

# Aurora experiment: Final results of studies of $^{116}\text{Cd}$ $2\beta$ decay with enriched $^{116}\text{CdWO}_4$ crystal scintillators

A.S. Barabash<sup>1</sup>, P. Belli<sup>2,3</sup>, R. Bernabei<sup>2,3</sup>, F. Cappella<sup>4</sup>, V. Caracciolo<sup>5</sup>,  
R. Cerulli<sup>2,3</sup>, D.M. Chernyak<sup>6,7</sup>, F.A. Danevich<sup>6</sup>, S. d'Angelo<sup>2,3</sup>, A. Incicchitti<sup>4,8</sup>,  
D.V. Kasperovych<sup>6</sup>, V.V. Kobychev<sup>6</sup>, S.I. Konovalov<sup>1</sup>, M. Laubenstein<sup>5</sup>, D.V. Poda<sup>6,9</sup>,  
O.G. Polischuk<sup>6</sup>, V.N. Shlegel<sup>10</sup>, V.I. Tretyak<sup>6</sup>, V.I. Umatov<sup>1</sup>, Ya.V. Vasiliev<sup>10</sup>

<sup>1</sup> NRC “Kurchatov Institute”, ITEP, 117218 Moscow, Russia

<sup>2</sup> INFN, sezione di Roma “Tor Vergata”, I-00133 Rome, Italy

<sup>3</sup> Dip. di Fisica, Universita di Roma “Tor Vergata”, I-00133 Rome, Italy

<sup>4</sup> INFN, sezione di Roma, I-00185 Rome, Italy

<sup>5</sup> INFN, Laboratori Nazionali del Gran Sasso, I-67100 Assergi (AQ), Italy

<sup>6</sup> Institute for Nuclear Research, 03028 Kyiv, Ukraine

<sup>7</sup> Kavli Institute for the Phys. and Math. of the Universe, Kashiwa, 277-8583, Japan

<sup>8</sup> Dip. di Fisica, Universita di Roma “La Sapienza”, I-00185 Rome, Italy

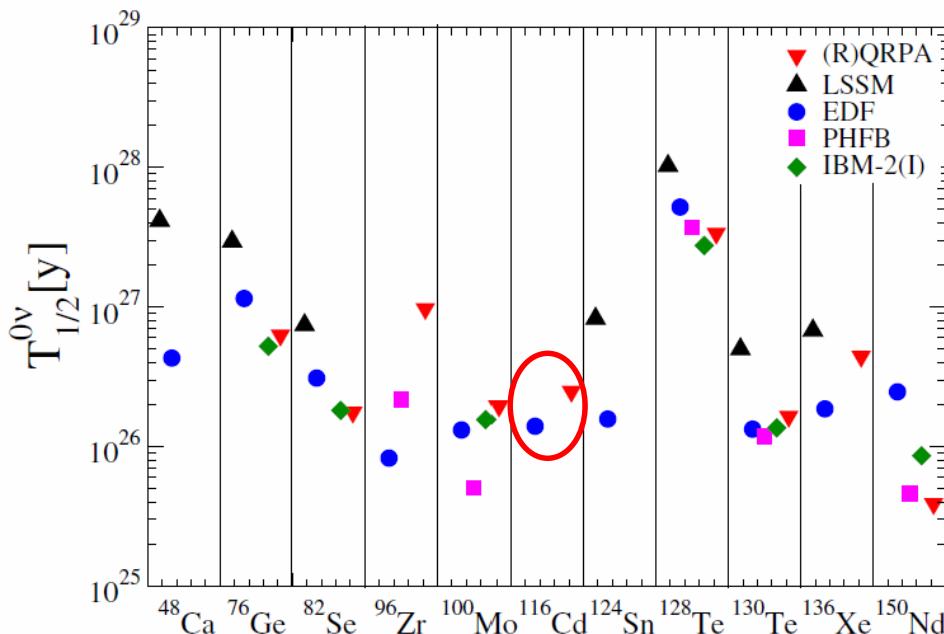
<sup>9</sup> CSNSM, Univ. Paris-Sud, CNRS/IN2P3, Univ. Paris-Saclay, 91405 Orsay, France

<sup>10</sup> Nikolaev Institute of Inorganic Chemistry, 630090 Novosibirsk, Russia

## Introduction

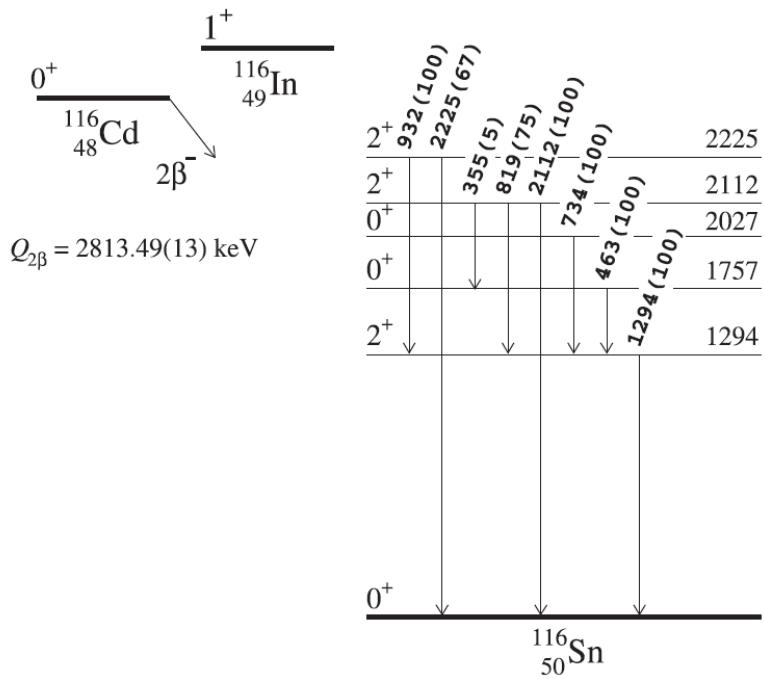
$^{116}\text{Cd}$  is one of the best isotopes to search for  $2\beta 0\nu$  decay:

- (1)  $Q_{2\beta} = 2813.49(13)$  keV:
  - (a) exp. point of view:  $> 2615$  keV of  $^{208}\text{TI}$ ;
  - (b) th. point of view:  $\Gamma(2\beta 2\nu) \sim Q_{2\beta}^{11}$ ,  $\Gamma(2\beta 0\nu) \sim Q_{2\beta}^5$ ;
- (2) favorable th. estimations of NMEs for  $2\beta 0\nu$ ;
- (3) quite high isotopic abundance  $\delta = 7.512(54)\%$  and availability of enrichment by centrifugation (cheap) in large amounts;
- (4) possibility to use “source = detector” approach with  $\text{CdWO}_4 / \text{CdTe}$  / ... which ensures high (close to 1) efficiency.

