QUANTIFYING UNCERTAINTIES IN NUCLEAR MATRIX ELEMENTS FOR DARK MATTER SEARCHES

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MANY THANKS TO MY COLLABORATORS

- Christian Forssén,
  Ricardo Catena
  (Chalmers, Sweden)
- Ragnar Stroberg
  (University of Washington, USA)
- Jason Holt,
  Petr Navrátil
  (TRIUMF, Canada)

OUTLINE

- **MOTIVATION**
  - Dark matter
  - Dark matter searches
- **NUCLEAR STRUCTURE INPUTS FOR DARK MATTER SEARCHES**
  - Nuclear response functions (form factors, matrix elements)
  - Ab initio no-core shell model
  - Theoretical uncertainties
- **SUMMARY & OUTLOOK**

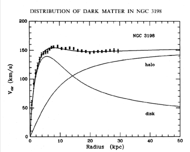
MOTIVATION

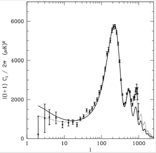
Evidence for dark matter

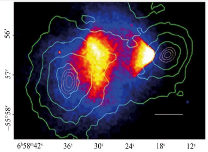
Dark matter makes up about 85% of the total matter in the Universe.

- rotational curves of galaxies
- cosmic microwave background, large structure formation
- gravitational lensing, Bullet Cluster
- ...

New type of particle provides simple explanation. WIMP is a well motivated candidate.





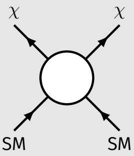


DARK MATTER SEARCHES

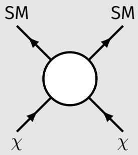
WIMP

- particle with $m_\chi \sim 100$ GeV
- interacts with Standard Model fields at $\sim$ EW scale

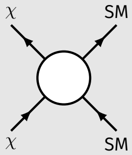
WIMP searches



production
collider searches

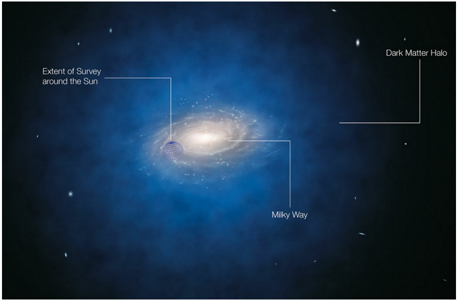


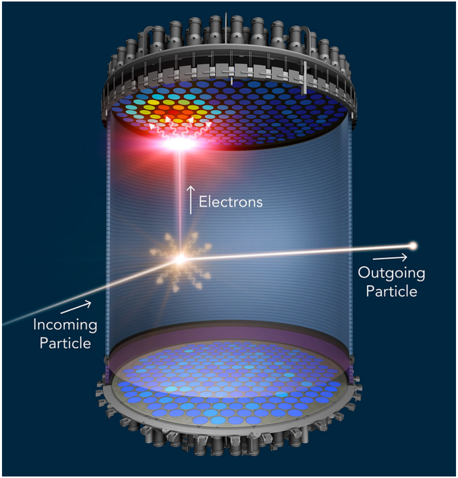
annihilation
indirect searches
γ, ν, CR telescopes



scattering
direct detection
deep-underground detectors

DARK MATTER DIRECT DETECTION

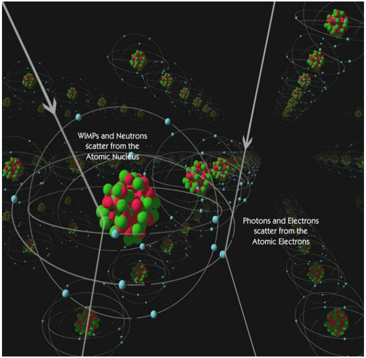




[Taken from: LZ collaboration.]

