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Introduction on neutrinos and cosmology

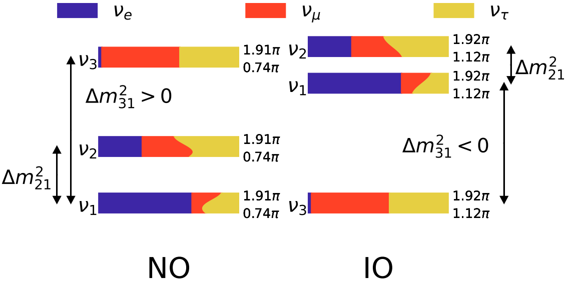
Some basic concepts on neutrinos
and on the history of the Universe

EuCAPT Astroneutrino Theory Workshop 2021, Prague (CZ) / online, 20/09/2021

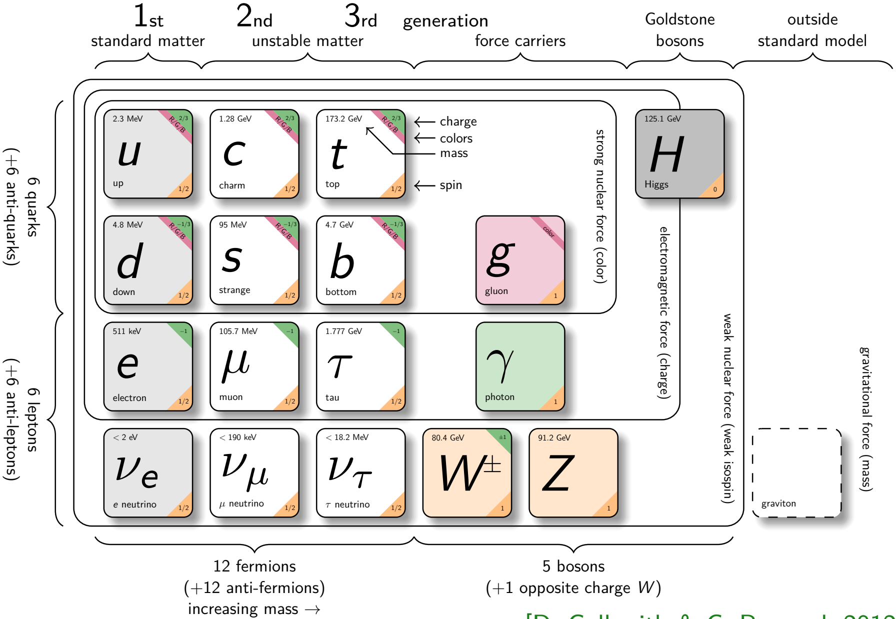
N Neutrinos

Based on:

- Giunti & Kim
- JHEP 02 (2021) 071
- KATRIN
- de Salas+, Frontiers 5 (2018) 36
- SG+, JPG 43 (2016) 033001

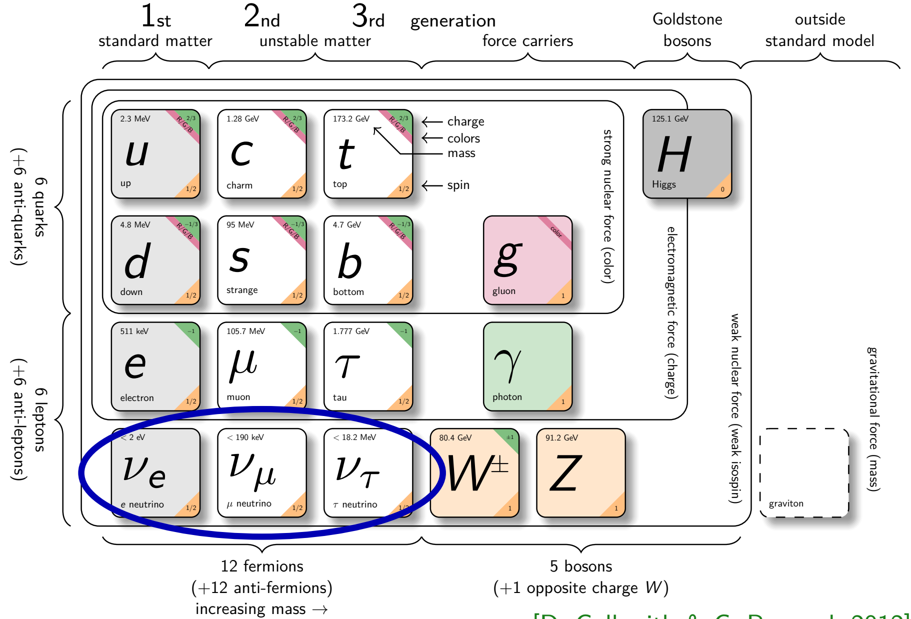


The Standard Model of Particle Physics



[D. Galbraith & C. Burgard, 2012]

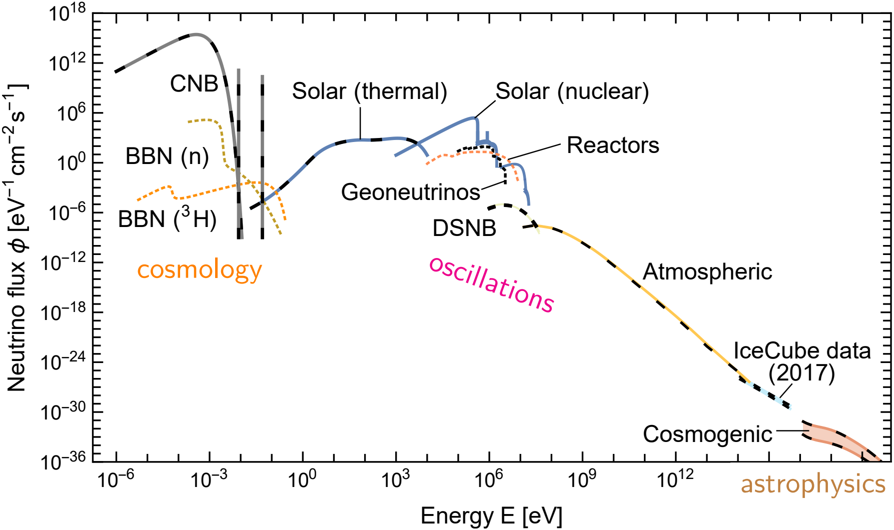
The Standard Model of Particle Physics



[D. Galbraith & C. Burgard, 2012]

Neutrino spectrum

[Vitagliano+, RMP 92 (2020)]

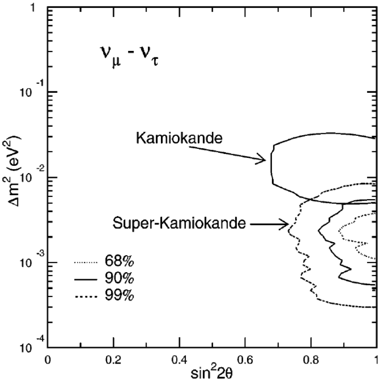


neutrinos at all energies provide valuable information!