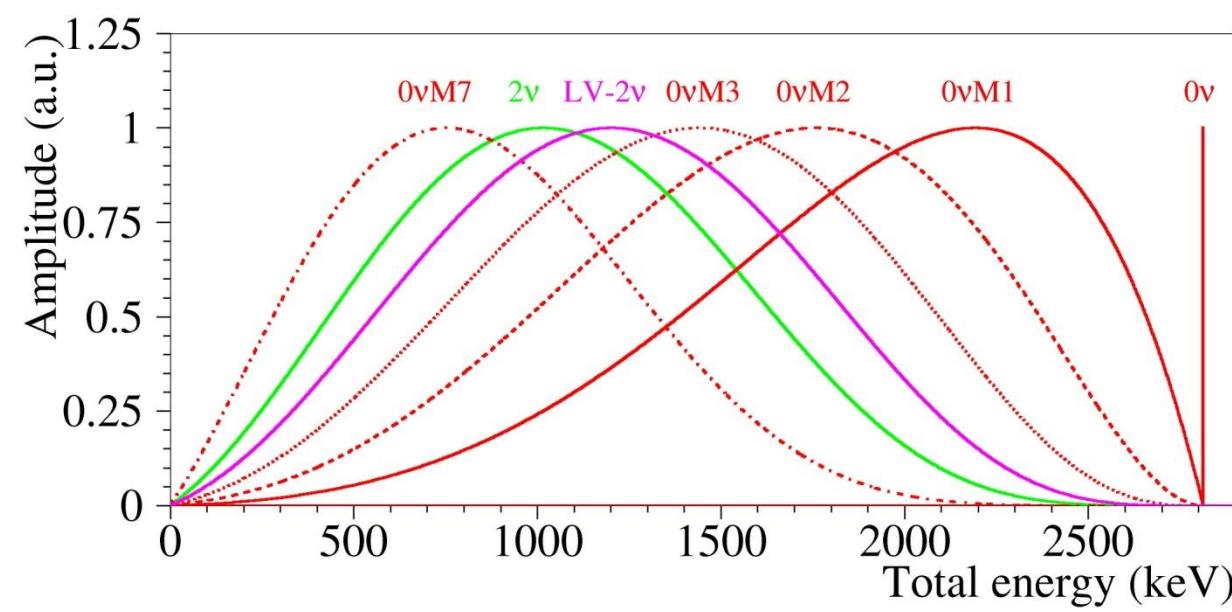


$$1/T_{1/2}(2\beta 0\nu) = \eta^2 |NME^{0\nu}|^2 G^{0\nu}(Q_{2\beta}, Z)$$

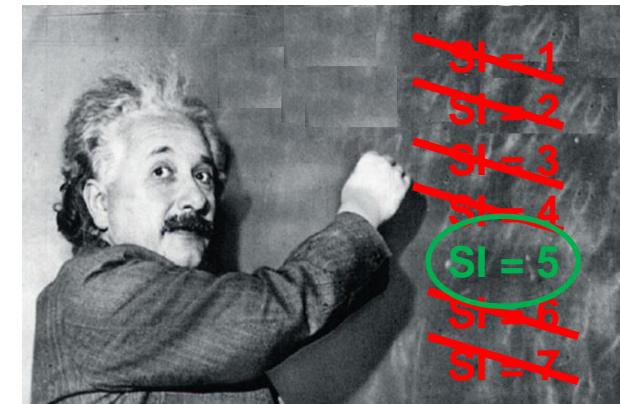
$\eta = m_\nu/m_e$  for light  $\nu$  mass mechanism



**Scheme of  $2\beta$  decay  $^{116}\text{Cd} \rightarrow ^{116}\text{Sn}$**



**Energy spectra ( $E_1+E_2$ ) for different  $2\beta$  modes**



## The most stringent previous limits (90% C.L.) for $^{116}\text{Cd}$ $2\beta0\nu$ :

$T_{1/2} > 1.7 \times 10^{23}$  yr (Solotvina, F.A. Danevich et al., PRC 68 (2003) 035501)

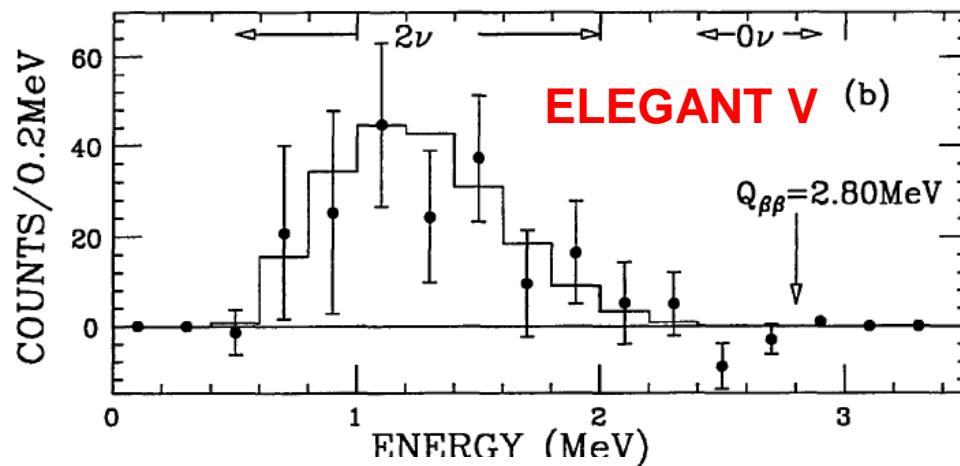
$T_{1/2} > 1.0 \times 10^{23}$  yr (NEMO-3, R. Arnold et al., PRD 95 (2017) 012007)

## Positive observations of $2\beta2\nu$ :

TABLE I. Experiments where  $2\nu2\beta$  decay of  $^{116}\text{Cd}$  was observed.