- Galaxies submerged in halos of DM particles
- Deep-underground low-background experiments searching for WIMP-induced nuclear/electronic recoils

DARK MATTER DIRECT DETECTION & NUCLEAR PHYSICS



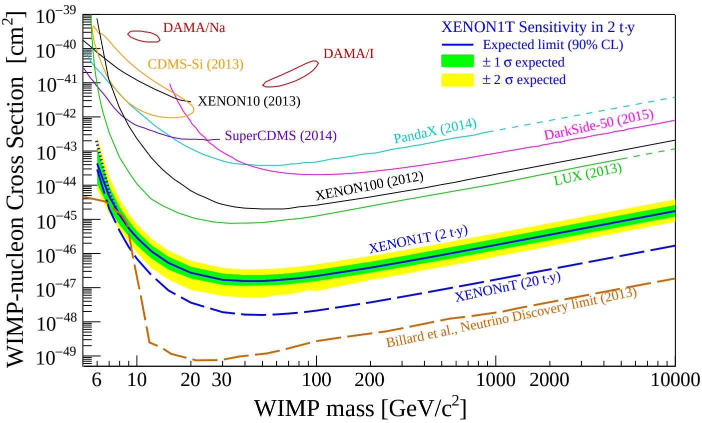
[Taken from: CDMS collaboration]

Typical (expected) nuclear recoil momentum can reach

$$q \approx 200 \text{ MeV} \sim m_\pi \longleftrightarrow \text{length scale } r \sim \frac{1}{q} \approx 1 \text{ fm}$$

Nuclear structure is resolved!

CURRENT STATUS OF DARK MATTER DIRECT DETECTION



- **2017**: PandaX-II (arXiv:1708.06917 [astro-ph.CO])
- **2018**: XENON1T Phys. Rev. Lett. 121, 111302 (2018)
- **2019**: XENONnT, **2019+** LZ
- **2020+**: DARWIN (will reach the neutrino floor)

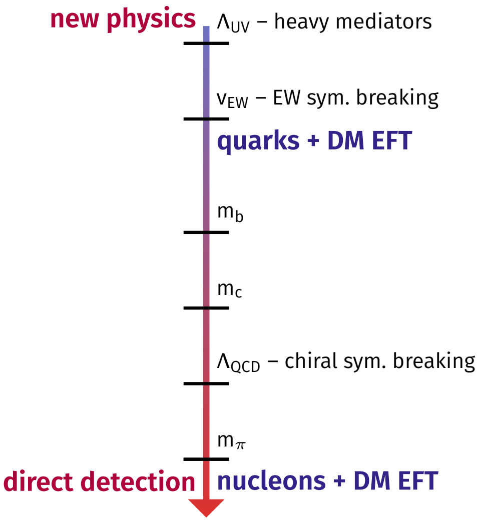
- Null results: upper bounds on dark matter–nucleon cross section

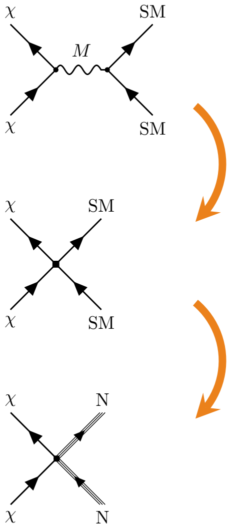
LARGE-SCALE AB INITIO NUCLEAR STRUCTURE COMPUTATIONS FOR DARK MATTER SEARCHES

AIMS

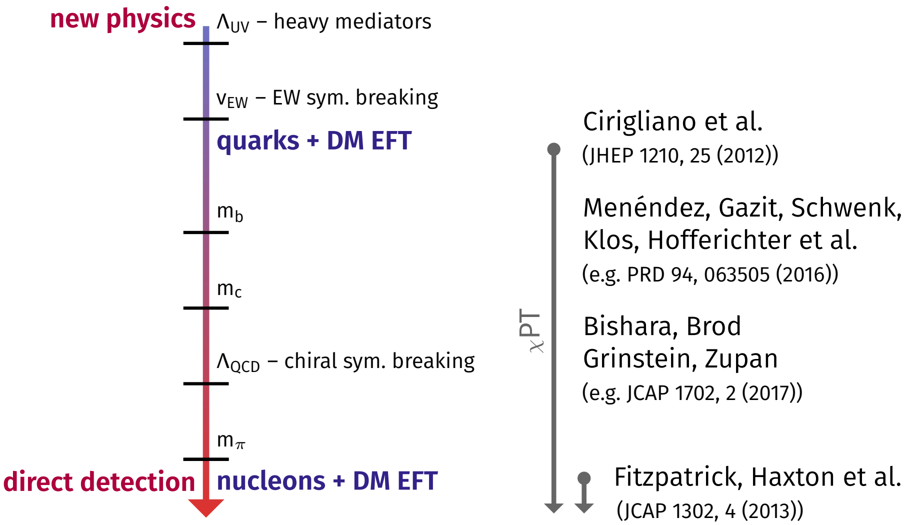
- Establish a new ab initio framework for nuclear structure calculations in the context of dark matter searches
- Apply ab initio nuclear many-body methods in calculations of WIMP scattering off:
    - ^{3}**He**, ^{4}**He** (detectors in R&D phase)
      Jacobi-coordinate-NCSM [Phys. Rev. D 95, 103011 (2017)]
    - ^{16}**O** (CRESST-II), ^{19}**F** (PICO),
      Slater-Derminant-NCSM [in progress, unpublished]
    - ^{23}**Na** (DAMA/LIBRA, COSINE-100, COSINUS), ^{40}**Ca** (CRESST-II), **Ge** (SuperCDMS), ... **Xe** (XENON)
      IMSRG valence-space interactions + shell model [in progress, unpublished]
- Quantify the impact of nuclear structure uncertainties on the interpretation of data from dark matter searches.