Neutrino oscillations

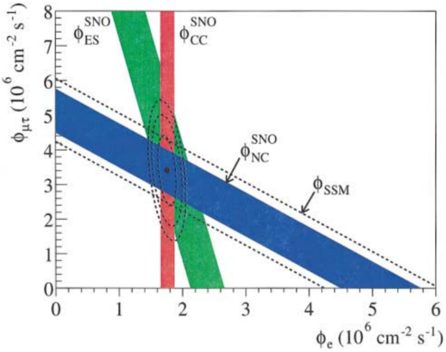
Major discoveries:

[SuperKamiokande, 1998]



first discovery of $\nu_\mu \to \nu_\tau$ oscillations from atmospheric ν

[SNO, 2001-2002]



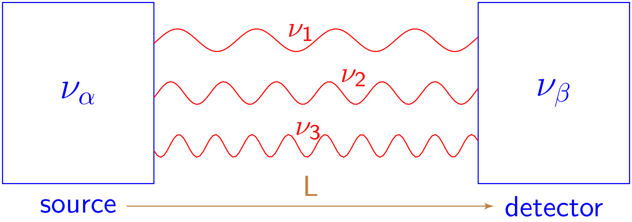
first discovery of $\nu_e \to \nu_\mu, \nu_\tau$ oscillations from solar ν

Nobel prize in 2015

Two neutrino bases

flavor neutrinos ν_α | $|\nu_\alpha\rangle = \sum_k U_{\alpha k} |\nu_k\rangle$ | massive neutrinos ν_k

$$|\nu(t=0)\rangle = |\nu_\alpha\rangle = U_{\alpha 1}|\nu_1\rangle + U_{\alpha 2}|\nu_2\rangle + U_{\alpha 3}|\nu_3\rangle$$



$$|\nu(t>0)\rangle = |\nu_\beta\rangle = U_{\alpha 1}\, e^{-iE_1 t}|\nu_1\rangle + U_{\alpha 2}\, e^{-iE_2 t}|\nu_2\rangle + U_{\alpha 3}\, e^{-iE_3 t}|\nu_3\rangle \neq |\nu_\alpha\rangle$$

$E_k^2 = p^2 + m_k^2$ ⟵ define ⟶ $t = L$

$$P_{\nu_\alpha \to \nu_\beta}(L) = |\langle \nu_\alpha | \nu(L) \rangle|^2 = \sum_{k,j} U_{\beta k} U^*_{\alpha k} U^*_{\beta j} U_{\alpha j} \exp\left(-i \frac{\Delta m^2_{kj} L}{2E}\right)$$

$$\Delta m^2_{ij} = m_i^2 - m_j^2$$

Oscillations in matter

presence of a medium affects neutrino propagation

↓

altered oscillation probabilities $|\langle\nu_\beta|\nu_\alpha(t)\rangle|^2$

Schrödinger eq: $i\frac{d}{dt}|\nu_\alpha(t)\rangle = (\mathcal{H}_0 + \mathcal{H}_I)\,|\nu_\alpha(t)\rangle$

vacuum Hamiltonian:

$$\mathcal{H}_0|\nu_k\rangle = E_k|\nu_k\rangle$$

$k \in [1,2,3]$, $E_k = \sqrt{|\vec{p}|^2 + m_k^2}$ neutrino energy

interaction Hamiltonian:

$$\mathcal{H}_I|\nu_\alpha\rangle = V_\alpha|\nu_\alpha\rangle$$

$\alpha \in [e,\mu,\tau]$, V_α matter potential

Notice: $\mathcal{H}_0$ and $\mathcal{H}_I$ defined in different basis!

Vacuum mixing parameters must enter the equations

matter potential depends on the medium where the neutrino propagates

Oscillations in matter

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↓

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Example with two neutrinos: mass splitting Δm^2 and angle θ in vacuum

$$\Delta m_M^2 = \sqrt{(\Delta m^2 \cos 2\theta - 2|\vec{p}|V_{CC})^2 + (\Delta m^2 \sin 2\theta)^2}$$

$$\tan 2\theta_M = \tan 2\theta \left[1 - \frac{2|\vec{p}|V_{CC}}{\Delta m^2 \cos 2\theta}\right]^{-1}$$

Matter effect particularly important for neutrinos in dense medium

(including early universe!)

The mixing matrix

U can be parameterized using 3 angles (θ_{12}, θ_{13}, θ_{23}) and max 3 (1 Dirac δ, 2 Majorana [∃ only for Majorana ν]) phases

$$U = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}}_{\text{mainly atmospheric and LBL accelerator disappearance}} \underbrace{\begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix}}_{\text{mainly SBL reactors and LBL accelerator appearance}} \underbrace{\begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\text{mainly solar and LBL reactors}} M$$

M → Majorana phases irrelevant for oscillation experiments

Relevant for example in neutrinoless double-beta decay

$s_{ij} \equiv \sin\theta_{ij}$; $c_{ij} \equiv \cos\theta_{ij}$

SBL = short baseline; LBL = long baseline

Three-neutrino oscillation data

Solar + LBL reactors

Experiments:	Parameters:
SuperK	θ_{12}
SNO	Δm^2_{21}
Borexino	(θ_{13})
KamLAND	
...	

baseline defined by $\Delta m^2_{kj} \cdot L/E$ LBL: long baseline ($E/L \gtrsim \Delta m^2_{31}$) SBL: short baseline ($E/L \sim \Delta m^2_{21}$)

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SBL reactors

Experiments:	Parameters:
DayaBay	θ_{13}
RENO	Δm^2_{31}
DoubleChooz	(θ_{12})
...	(Δm^2_{21})

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Atmospheric

Experiments:	Parameters:
Antares	θ_{23}
IceCube	Δm^2_{31}
SuperK	(θ_{13})
...	$(\delta_{\rm CP})$

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Experiments:	Parameters:
Antares	θ_{23}
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SuperK	(θ_{13})
...	$(\delta_{\rm CP})$

LBL accelerators

Experiments:	Parameters:
NOνA	θ_{13}
T2K	θ_{23}
MINOS	Δm^2_{31}
...	$\delta_{\rm CP}$

baseline defined by $\Delta m^2_{kj} \cdot L/E$ LBL: long baseline ($E/L \gtrsim \Delta m^2_{31}$) SBL: short baseline ($E/L \sim \Delta m^2_{21}$)

Three Neutrino Oscillations

$$\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k}\nu_k \quad (\alpha = e, \mu, \tau)$$

$U_{\alpha k}$ described by 3 mixing angles θ_{12}, θ_{13}, θ_{23} and one CP phase δ

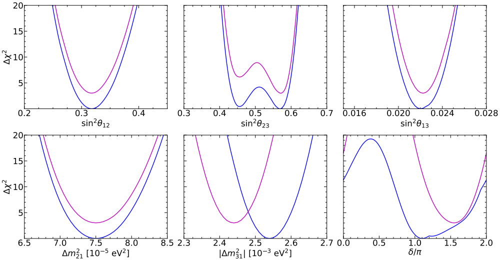
Current knowledge of the 3 active ν mixing: [JHEP 02 (2021) update]