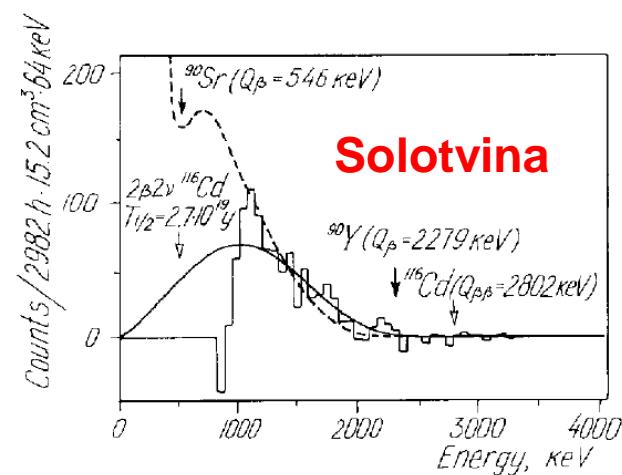
Experiment	$T_{1/2} (\times 10^{19} \text{ yr})$	Year, Reference
ELEGANT V, $^{116}\text{Cd}$ foil, drift chambers, plastic scintillators	$2.6_{-0.5}^{+0.9}$	1995 [40]
Solotvina, $^{116}\text{CdWO}_4$ scintillators	$2.7_{-0.4}^{+0.5}(\text{stat})_{-0.6}^{+0.9}(\text{sys})$	1995 [41]
NEMO-2, $^{116}\text{Cd}$ foils, track reconstruction by Geiger cells, plastic scintillators	$3.75 \pm 0.35(\text{stat}) \pm 0.21(\text{sys})^a$	1995 [43,44]
Solotvina, $^{116}\text{CdWO}_4$ scintillators	$2.6 \pm 0.1(\text{stat})_{-0.4}^{+0.7}(\text{sys})$	2000 [42]
Solotvina, $^{116}\text{CdWO}_4$ scintillators	$2.9 \pm 0.06(\text{stat})_{-0.3}^{+0.4}(\text{sys})$	2003 [32]
NEMO-3, $^{116}\text{Cd}$ foils, track reconstruction by Geiger cells, plastic scintillators	$2.74 \pm 0.04(\text{stat}) \pm 0.18(\text{sys})$	2017 [45]
$^{116}\text{CdWO}_4$ scintillators	$2.63 \pm 0.01(\text{stat})_{-0.12}^{+0.11}(\text{sys})$	2018, Present work

<sup>a</sup>The result of NEMO-2 was re-estimated as  $T_{1/2} = [2.9 \pm 0.3(\text{stat}) \pm 0.2(\text{sys})] \times 10^{19}$  yr in [46].

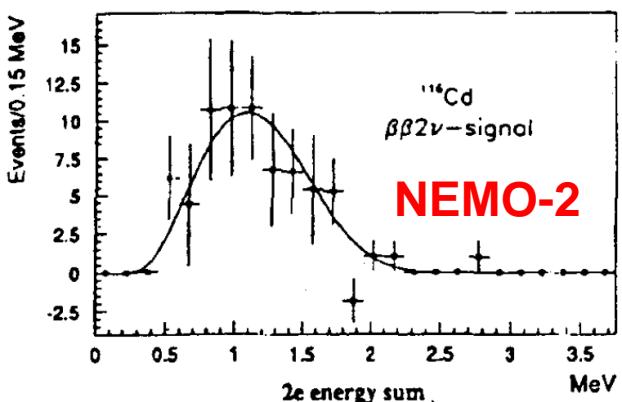


First observations (1995) of  $2\beta 2\nu$  decay in  $^{116}\text{Cd}$

ELEGANT V: H.Ejiri et al., J. Phys. Soc. Japan 64 (1995) 339:  
foil  $^{116}\text{Cd}$  (90.7%), 33  $\mu\text{m}$ , 91 g, 1875 h, ~200 events



Solotvina: F.A. Danevich et al., PLB 344 (1995) 72:  
 $^{116}\text{CdWO}_4$  19 cm<sup>3</sup> (83%), 2982 h, ~600 events

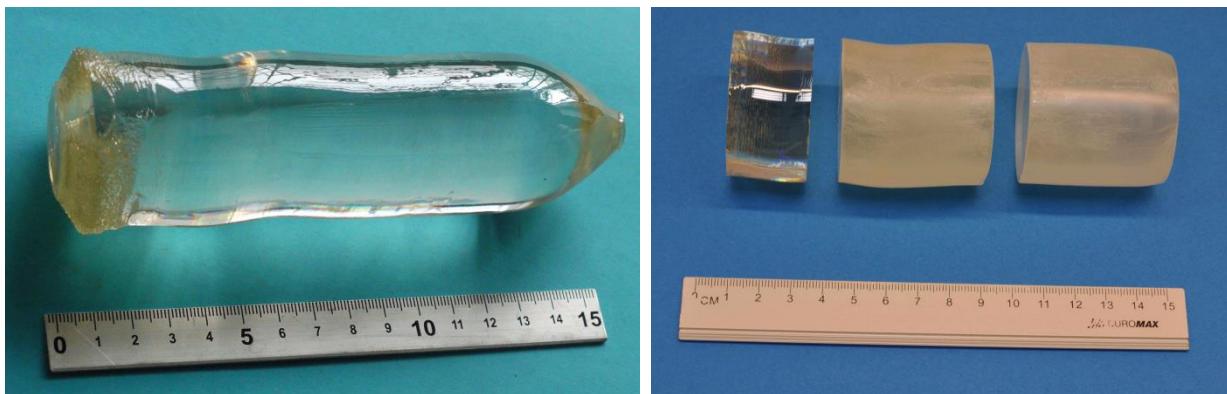


NEMO-2: R. Arnold et al., JETP Lett. 61 (1995) 170:  
foil  $^{116}\text{Cd}$  (93.2%), 40  $\mu\text{m}$ , 152 g, 2460 h, 69 events

(Aurora experiment: 92,923 events observed)