DARK MATTER–NUCLEUS INTERACTION





DARK MATTER–NUCLEUS INTERACTION



NONRELATIVISTIC EFT FOR DM–NUCLEON INTERACTION

- The most general form of dark matter–nucleon interaction
  [Fitzpatrick et al., JCAP 1302, 4 (2013)]
  - No evidence to justify a simple form!

$$\mathcal{L}_{\text{int}} = \sum_{N=n,p} c_i^{(N)}\, \hat{O}_i\, \chi^\dagger \chi\, N^\dagger N$$

  - All possible DM–nucleon interaction terms (up to $\mathbf{q}^2$):

$$\hat{O}_1 = 1_{\chi N}$$
$$\hat{O}_3 = i\hat{\mathbf{S}}_N \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right)$$
$$\hat{O}_4 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{S}}_N$$
$$\hat{O}_5 = i\hat{\mathbf{S}}_\chi \cdot \left(\frac{\hat{\mathbf{q}}}{m_N} \times \hat{\mathbf{v}}^\perp\right)$$
$$\hat{O}_6 = \left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)$$
$$\hat{O}_7 = \hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp$$
$$\hat{O}_8 = \hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp$$
$$\hat{O}_9 = i\hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \frac{\hat{\mathbf{q}}}{m_N}\right)$$
$$\hat{O}_{10} = i\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}$$
$$\hat{O}_{11} = i\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}$$
$$\hat{O}_{12} = \hat{\mathbf{S}}_\chi \cdot \left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right)$$
$$\hat{O}_{13} = i\left(\hat{\mathbf{S}}_\chi \cdot \hat{\mathbf{v}}^\perp\right)\left(\hat{\mathbf{S}}_N \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)$$
$$\hat{O}_{14} = i\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left(\hat{\mathbf{S}}_N \cdot \hat{\mathbf{v}}^\perp\right)$$
$$\hat{O}_{15} = -\left(\hat{\mathbf{S}}_\chi \cdot \frac{\hat{\mathbf{q}}}{m_N}\right)\left[\left(\hat{\mathbf{S}}_N \times \hat{\mathbf{v}}^\perp\right) \cdot \frac{\hat{\mathbf{q}}}{m_N}\right]$$

  - Chiral EFT: hierarchy + relations between $\hat{O}_i$, meson-exchange currents

NONRELATIVISTIC EFT FOR DM–NUCLEUS INTERACTION

Rate of nuclear scattering events in direct detection experiments:

$$\frac{d\mathcal{R}}{dq^2} = \frac{\rho_\chi}{m_A m_\chi} \int d^3\vec{v}\, f(\vec{v} + \vec{v}_e) v \frac{d\sigma}{dq^2}$$

- astrophysics $\to$ m_χ, ρ_χ, f - dark matter mass, density, velocity distributions
- particle and nuclear physics $\to$ $\frac{d\sigma}{dq^2}$

Scattering cross section:

$$\frac{d\sigma}{dq^2} = \frac{1}{(2J+1)v^2} \sum_{\tau,\tau'} \left[\sum_{\ell=M,\Sigma',\Sigma''} R_\ell^{\tau\tau'} W_\ell^{\tau\tau'} + \frac{q^2}{m_N^2} \sum_{\substack{\ell=\Phi'',\Phi''M,\\ \tilde{\Phi}',\Delta,\Delta\Sigma'}} R_\ell^{\tau\tau'} W_\ell^{\tau\tau'} \right]$$

- dark matter response functions $R_m^{\tau\tau'}\left(v_T^{\perp 2}, \frac{q^2}{m_N^2}, c_i^\tau c_j^{\tau'}\right)$
- nuclear response functions $W_\ell^{\tau\tau'}(q^2)$

Uncertainties?