NO/NH: Normal Ordering/Hierarchy, $m_1 < m_2 < m_3$ IO/IH: Inverted O/H, $m_3 < m_1 < m_2$

$$\Delta m^2_{21} = (7.50^{+0.22}_{-0.20}) \cdot 10^{-5}\ \mathrm{eV}^2$$
$$|\Delta m^2_{31}| = (2.54 \pm 0.03) \cdot 10^{-3}\ \mathrm{eV}^2\ (\mathrm{NO})$$
$$= (2.44 \pm 0.03) \cdot 10^{-3}\ \mathrm{eV}^2\ (\mathrm{IO})$$

$$10 \sin^2(\theta_{12}) = 3.18 \pm 0.16$$
$$10^2 \sin^2(\theta_{13}) = 2.200^{+0.069}_{-0.062}\ (\mathrm{NO})$$
$$= 2.225^{+0.064}_{-0.070}\ (\mathrm{IO})$$
$$10 \sin^2(\theta_{23}) = 4.55 \pm 0.13 U 5.71 \pm 0.12\ (\mathrm{NO})$$
$$= 5.71^{+0.14}_{-0.17}\ (\mathrm{IO})$$

$$\delta/\pi = 1.10^{+0.27}_{-0.12}\ (\mathrm{NO})$$
$$= 1.54 \pm 0.14\ (\mathrm{IO})$$



mass ordering still unknown

δ still unknown

see also: `http://globalfit.astroparticles.es`

Neutrinos and their masses

[de Salas+, Frontiers 5 (2018) 36]

Normal ordering (NO)

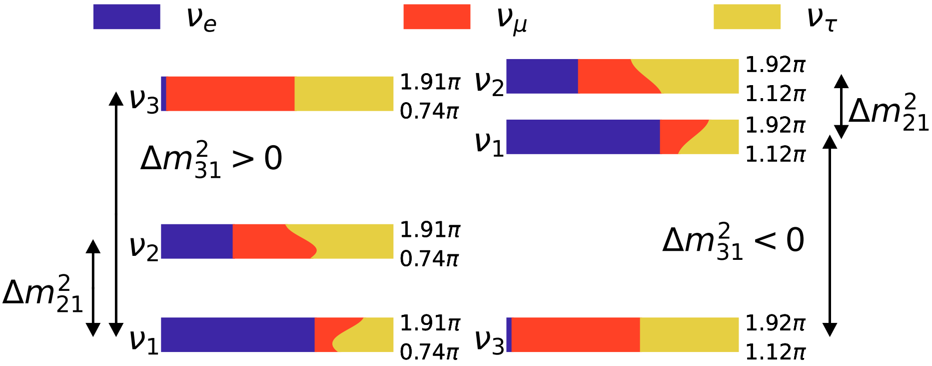
$m_1 < m_2 < m_3$

$\sum m_k \gtrsim 0.06$ eV

Inverted ordering (IO)

$m_3 < m_1 < m_2$

$\sum m_k \gtrsim 0.1$ eV



Absolute scale unknown!

Can we constrain the mass ordering using bounds on $\sum m_\nu$?

Neutrino masses from neutrinoless double β decay

(if neutrino is Majorana)

[Schechter&Valle, 1982]

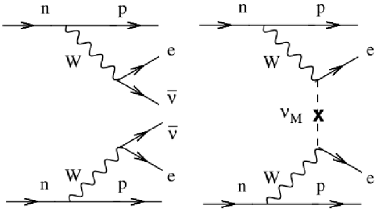
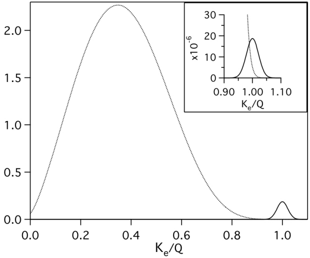
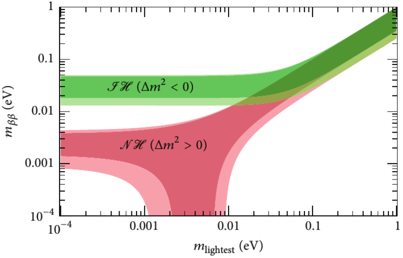


figure from [NEXT] webpage





[Dell'Oro+, 2016]

Measure $T^{0\nu}_{1/2}$

m_e electron mass,
$G_{0\nu}$ phase space,
$\mathcal{M}'^{\nu}$ matrix element

convert into $m_{\beta\beta} = \dfrac{m_e}{\mathcal{M}'^{\nu}\sqrt{G_{0\nu}\, T^{0\nu}_{1/2}}}$

α_k Majorana phases

and then use $m_{\beta\beta} = \left|\sum_k e^{i\alpha_k} U^2_{ek}\, m_k\right|$

β decay

β decay: $\mathcal{N}(A,Z) \longrightarrow \mathcal{N}(A,Z+1) + e^- + \bar{\nu}_e$

$$Q_\beta = M_i - M_f - m_e$$

total available energy

$$E_\nu = Q_\beta - T = Q_\beta - (E_e - m_e)$$

neutrino energy

notice that max electron energy is:

$$T_{\rm max} = Q_\beta - m_{\bar{\nu}_e}$$

Kurie function: (degenerate ν masses)

$$K(T) = \left[(Q_\beta - T)\sqrt{(Q_\beta - T)^2 - m^2_{\bar{\nu}_e}}\right]^{1/2}$$

Useful to describe the e^- spectrum near the endpoint

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notice: flavor neutrinos have no definite mass!

$$m^2_{\bar{\nu}_e} = \sum |U_{ei}|^2 m_i^2$$

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notice: flavor neutrinos have no definite mass!

$$m^2_{\bar{\nu}_e} = \sum |U_{ei}|^2 m_i^2$$

Full expression:

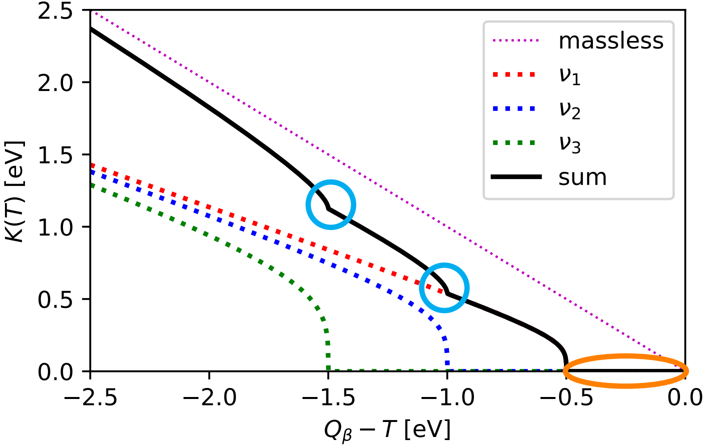
$$K(T) = \left[(Q_\beta - T)\sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2}\right]^{1/2}$$