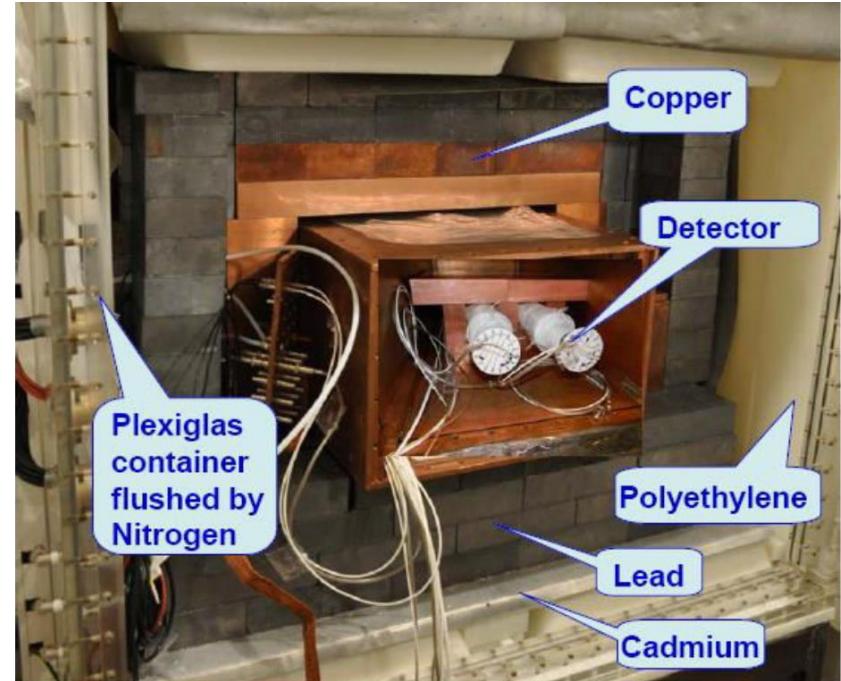
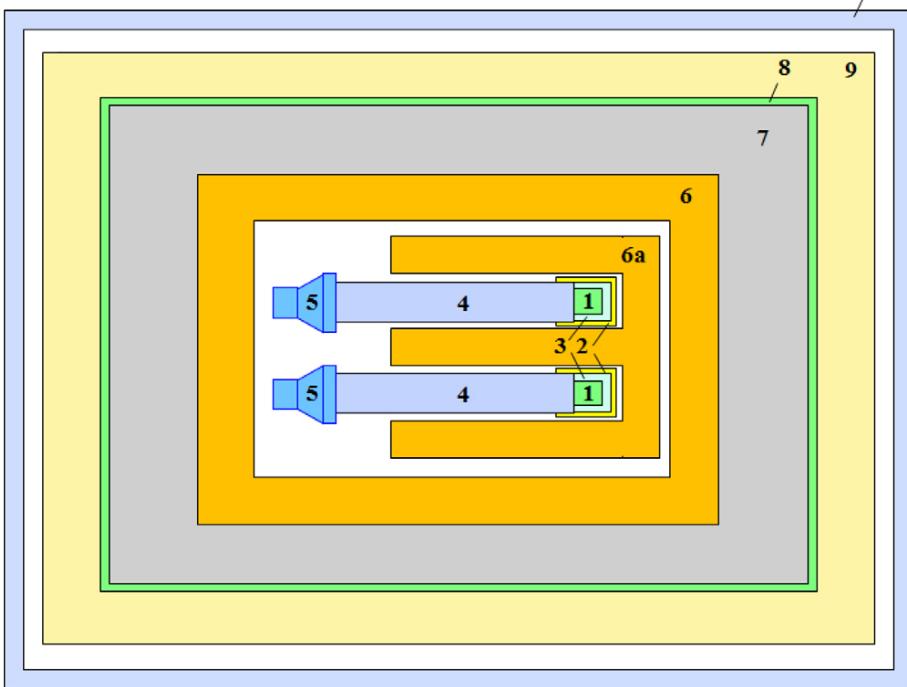
## Experiment

**Two enriched  $\text{CdWO}_4$  scintillating crystals (580 and 582 g), 82% of  $^{116}\text{Cd}$ , produced by low-thermal-gradient Czochralski crystal growth technique from highly purified Cd**



**Crystal boule 1868 g and scintillating elements  
(326, 582 and 586 g, JINST 06 (2011) P08011)**

**LNGS (3600 m w.e.), DAMA/R&D low background set-up**



Few upgrades. Final stage (since March 2014):

- (1)  $^{116}\text{CdWO}_4$  crystal scintillators
- (2) teflon containers
- (3) liquid scintillator
- (4) quartz light guides ( $\varnothing 7 \times 40$  cm)
- (5) photomultipliers (3" Hamamatsu R6233MOD)
- (6) high-purity copper (10 cm)
- (7) low radioactive lead (15 cm)
- (8) cadmium (1.5 mm)
- (9) polyethylene/paraffin (4 to 10 cm)
- (10) plexiglas box (flushed by HP N<sub>2</sub>)

**DAQ:**  
amplitude  
arrival time  
pulse shape  
(50  $\mu\text{s}$  with 20 ns bin)

**Calibration:**  
 $^{22}\text{Na}$ ,  $^{60}\text{Co}$ ,  $^{133}\text{Ba}$ ,  
 $^{137}\text{Cs}$ ,  $^{228}\text{Th}$