- ρ_χ: $\pm$30%, $f(\vec{v})$: $\pm$? (important only for light DM), $W_l^{\tau\tau'}$: $\pm$?

- nuclear response functions:

$$W_{AB}^{\tau\tau'}(q^2) = \sum_{L\leq 2J} \langle \Psi \| \hat{A}_{L;\tau}(q) \| \Psi \rangle \langle \Psi \| \hat{B}_{L;\tau'}(q) \| \Psi \rangle$$

  - $\hat{A}_{L;\tau}$, $\hat{B}_{L;\tau}$ – nuclear response operators:

$$M_{LM;\tau}(q) = \sum_{i=1}^{A} M_{LM}(q\boldsymbol{\rho}_i)\, t_{(i)}^{\tau}, \quad \Sigma'_{LM;\tau}(q) = -i \sum_{i=1}^{A} \left[\frac{1}{q} \overrightarrow{\nabla}_{\rho_i} \times \mathbf{M}_{LL}^{M}(q\boldsymbol{\rho}_i) \right] \cdot \vec{\sigma}_{(i)} t_{(i)}^{\tau},$$

$$\Sigma''_{LM;\tau}(q) = \sum_{i=1}^{A} \left[\frac{1}{q} \overrightarrow{\nabla}_{\rho_i} M_{LM}(q\boldsymbol{\rho}_i) \right] \cdot \vec{\sigma}_{(i)} t_{(i)}^{\tau}, \quad \Delta_{LM;\tau}(q) = \sum_{i=1}^{A} \mathbf{M}_{LL}^{M}(q\boldsymbol{\rho}_i) \cdot \frac{1}{q} \overrightarrow{\nabla}_{\rho_i} t_{(i)}^{\tau},$$

$$\tilde{\Phi}'_{LM;\tau}(q) = \sum_{i=1}^{A} \left[\left(\frac{1}{q} \overrightarrow{\nabla}_{\rho_i} \times \mathbf{M}_{LL}^{M}(q\boldsymbol{\rho}_i) \right) \cdot \left(\vec{\sigma}_{(i)} \times \frac{1}{q} \overrightarrow{\nabla}_{\rho_i} \right) + \frac{1}{2} \mathbf{M}_{LL}^{M}(q\boldsymbol{\rho}_i) \cdot \vec{\sigma}_{(i)} \right] t_{(i)}^{\tau},$$

$$\Phi''_{LM;\tau}(q) = i \sum_{i=1}^{A} \left(\frac{1}{q} \overrightarrow{\nabla}_{\rho_i} M_{LM}(q\boldsymbol{\rho}_i) \right) \cdot \left(\vec{\sigma}_{(i)} \times \frac{1}{q} \overrightarrow{\nabla}_{\rho_i} \right) t_{(i)}^{\tau}$$

  - many-body nuclear wave function $|\Psi\rangle$

AB INITIO NO-CORE SHELL MODEL

Given a Hamiltonian operator solve the eigenvalue problem of A nucleons

$$\left[\sum_{i\leq A}\frac{\hat{\mathbf{p}}_i^2}{2m}+\sum_{i<j\leq A}\hat{V}_{NN;ij}+\sum_{i<j<k\leq A}\hat{V}_{NNN;ijk}\right]\Psi=E\Psi$$

Ab initio

- all particles are active (no rigid core)
- exact Pauli principle
- realistic internucleon interactions
- controllable approximations