N_ν neutrinos with different masses m_i

mixing angles enter ($|U_{ei}|^2$)

β decay

$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$



Fake case:
3 neutrinos

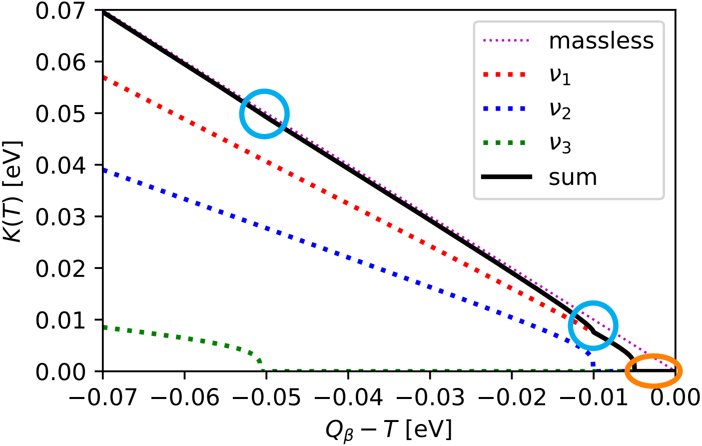
masses:
$m_i = i \cdot 0.5$ eV,

mixings:
$|U_{ei}|^2 = 1/3$

endpoint shifted + one kink for each mass eigenstate

β decay

$$K(T) = \left[(Q_\beta - T) \sum_{i=1}^{N_\nu} |U_{ei}|^2 \sqrt{(Q_\beta - T)^2 - m_i^2} \right]^{1/2}$$



Realistic case:
3 neutrinos, normal ordering

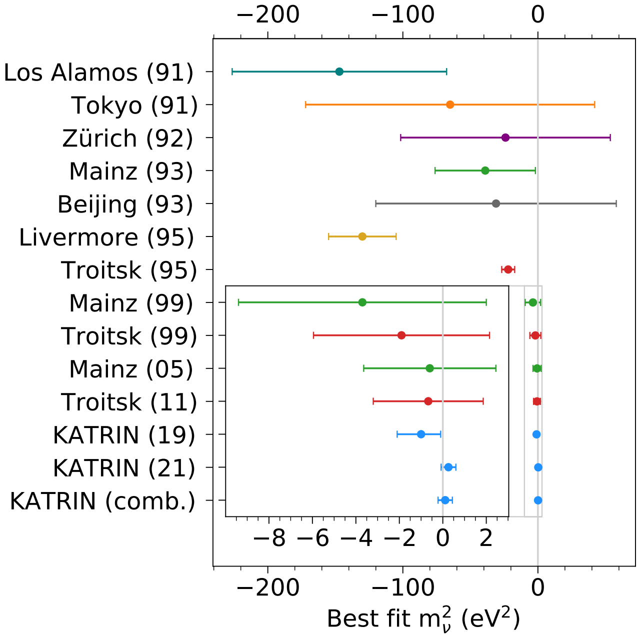
masses: $m_i = [5, 10, 51]$ meV,

mixings: $|U_{ei}|^2 = [0.67, 0.31, 0.02]$

Much harder to see the endpoint shift and kinks!

KATRIN results

[KATRIN, arxiv:2105.08533]

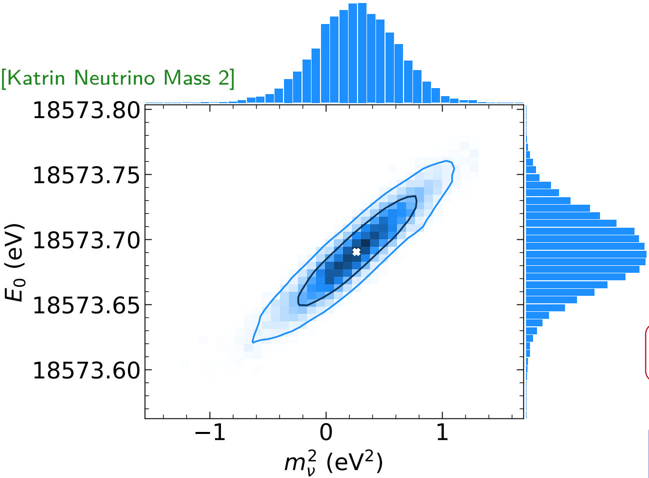


KATRIN results

[KATRIN, arxiv:2105.08533]

strongest bound on $m_\nu (\equiv m_{\bar{\nu}_e})$ are from KATRIN

[Katrin Neutrino Mass 2]



KNM1+KNM2:
$m_\nu^2 = (0.1 \pm 0.3)$ eV^2

Upper limit 90%:
$m_\nu < 0.8$ eV

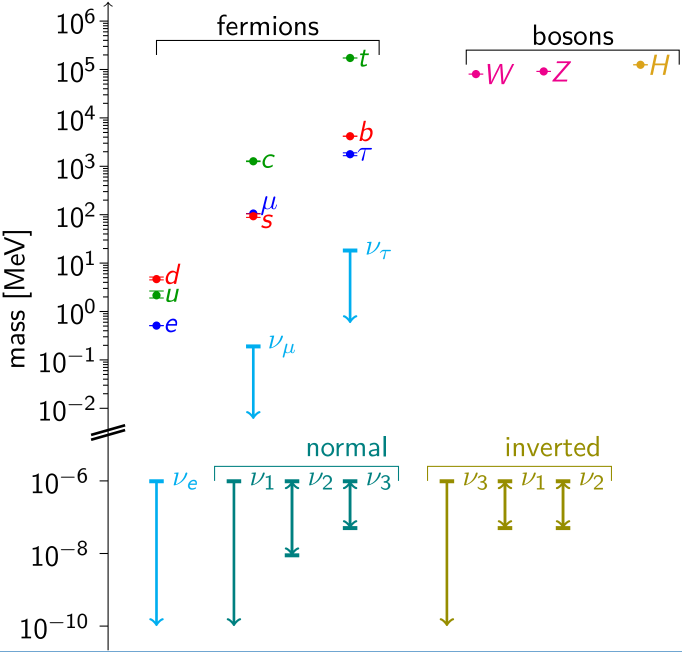
Bayesian 90%:
$m_\nu < 0.7$ eV

statistics dominated!

expected final sensitivity (90%):
$m_\nu \lesssim 0.2$ eV

Masses in the Standard Model

[masses from PDG 2020]



Short Baseline (SBL) anomalies

[SG+, JPG 43 (2016) 033001]

Do three-neutrino oscillations explain all experimental results?