$$\text{FWHM}_{\gamma} = (10.2E_{\gamma})^{1/2}$$

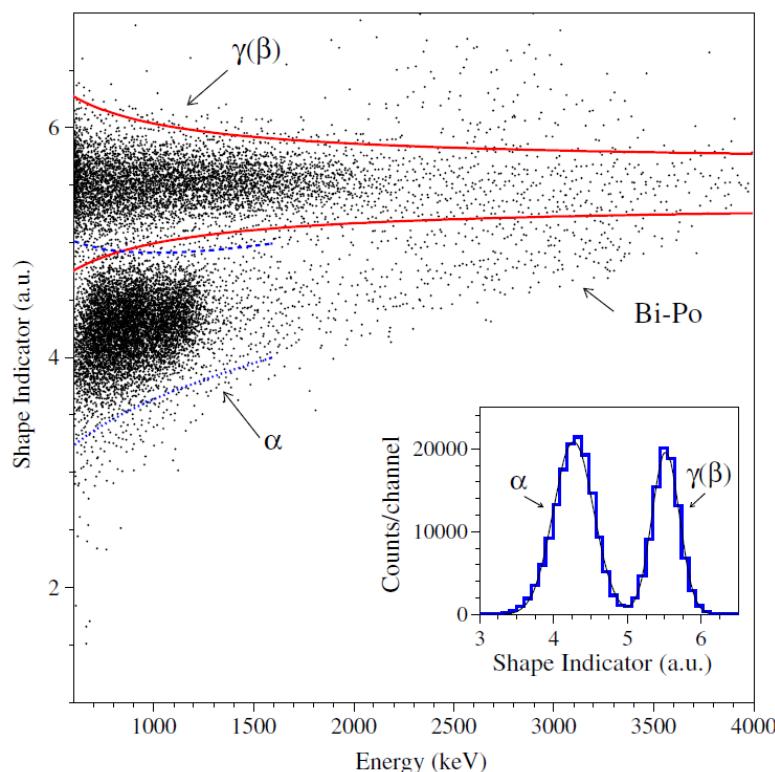
# Data analysis

## 1. Pulse-shape discrimination

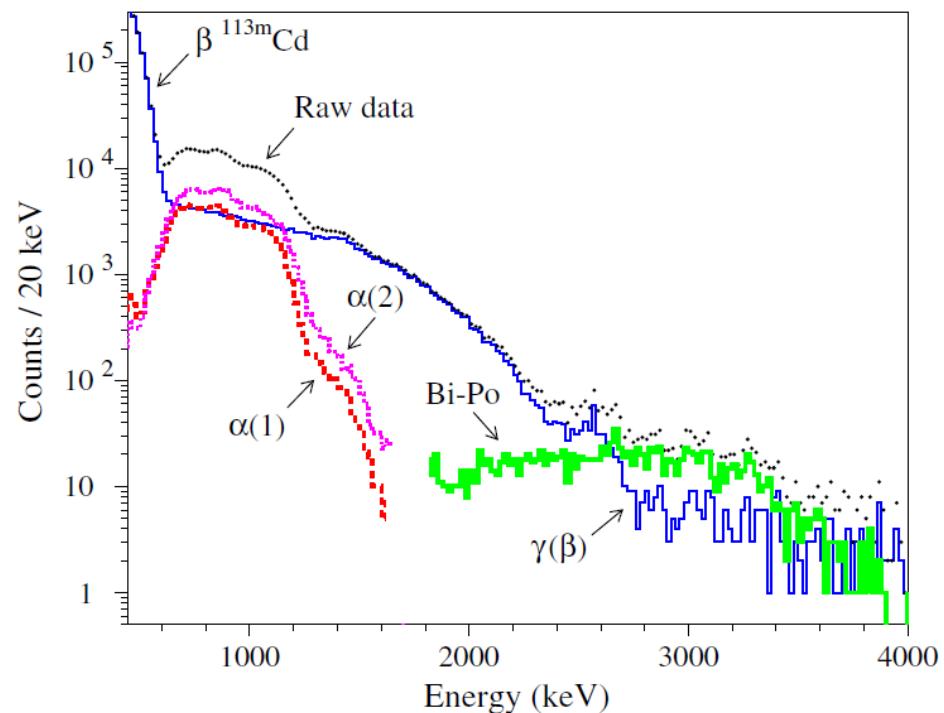
For each signal  $f(t)$ , shape indicator (SI) is calculated:

$$SI = \sum f(t_k) \times P(t_k) / \sum f(t_k)$$

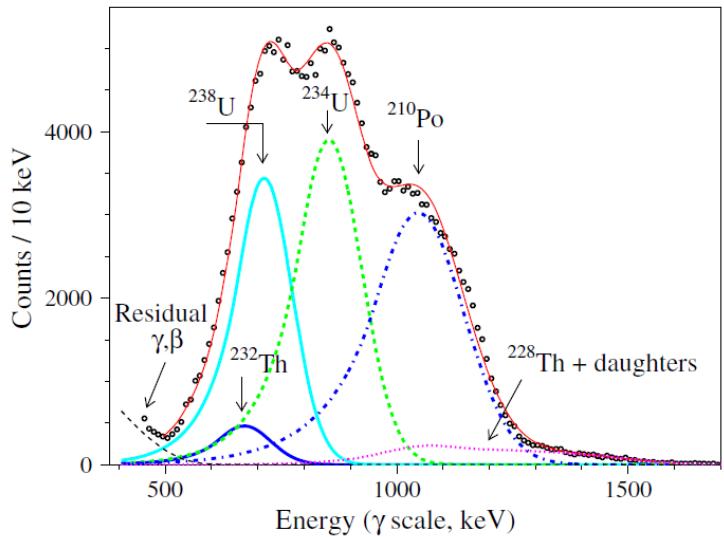
$$P(t) = |f_\alpha(t) - f_\gamma(t)| / |f_\alpha(t) + f_\gamma(t)|$$



$^{116}\text{CdWO}_4$  detector #2, 26831 h.  
Good discrimination ability.



$^{116}\text{CdWO}_4$  detectors #1+2, 26831 h.  
Raw data,  $\gamma(\beta)$  and  $\alpha$  components  
(in CWO-1 and CWO-2), 212Bi-Po<sub>8</sub>  
events



## Spectrum of $\alpha$ events (26831 h, CWO-1 and CWO-2) and its individual components

TABLE II. Radioactive contamination of the  $^{116}\text{CdWO}_4$  crystals. Reference date is February 2016.

Chain	Nuclide	Activity (mBq/kg)
$^{232}\text{Th}$	$^{40}\text{K}$	0.22(9)
	$^{90}\text{Sr} - ^{90}\text{Y}$	$\leq 0.02$
	$^{110m}\text{Ag}$	$\leq 0.007$
	$^{116}\text{Cd}$	1.138(5)
	$^{232}\text{Th}$	0.07(2)
	$^{228}\text{Ra}$	$\leq 0.005$
	$^{228}\text{Th}$	0.020(1)
$^{235}\text{U}$	$^{227}\text{Ac}$	$\leq 0.002$
$^{238}\text{U}$	$^{238}\text{U}$	0.58(4)
	$^{234}\text{U}$	0.6(1)
	$^{230}\text{Th}$	$\leq 0.13$
	$^{226}\text{Ra}$	$\leq 0.006$
	$^{210}\text{Pb}$	0.70(4)
Total $\alpha$		2.14(2)

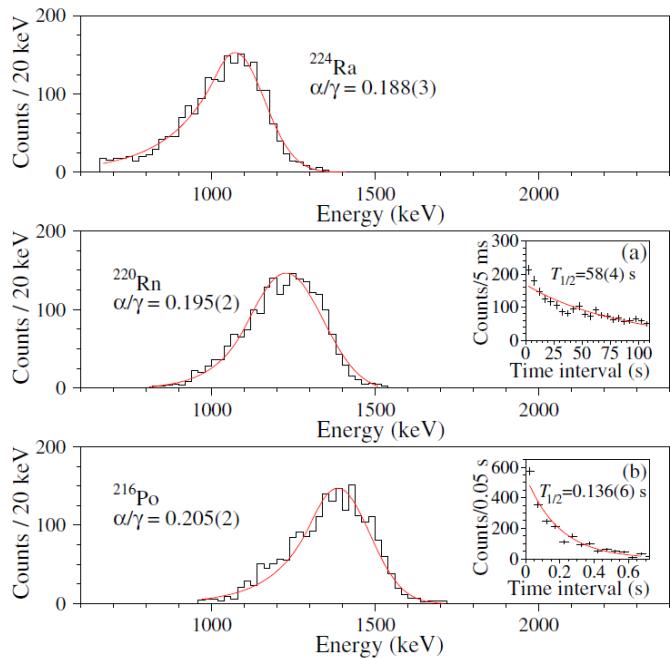
## 2. Time-amplitude analysis of fast subchains

### Selection of subchains: events with known energies and time differences

$^{224}\text{Ra}$  ( $Q_\alpha = 5789 \text{ keV}; T_{1/2} = 3.632 \text{ d}$ )