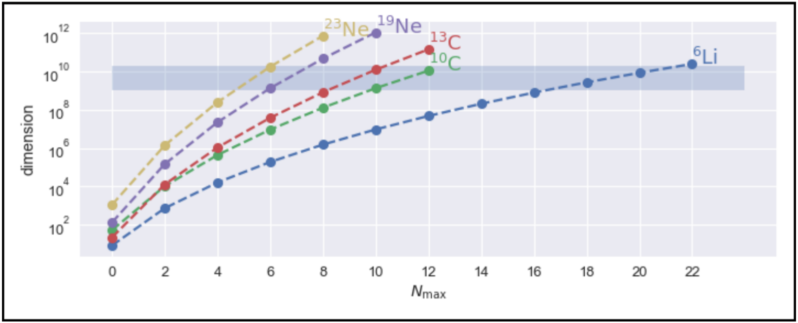
- Hamiltonian is diagonalized in a finite A-particle harmonic oscillator basis

$$\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_A)=\sum_{n\leq N_{tot}}\Phi_n^{HO}(\mathbf{r}_1,\ldots,\mathbf{r}_A)$$

(matrix dimensions up to $\sim 10^{10}$ with $\sim 10^{14}$ nonzero elements)

AB INITIO NO-CORE SHELL MODEL: THE CURSE OF DIMENSIONALITY

- Basis dimensions for p- and sd-shell nuclei:



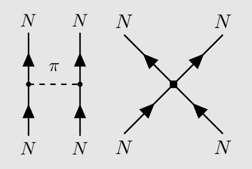
Taken from: C. Forssén, `pAntoine` code.

- Symmetry-Adapted-NCSM (T. Dytrych et al. @ LSU)
    - Exploits dynamical symmetries to select relevant basis states.
    - Calculations up to pf-shell nuclei possible! (^{48}Ca, ^{48}Ti – 8 shells)

INPUT HAMILTONIANS

- **V_{NN} + V_{NNN} potentials derived from chiral EFT**
  - long-range part of the interaction, π-exchange, predicted by chiral perturbation theory
  - short-range part parametrized by contact interactions, LECs fitted to experimental data

At LO



  - **NNLO$_{sim}$** [Carlsson et al., PRX 6, 011019 (2016)]
    - parameters fitted to reproduce simultaneously πN, NN, and NNN low-energy observables
    - family of 42 Hamiltonians where the experimental uncertainties propagate into LECs
    - all Hamiltonians give equally good description on the fit data
  - **NNLO$_{opt}$** [A. Ekström et al., PRL 110, 192502 (2013)]
    optimized 2-nucleon V_{NN}; found to minimize the effect of V_{NNN}

RESULTS
Nuclear response functions

^{4}He TARGET: NUCLEAR RESPONSE FUNCTIONS AND RECOIL RATES

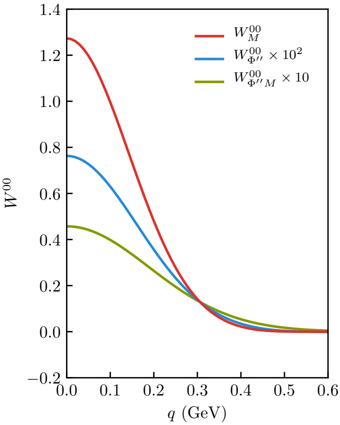


Figure 1: Isoscalar nuclear response functions of ^{4}He as functions of the recoil momentum q calculated within ab initio NCSM using NNLO$_{\text{sim}}$.

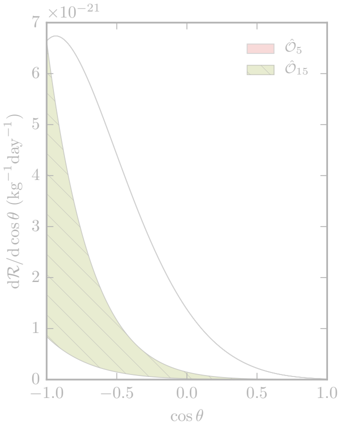
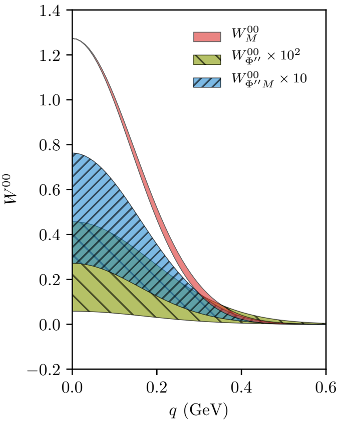


Figure 2: Differential rate of nuclear recoil events as a function of the recoil direction.

