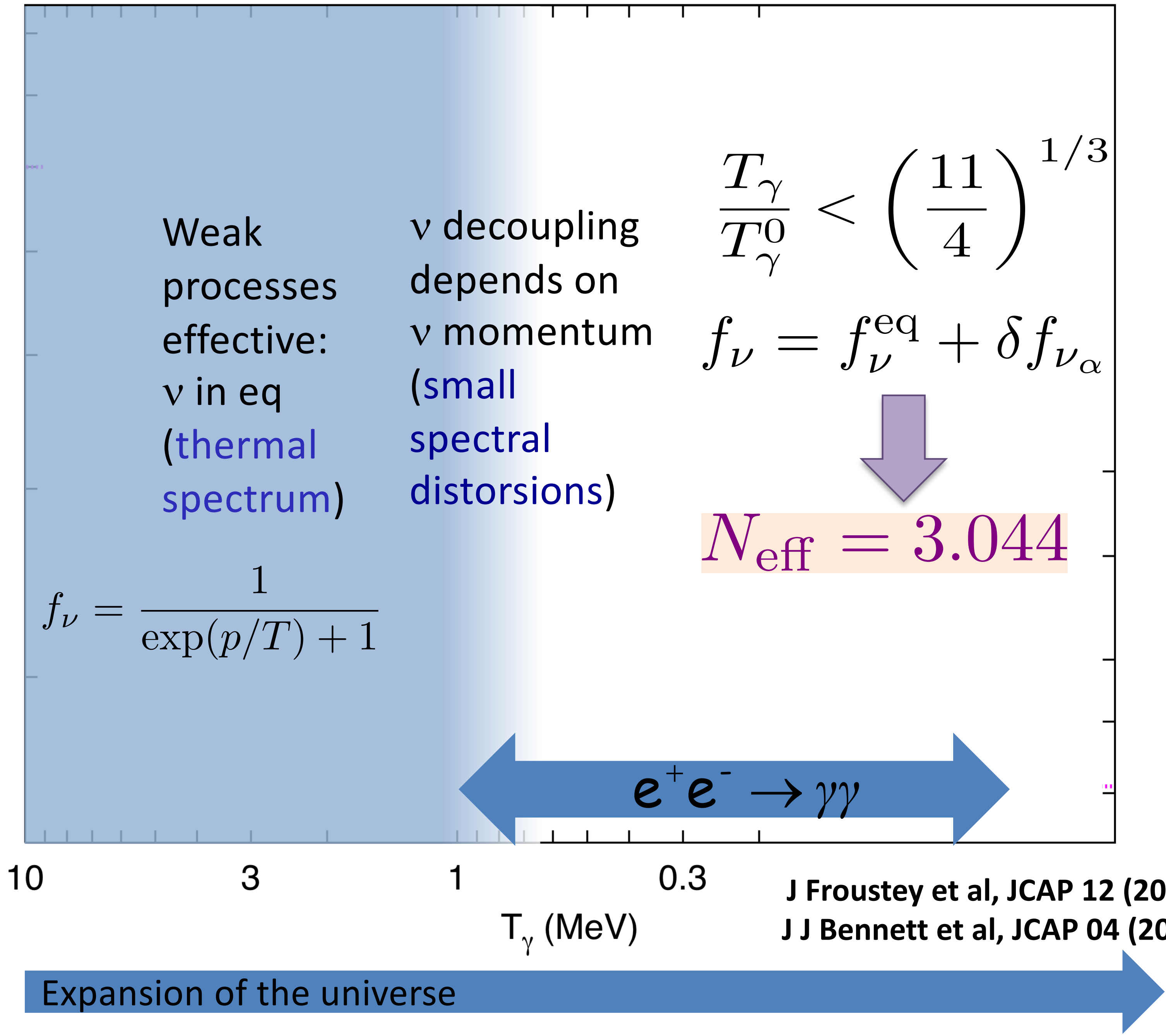


Standard scenario: final distortions



P F de Salas & SP, JCAP 07 (2016) 051

N_{eff} > 3 : small neutrino heating



J Froustey et al, JCAP 12 (2020) 015
J J Bennett et al, JCAP 04 (2021) 073

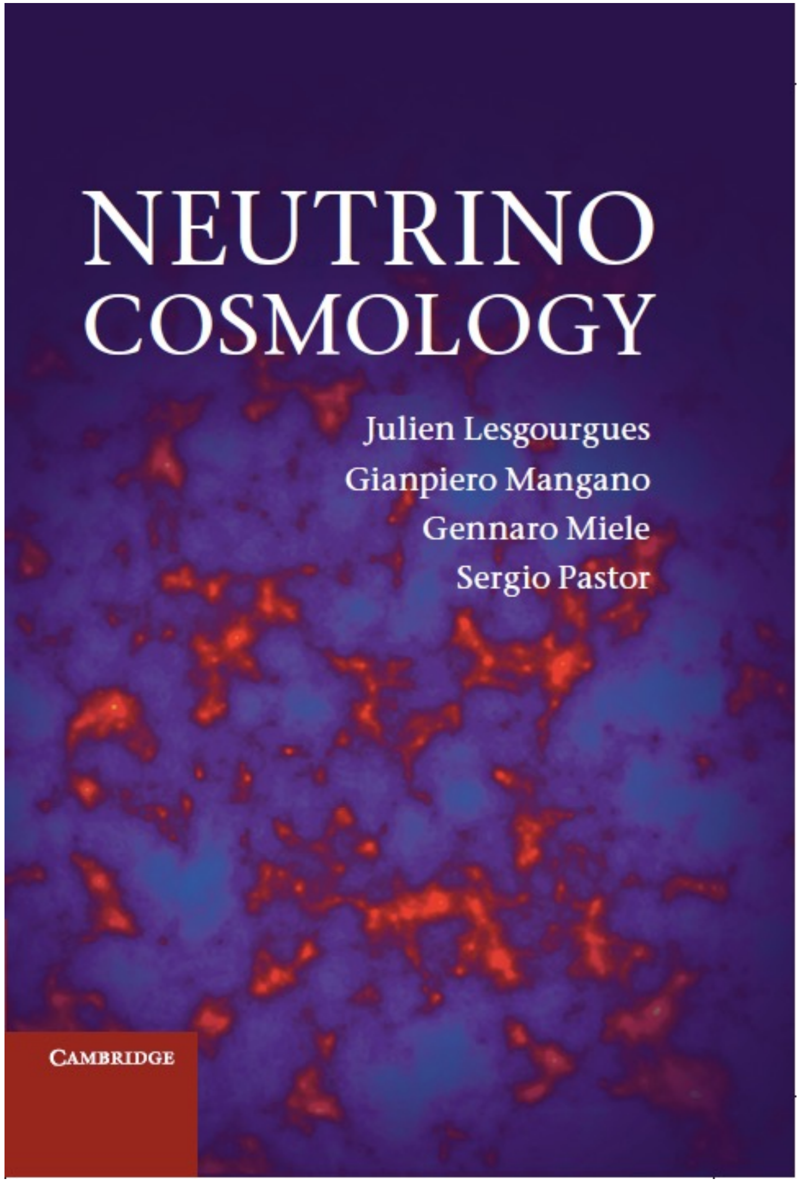
End

Exercises: try to calculate…

- The present number density of massive/massless neutrinos n_ν^0 in cm^{-3}
- The present energy density of massive/massless neutrinos Ω_ν^0 and find the limits on the total neutrino mass from $\Omega_\nu^0<1$ and $\Omega_\nu^0 < \Omega_m^0$
- The final ratio T_γ/T_ν using the conservation of entropy density before/after $e^\pm$ annihilations
- The decoupling temperature of relic neutrinos using $\Gamma_W \approx H$

- The photon temperature / redshift of the matter radiation equality for $m_\nu = 1$ eV

For more details…



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