$\rightarrow ^{220}\text{Rn}$  ( $Q_\alpha = 6405 \text{ keV}; T_{1/2} = 55.6 \text{ s}$ )  $\rightarrow ^{216}\text{Po}$

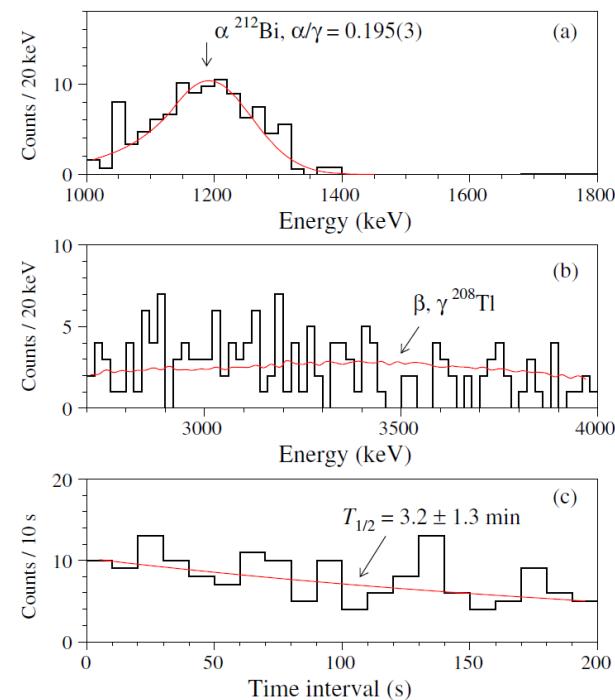
( $Q_\alpha = 6906 \text{ keV}; T_{1/2} = 0.145 \text{ s}$ )  $\rightarrow ^{212}\text{Pb}$ .



**$\alpha$  peaks of  $^{224}\text{Ra}$ ,  $^{220}\text{Rn}$ ,  $^{216}\text{Po}$ .**  
 $T_{1/2}$ :  $^{220}\text{Rn} = 58(4) \text{ s}$ ;  $^{216}\text{Po} = 0.136(6) \text{ s}$

$^{212}\text{Bi}(Q_\alpha = 6207 \text{ keV}) \rightarrow$

$^{208}\text{Tl}(Q_\beta = 4999 \text{ keV}, T_{1/2} = 3.053 \text{ min}) \rightarrow ^{208}\text{Pb}$

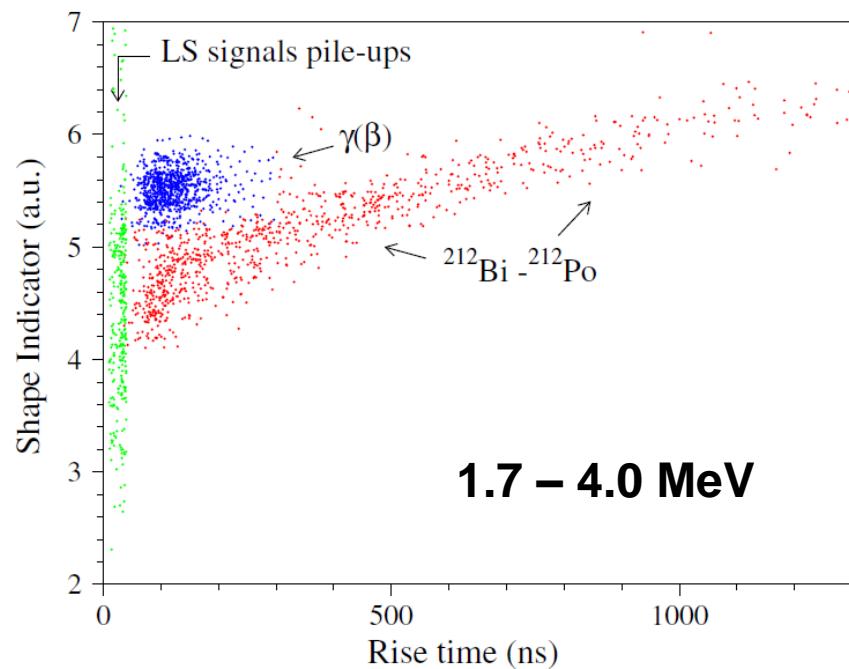
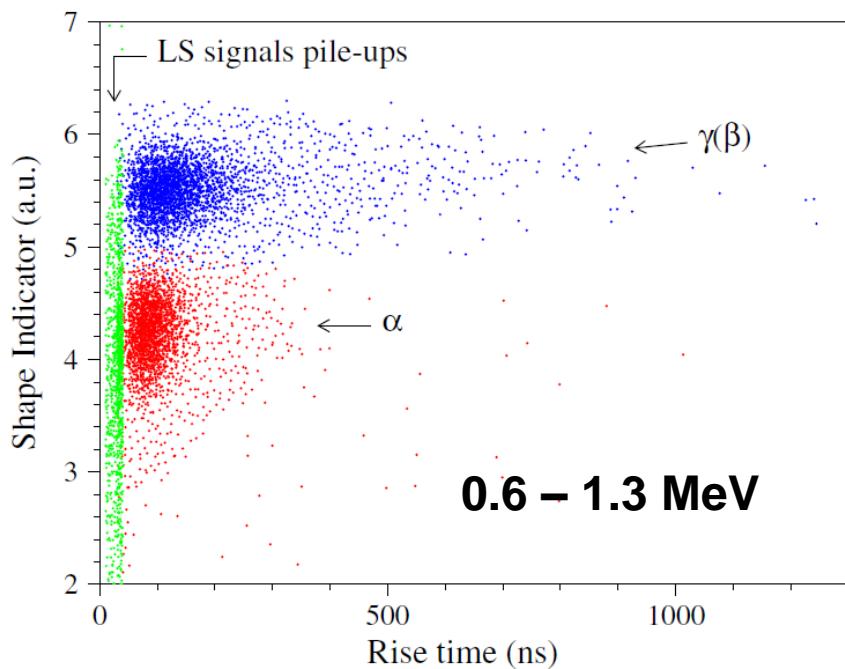