- only W^{00}_M, $W^{00}_{\Phi''}$ and $W^{00}_{\Phi'' M}$ due to $J = T = 0$
- for $q \to 0$: $W^{00}_M \propto A^2$ and $W^{00}_{\Phi''} \propto \langle \sum_i^A \mathbf{l}_{(i)} \cdot \boldsymbol{\sigma}_{(i)} \rangle^2$

^{4}He TARGET: NUCLEAR RESPONSE FUNCTIONS AND RECOIL RATES



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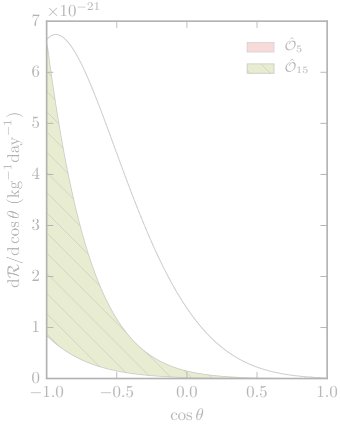

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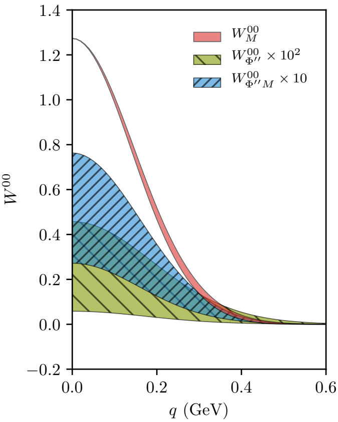
^{4}He TARGET: NUCLEAR RESPONSE FUNCTIONS AND RECOIL RATES



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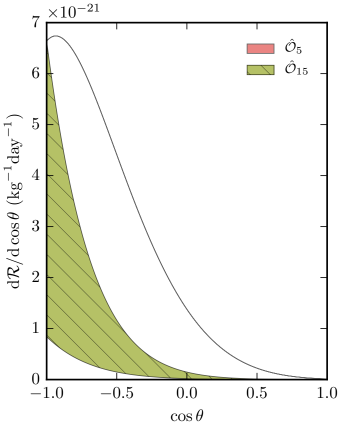

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^{3}He TARGET: NUCLEAR RESPONSE FUNCTIONS AND RECOIL RATES

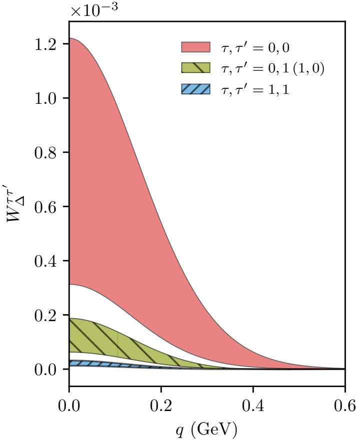


Figure 3: Nuclear response functions of ^{3}He as functions of the recoil momentum q calculated within ab initio NCSM.

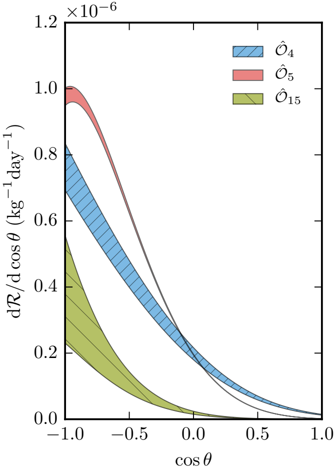


Figure 4: Differential rate of nuclear recoil events as a function of the recoil direction.

- for $q \to 0$: $W_\Delta^{00} \propto \langle \sum_i^A \mathbf{l}_{(i)} \rangle^2$