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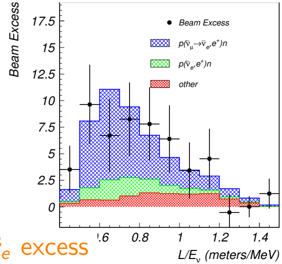
LSND



[Aguilar+, 2001]

3.8σ

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

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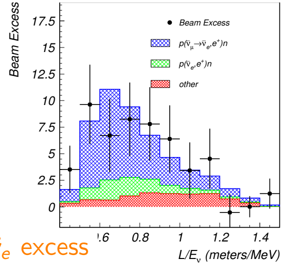

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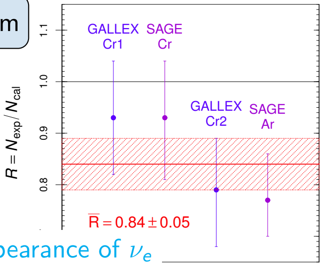
Gallium



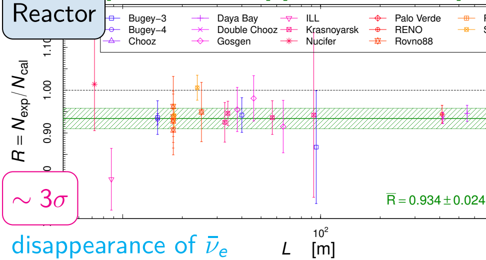
[Giunti, Laveder, 2011]

2.7σ

disappearance of ν_e

Reactor

[Mention+, 2011] [Azabajian+, 2012]



$\sim 3\sigma$

disappearance of $\bar{\nu}_e$

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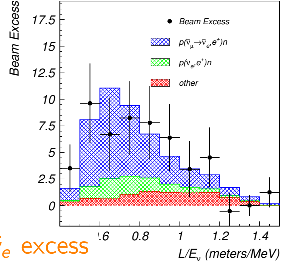

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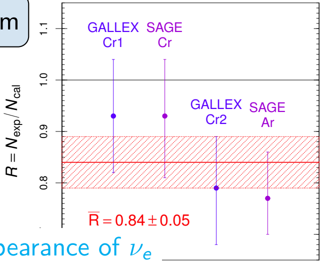

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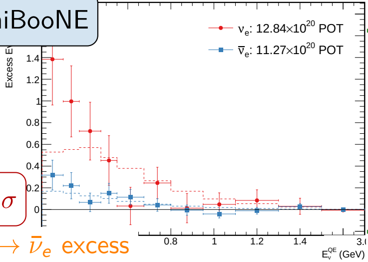
MiniBooNE



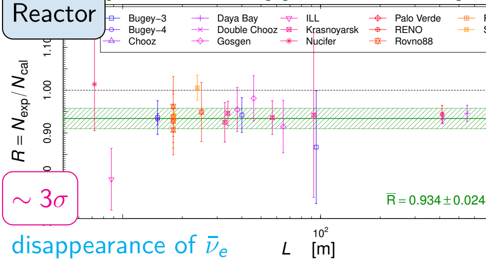
[Aguilar+, 2008-2018]

$\sim 5\sigma$

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

Reactor

[Mention+, 2011] [Azabajian+, 2012]



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disappearance of $\bar{\nu}_e$

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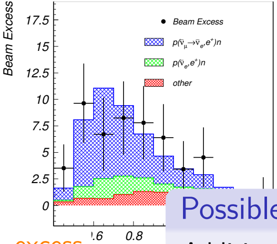
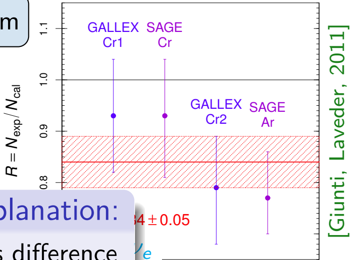


3.8σ

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

Gallium

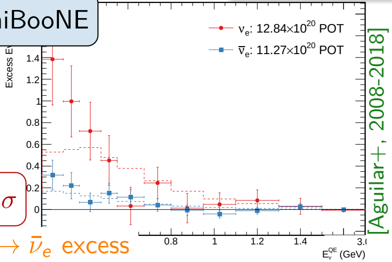


Possible common explanation:

Additional squared mass difference $\Delta m^2_{SBL} \simeq 1\ \mathrm{eV}^2$

[Mention+, 2011] [Azabajian+, 2012]

MiniBooNE



$\sim 5\sigma$

$\bar{\nu}_\mu \to \bar{\nu}_e$ excess

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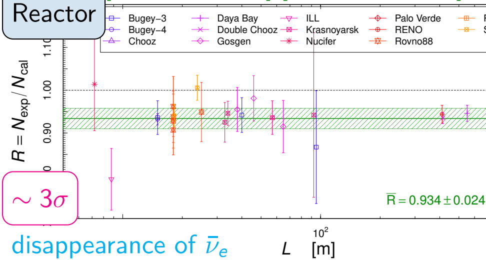


$\sim 3\sigma$

disappearance of $\bar{\nu}_e$

A large family

In principle, previous discussion is valid for N neutrinos

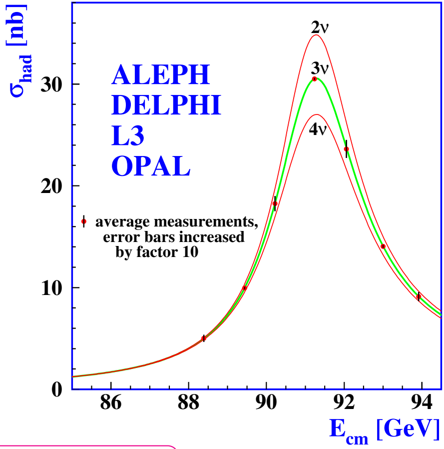
only constraint: there are exactly three flavor neutrinos in the SM

[LEP, Phys. Rept. 427 (2006) 257]
$N_\nu^{(Z)} = 2.9840 \pm 0.0082$

[Janot+, PLB 2020]
$N_\nu^{(Z)} = 2.9963 \pm 0.0074$

from measurement of Z resonance

$$e^+e^- \longrightarrow Z \longrightarrow \sum_{a=e,\mu,\tau} \nu_a\bar{\nu}_a$$



neutrinos $\alpha > 3$ must be sterile

sterile neutrino = SM singlet: no couplings with other SM particles

A large family

In principle, previous discussion is valid for N neutrinos

$N \times N$ mixing matrix, N flavor neutrinos, N massive neutrinos

$$\begin{pmatrix} |\nu_e\rangle \\ |\nu_\mu\rangle \\ |\nu_\tau\rangle \\ |\nu_{s_1}\rangle \\ \dots \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} & U_{e4} & \vdots \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \\ U_{s_1 1} & U_{s_1 2} & U_{s_1 3} & U_{s_1 4} & \\ \dots & & & & \ddots \end{pmatrix} \begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \\ \dots \end{pmatrix}$$