**$\alpha$  peak of  $^{212}\text{Bi}$  and  
 $\beta$  distribution of  $^{208}\text{Tl}$**

$\alpha/\beta$  ratio =  $0.114(7) + 0.0133(12)E_\alpha^{10}$

### 3. Front-edge analysis

**Front-edge parameter (rise time) = time between the signal origin and time of 0.7 of max value**

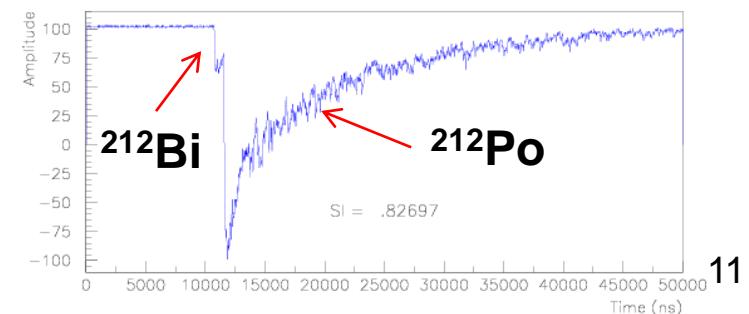


**In this way  $^{212}\text{Bi}$ - $^{212}\text{Po}$  events are selected**

$^{212}\text{Bi}$  ( $Q_\beta = 2252 \text{ keV}; T_{1/2} = 60.55 \text{ m}$ )

$\rightarrow ^{212}\text{Po}$  ( $Q_\alpha = 8954 \text{ keV}; T_{1/2} = 0.299 \mu\text{s}$ )  $\rightarrow ^{208}\text{Pb}$

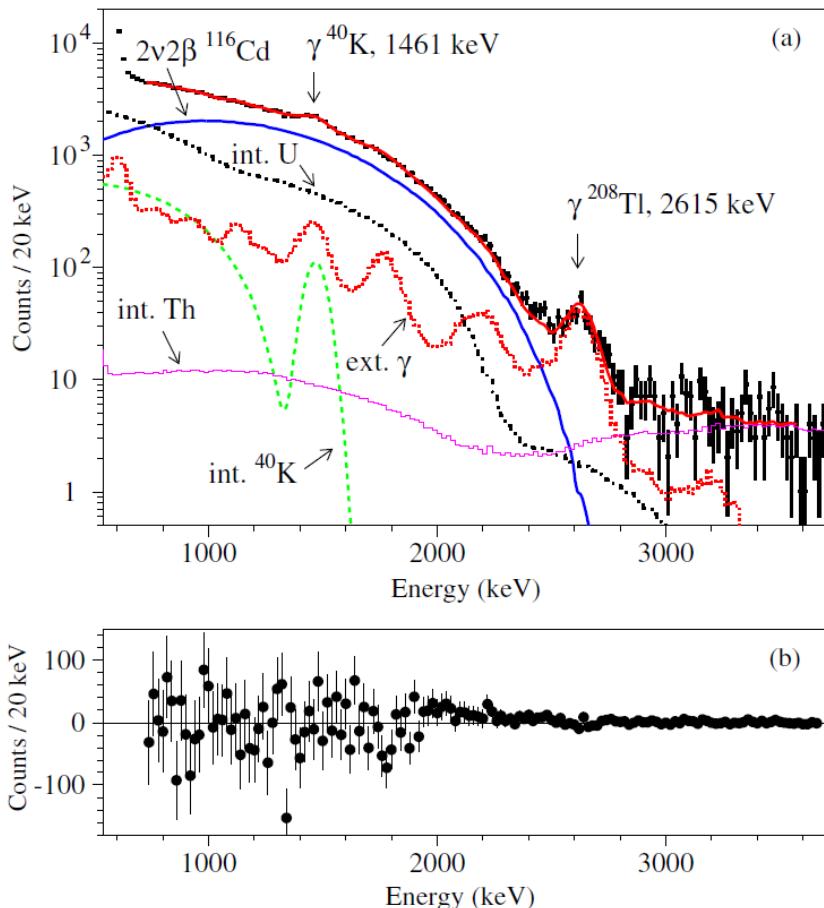
(also pile-ups of CWOs with LS)



# Results

## 1. $2\beta 2\nu$ decay of $^{116}\text{Cd}$ (g.s. to g.s.)

Selection of events: PSD and FE



$\gamma(\beta)$  energy spectrum, CWO-1 and CWO-2, 26831 h together with the main components

Background model:

- (1) internal contaminations of CWOs by  $^{40}\text{K}$ ,  $^{90}\text{Sr}/^{90}\text{Y}$ ,  $^{110m}\text{Ag}$ ,  $^{232}\text{Th}$ ,  $^{238}\text{U}$
- (2) external  $\gamma$ 's from Cu shield, PMTs, quartz light-guides ( $^{40}\text{K}$ , Th/U)

Initial kinematics: DECA0 generator  
Simulations: EGS4

Starting point: 640–1600 keV (20 keV step)

Final point: 2800–3600 keV

$$\chi^2/\text{ndf} = 1.15 - 1.75$$

Best fit (720 – 3560 keV,  $\chi^2/\text{ndf} = 1.15$ ):  
 $92923 \pm 388$   $2\beta 2\nu$  events (126341  $\pm 527$  in the whole spectrum)

$$T_{1/2}(2\beta 2\nu) = (2.630 \pm 0.011(\text{stat})) \times 10^{19} \text{ yr}$$

## Examples of simulations of $2\beta$ processes

$^{116}\text{CdWO}_4$  response to  $2\beta$  processes in  $^{116}\text{Cd}$  (EGS4 + DECAY0)