^{16}O NUCLEAR RESPONSE FUNCTIONS

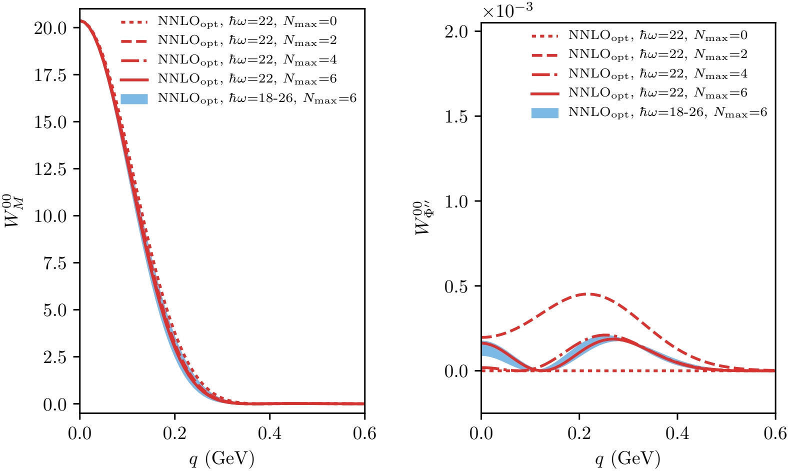


Figure 5: Isoscalar nuclear response functions W_M^{00} and $W_{\Phi''}^{00}$ of ^{16}O as functions of the recoil momentum q calculated within ab initio NCSM using NNLO$_{\rm opt}$.

- only W_M^{00}, $W_{\Phi''}^{00}$ and $W_{\Phi''M}^{00}$ due to $J = T = 0$
- for $q \to 0$: $W_M^{00} \propto A^2$ and $W_{\Phi''}^{00} \propto \langle \sum_i^A \mathbf{l}(i) \cdot \boldsymbol{\sigma}(i) \rangle^2$

^{19}F NUCLEAR RESPONSE FUNCTIONS

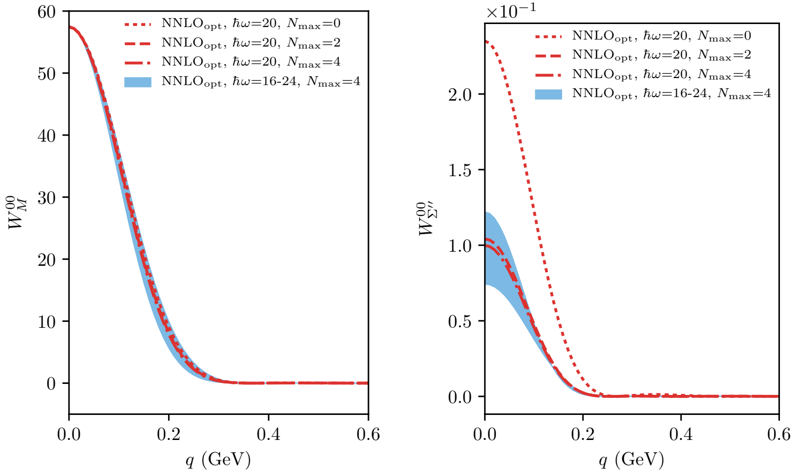
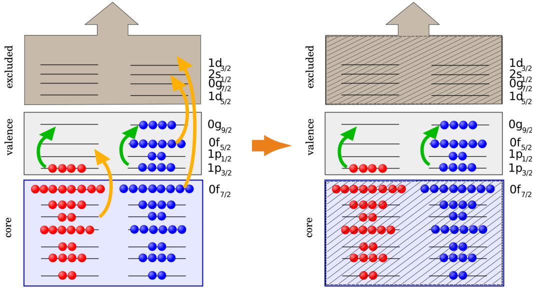


Figure 6: Isoscalar nuclear response functions W_M^{00} and $W_{\Sigma''}^{00}$ of ^{19}F as functions of the recoil momentum q calculated within ab initio NCSM using NNLO$_{opt}$.

- for $q \to 0$: $W_M^{00} \propto A^2$, $W_{\Sigma''}^{00} \propto \langle \sum_i^A \boldsymbol{\sigma}_{(i)} \rangle^2$

SHELL MODEL WITH IMSRG VALENCE-SPACE INTERACTIONS

- For large number of particles NCSM becomes intractable
- Unitary transformation $\tilde{H} = UHU^{\dagger}$ which decouples valence-space orbits and provides effective interaction and operators for shell-model calculations:



[Taken from: R. Stroberg]

- Broad range of applicability $2 \lesssim A \lesssim 100$
- Starts with realistic $V_{NN} + V_{NNN}$ interactions
- Consistent evolution of all operators (no “quenching” factors!)