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$$
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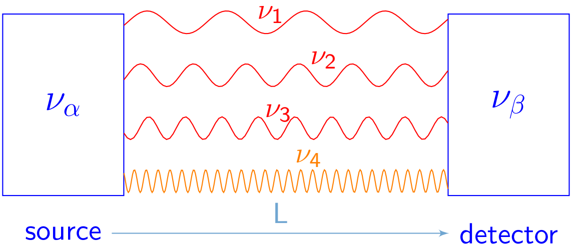
=
\begin{pmatrix}
U_{e1} & U_{e2} & U_{e3} & U_{e4} & \vdots \\
U_{\mu 1} & U_{\mu 2} & U_{\mu 3} & U_{\mu 4} & \\
U_{\tau 1} & U_{\tau 2} & U_{\tau 3} & U_{\tau 4} & \\
U_{s_1 1} & U_{s_1 2} & U_{s_1 3} & U_{s_1 4} & \\
\cdots & & & & \ddots
\end{pmatrix}
\begin{pmatrix} |\nu_1\rangle \\ |\nu_2\rangle \\ |\nu_3\rangle \\ |\nu_4\rangle \\ \cdots \end{pmatrix}
$$

Our case will be 3 (active)+1 (sterile), a perturbation of 3 neutrinos case

$m_4 \gg m_\ell$

$\ell = 1, 2, 3$

$\Delta m^2_{41} \simeq 1\ \text{eV}^2 \simeq \Delta m^2_{42} \simeq \Delta m^2_{43}$



6 angles

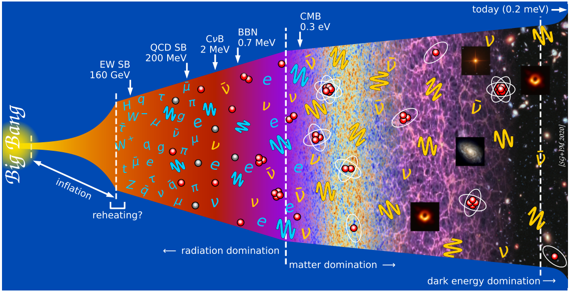
3 Dirac CP phases

3 Majorana CP phases

C Cosmology

Based on:

- Lesgourgues+,
  Neutrino Cosmology



A flash on general relativity

use metric $g_{\mu\nu}$ to define measure: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$

x^μ coordinates, $u^\mu = \frac{dx^\mu}{d\lambda}$ velocity, $P^\mu = m u^\mu$ momentum

short notation for derivatives: $\partial^\mu \equiv \frac{\partial}{\partial x_\mu}$, $\partial_\mu \equiv \frac{\partial}{\partial x^\mu}$

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Christoffel symbols (not tensors!): $\Gamma^\mu_{\nu\rho} = \frac{g^{\mu\sigma}}{2}\left(\partial_\nu g_{\sigma\rho} + \partial_\rho g_{\nu\sigma} - \partial_\sigma g_{\nu\rho}\right)$

Ricci tensor: $R_{\mu\nu} = \partial_\sigma \Gamma^\sigma_{\mu\nu} - \partial_\nu \Gamma^\sigma_{\mu\sigma} + \Gamma^\sigma_{\rho\sigma}\Gamma^\rho_{\mu\nu} - \Gamma^\sigma_{\rho\nu}\Gamma^\rho_{\mu\sigma}$

Ricci scalar: $R = R_{\mu\nu} g^{\mu\nu}$

Einstein equations:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G\, T_{\mu\nu} - \Lambda g_{\mu\nu}$$

$T_{\mu\nu}$ stress-energy tensor, symmetric, must satisfy $\nabla_\mu T^{\mu\nu} = 0$

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given lagrangian $\mathcal{L}(\phi_\alpha) \longrightarrow T^{\mu\nu}_{(\phi_\alpha)} = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi_\alpha)} \partial^\nu \phi_\alpha - g^{\mu\nu} \mathcal{L}$

ϕ_α set of fields

Homogeneous and isotropic universe

Metric defines the structure of the universe

One of the simplest assumptions: universe is

Homogeneous

universe properties do not change with position

Isotropic

universe properties do not change with direction

Friedmann-Lemaître-Robertson-Walker (FLRW) metric (polar coordinates):

$$g_{\mu\nu}dx^\mu dx^\nu = -dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right)$$

a scale factor, encodes expansion of the space-time

k spatial curvature of the universe (0→flat, ±1 → curved)

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Perfect fluid (energy density ρ, pressure P): $T^{\mu\nu} = (\rho + P)u^\mu u^\nu - Pg^{\mu\nu}$

in fluid rest frame: $u^\mu = (1,0,0,0) \longrightarrow T^0_0 = -\rho \qquad T^i_j = P\delta^i_j$

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Perfect fluid (energy density ρ, pressure P): $T^{\mu\nu} = (\rho + P) u^\mu u^\nu - P g^{\mu\nu}$

Use Einstein equations to obtain **Friedmann equations**:

$$H^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{\Lambda}{3}$$

expansion rate $H \equiv \dot{a}/a$

expansion depends on universe content!

Background evolution of the universe

conservation of stress-energy tensor:

$$\nabla_\mu T^{\nu\mu} \equiv \partial_\mu T^{\nu\mu} + \Gamma^\nu_{\mu\rho} T^{\mu\rho} + \Gamma^\mu_{\mu\rho} T^{\nu\rho} = 0$$

for a perfect fluid this leads to continuity equation:

$$\boxed{\dot{\rho} + 3H(\rho + P) = 0}$$

define w equation of state, so that $P = w\rho$:

continuity equation solved by $\rho(a) = a^{-3(1+w)}$

radiation	matter	Λ
(relativistic fluid)	(non-rel. fluid)	(dark energy)
$w = 1/3$, $\rho_{\rm R} \propto a^{-4}$	$w = 0$, $\rho_{\rm M} \propto a^{-3}$	$w = -1$, $\rho_\Lambda = const$

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Consider Friedmann equation: $H^2 = \frac{8\pi G}{3}\rho_\mathrm{tot} + \frac{\Lambda}{3} - \frac{k}{a^2}$

$\rho_\Lambda \equiv \frac{\Lambda}{8\pi G}$

$\rho_k \equiv \frac{3k}{8\pi G a^2}$

If one component dominates ($\rho_\mathrm{tot} \simeq \rho_i$, with $i \in [\mathrm{R}, \mathrm{M}, k, \Lambda, \ldots]$), we have:

$a(t) = t^{2/(3(1+w))}$ for $w \neq -1$ $a(t) = e^{Ht}$ for $w = -1$

Background evolution of the universe

conservation of stress-energy tensor:

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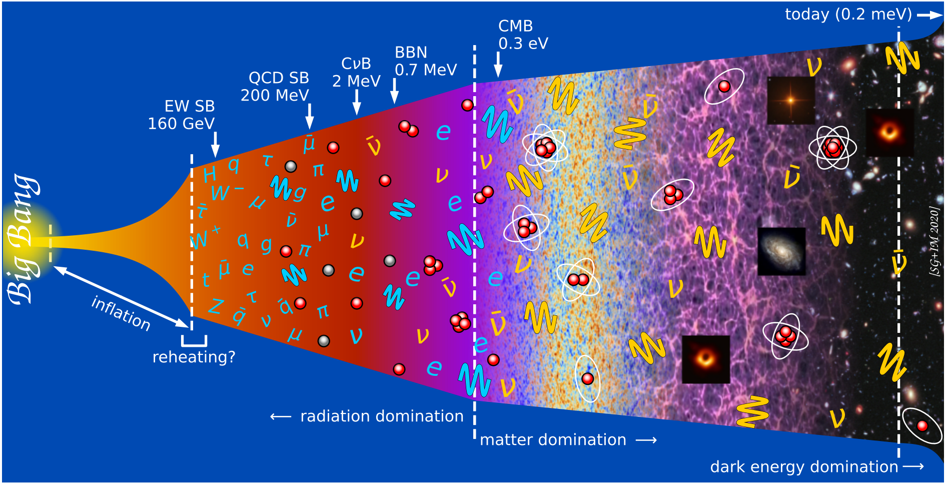
define critical density: $\rho_{\rm cr} \equiv \dfrac{3H^2}{8\pi G}$

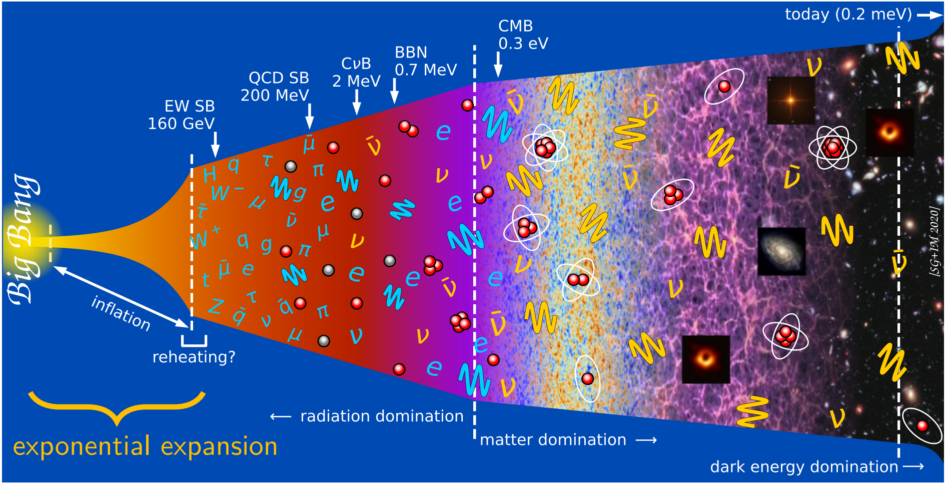
define fractional energy densities: $\Omega_i = \rho_{i,0}/\rho_{\rm cr,0}$ $\quad$ 0→today

Friedmann equation: $H(a)^2/H_0^2 = \Omega_{\rm R} a^{-4} + \Omega_{\rm M} a^{-3} + \Omega_k a^{-2} + \Omega_\Lambda$