^{23}Na, ^{40}Ca, ... NUCLEAR RESPONSE FUNCTIONS

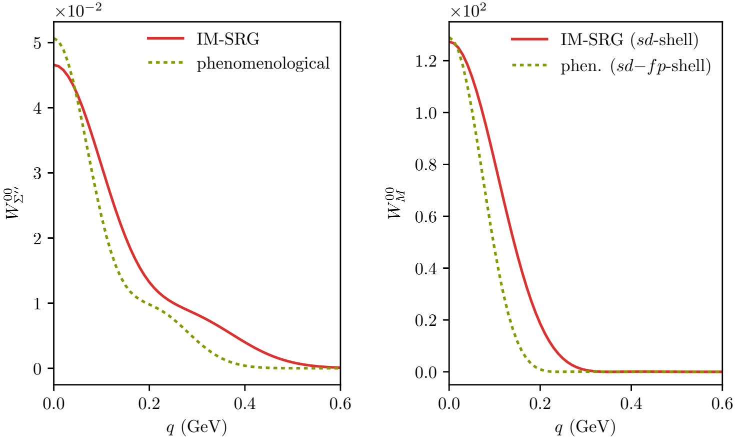


Figure 7: Isoscalar nuclear response functions $W^{00}_{\Sigma''}$ and W^{00}_{M} of ^{23}Na and ^{40}Ca as functions of the recoil momentum q calculated within SM (^{16}O core + sd-shell) using IMSRG (EM 1.8/2.0) and phenomenological (w) interactions.

- for $q \to 0$: $W^{00}_{\Sigma''} \propto \langle \sum_i^A \boldsymbol{\sigma}_{(i)} \rangle^2$, $W^{00}_{M} \propto A^2$
- TODO evolution of operators

SUMMARY & OUTLOOK

SUMMARY & OUTLOOK

Summary

- Ab initio framework for computation of nuclear response functions for dark matter scattering off nuclei have been developed.
- Certain nuclear response functions suffer from large uncertainties which propagate into physical observables.

[Phys. Rev. D 95, 103011 (2017)]

Outlook & future work

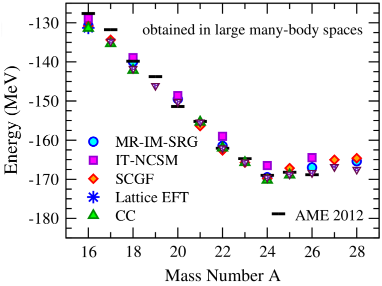
- Apply the framework in:
  - nuclear beta decays, $\beta - \nu$ correlations
  - nuclear double-beta decays

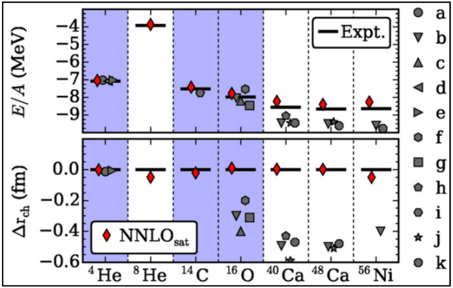
Thank you!

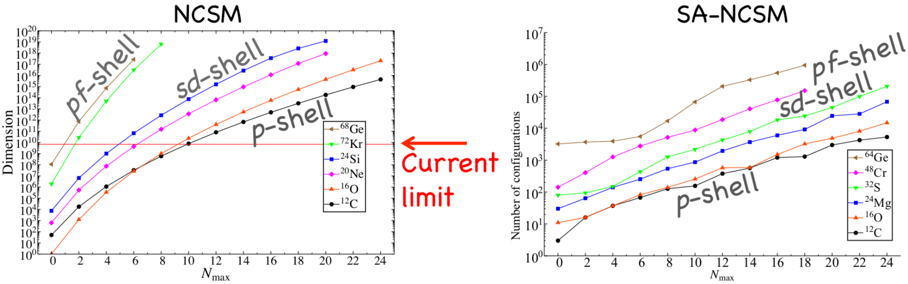
Backup slides

REACH OF AB INITIO METHODS

- Many-body methods with polynomial scaling (CC, SCGF, IMSRG) reach Ca, Ni region and even beyond
- Precision of computational methods exceeds accuracy of available nuclear interactions







[Taken from: K. Launey]

NCSM – SA-NCSM: $\hbar\omega$, N_{max}