Short history of the universe



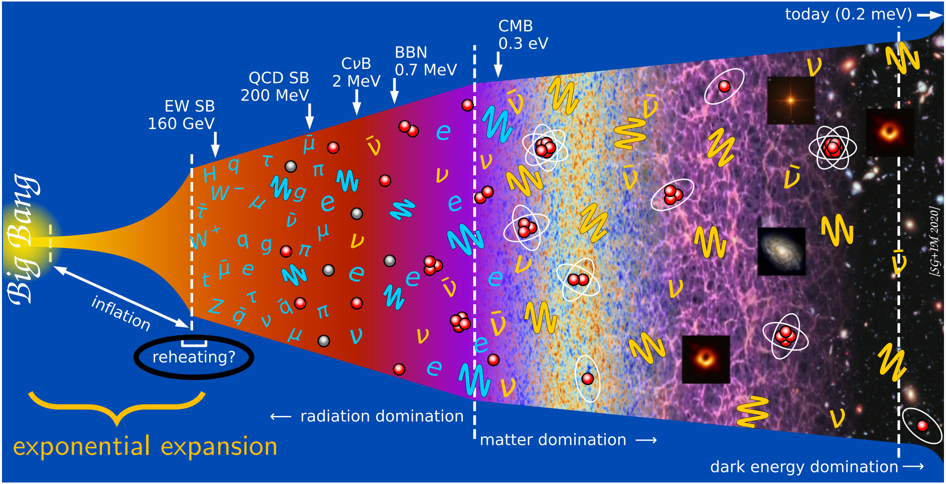
Short history of the universe



exponential expansion achieved with $w = -1$ ⟶ by scalar field? inflaton

inflation is needed to solve: flatness problem horizon problem

Short history of the universe

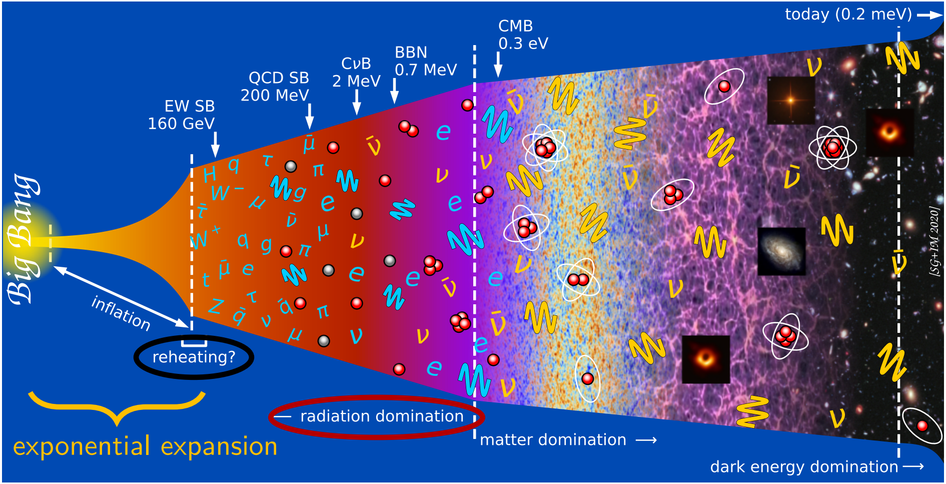


Reheating: inflation ends with energy transfer

from inflaton to (relativistic) standard model particles

which later reach thermal equilibrium

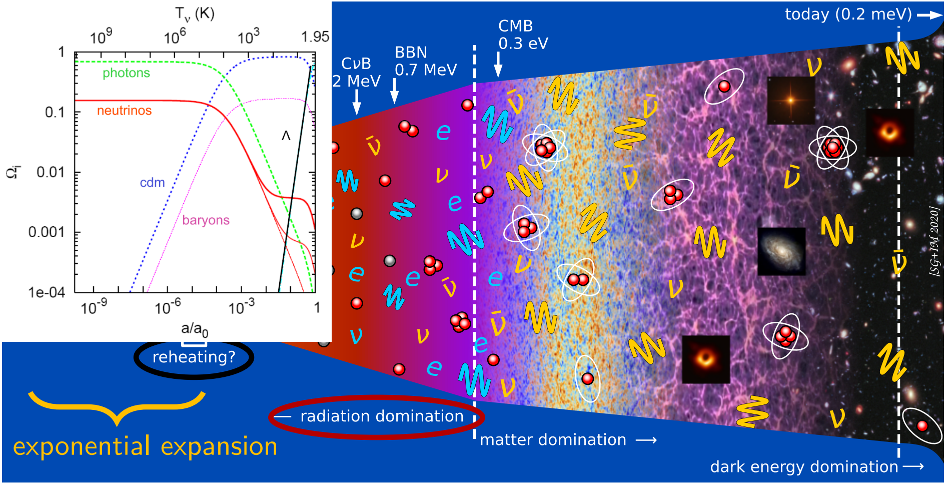
Short history of the universe



after **reheating**, relativistic particles (= radiation) start to dominate

while temperature decreases, several particles become non relativistic

Short history of the universe

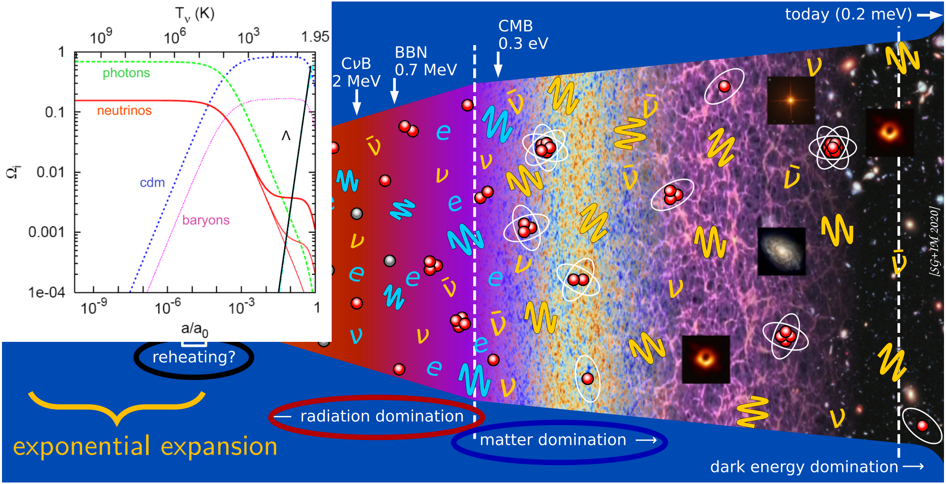


after **reheating**, relativistic particles (= radiation) start to dominate

while temperature decreases, several particles become non relativistic

last particles to remain in equilibrium are photons, electrons, neutrinos

Short history of the universe

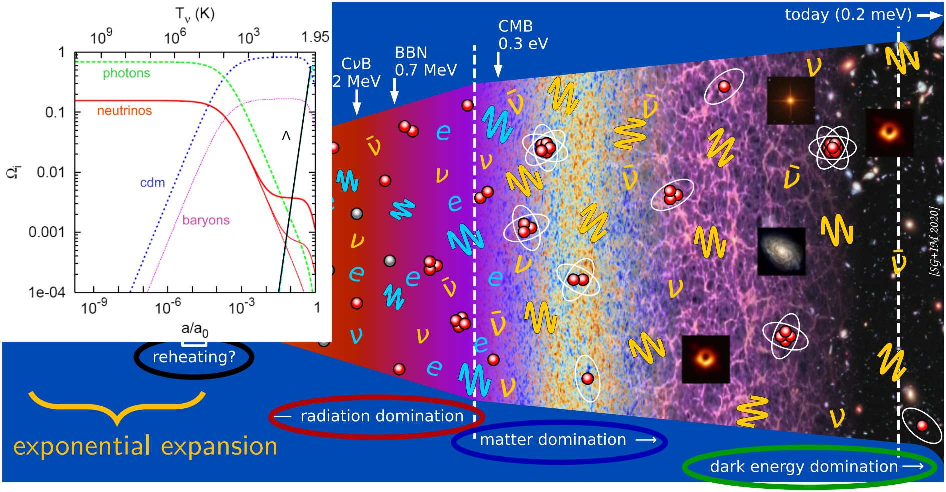


at some point, $\Omega_{\rm R}\, a^{-4}$ becomes smaller than $\Omega_{\rm M}\, a^{-3}$

matter domination!

gravity start to be stronger than radiation pressure ⟶ growth of structures!

Short history of the universe



Finally, $\Omega_M a^{-3}$ becomes smaller than Ω_Λ

dark energy domination!

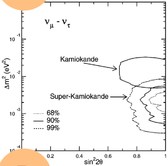
expansion starts to (exponentially) accelerate again

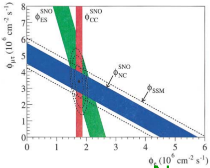
S Summary

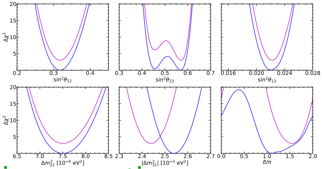
What did we learn about neutrinos and cosmology?

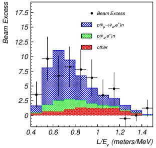
O

Neutrino oscillations



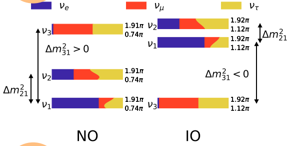


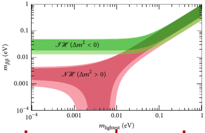


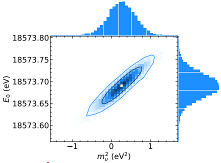


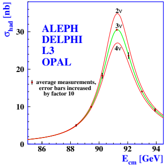
P

Neutrino properties



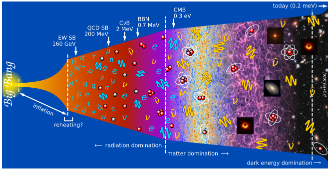


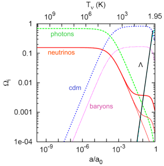




U

Basics on the universe history





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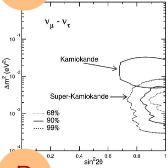

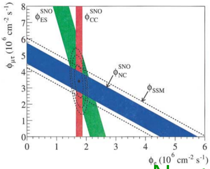

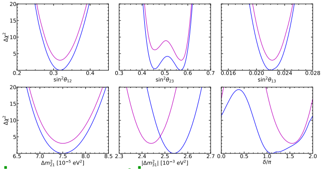

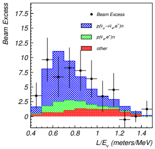

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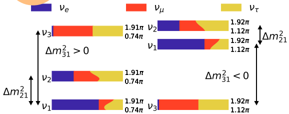
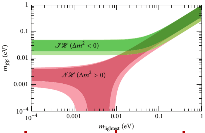
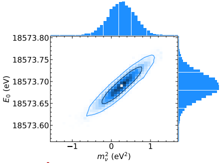
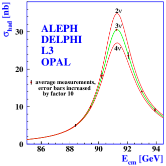


NO IO







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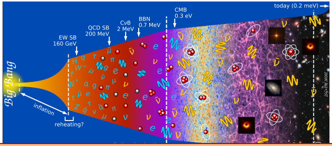

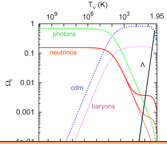

Thanks for your attention!