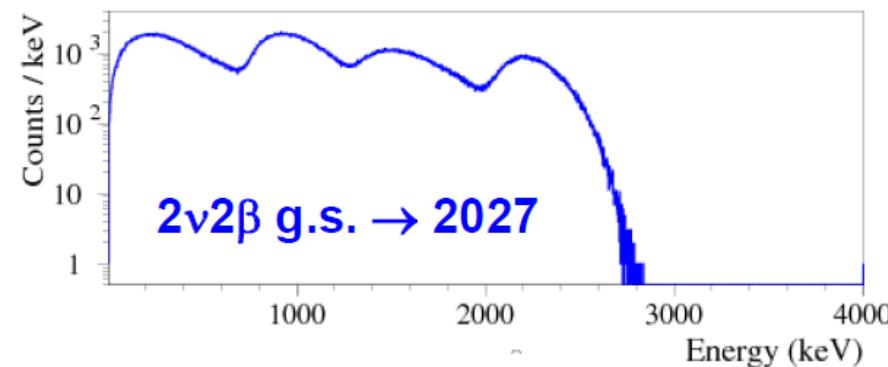
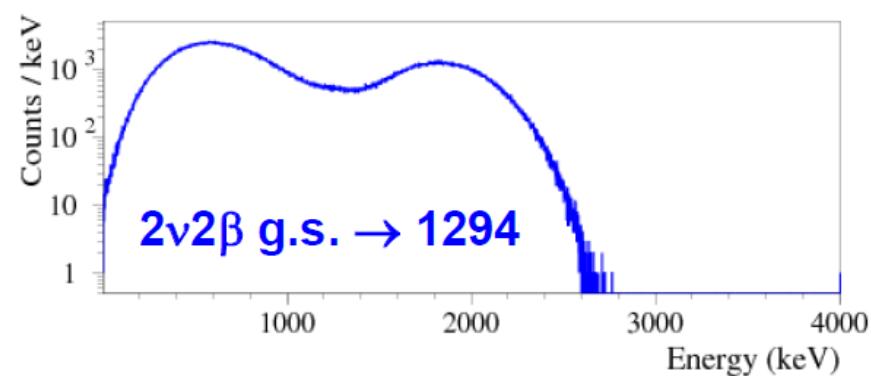
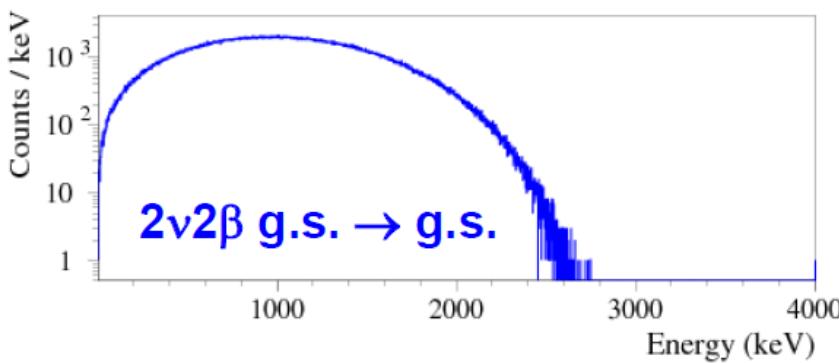
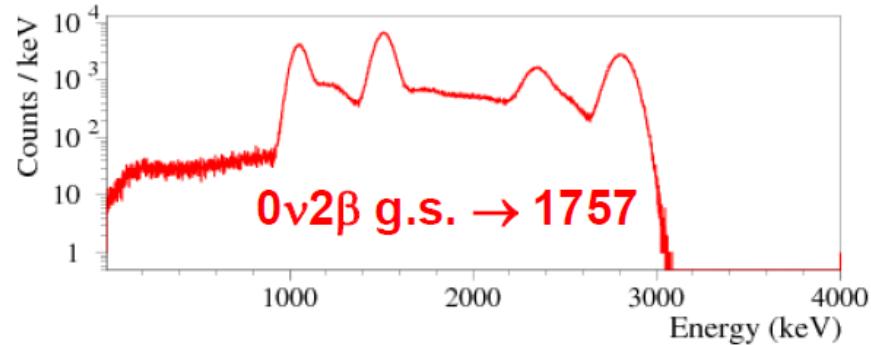
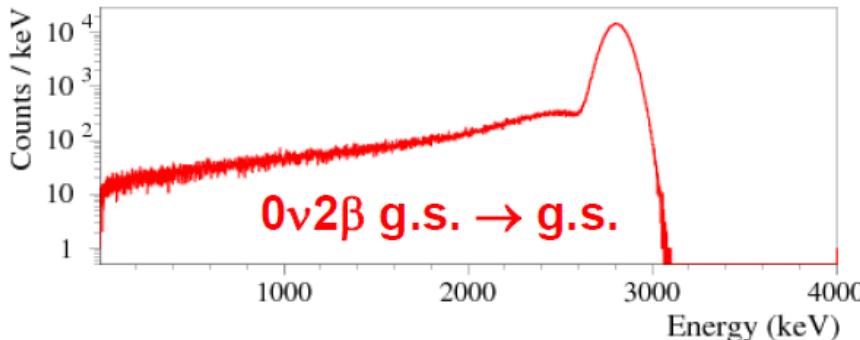
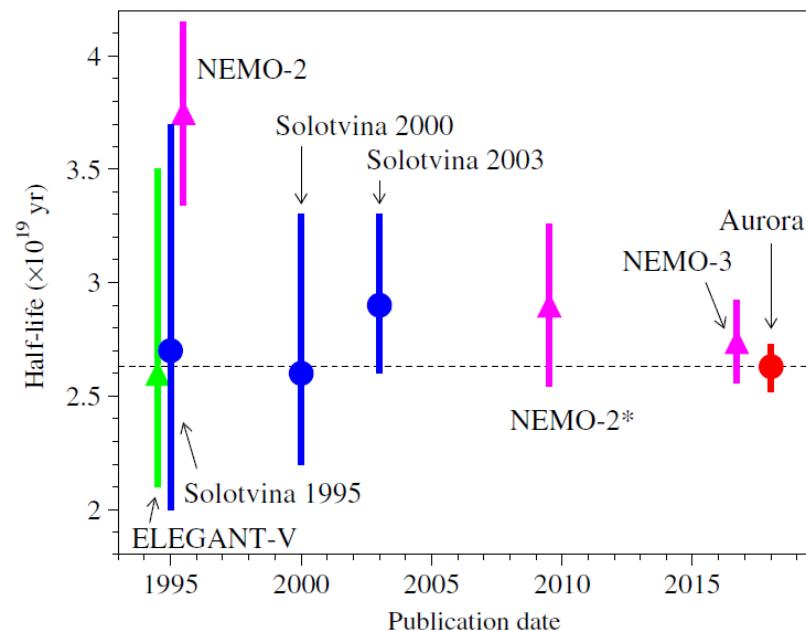


TABLE IV. Systematic uncertainties of  $T_{1/2}$  (%).

Source	Contribution
Number of $^{116}\text{Cd}$ nuclei	$\pm 0.12$
PSD and front-edge cuts efficiency	$\pm 1.2$
Model of background	$+3.25$ $-2.93$
Localization of radioactive contaminations	$+1.54$ $-2.63$
Interval of the fit	$+0.34$ $-1.02$
Energy scale instability	$\pm 1.72$
$2\nu 2\beta$ spectral shape	$\pm 1.0$
Total systematic error	$+4.30$ $-4.69$

$$T_{1/2}(2\beta 2\nu) = (2.630 \pm 0.011(\text{stat})^{+0.113}_{-0.123}(\text{sys})) \times 10^{19} \text{ yr}$$



$$NME_{\text{eff}} = 1/(G_{2\nu} \times T_{1/2})^{1/2}$$

 TABLE V. Effective nuclear matrix elements for  $2\nu 2\beta$  decay of  $^{116}\text{Cd}$  to the ground state of  $^{116}\text{Sn}$  obtained by using different calculations of the phase space factors.

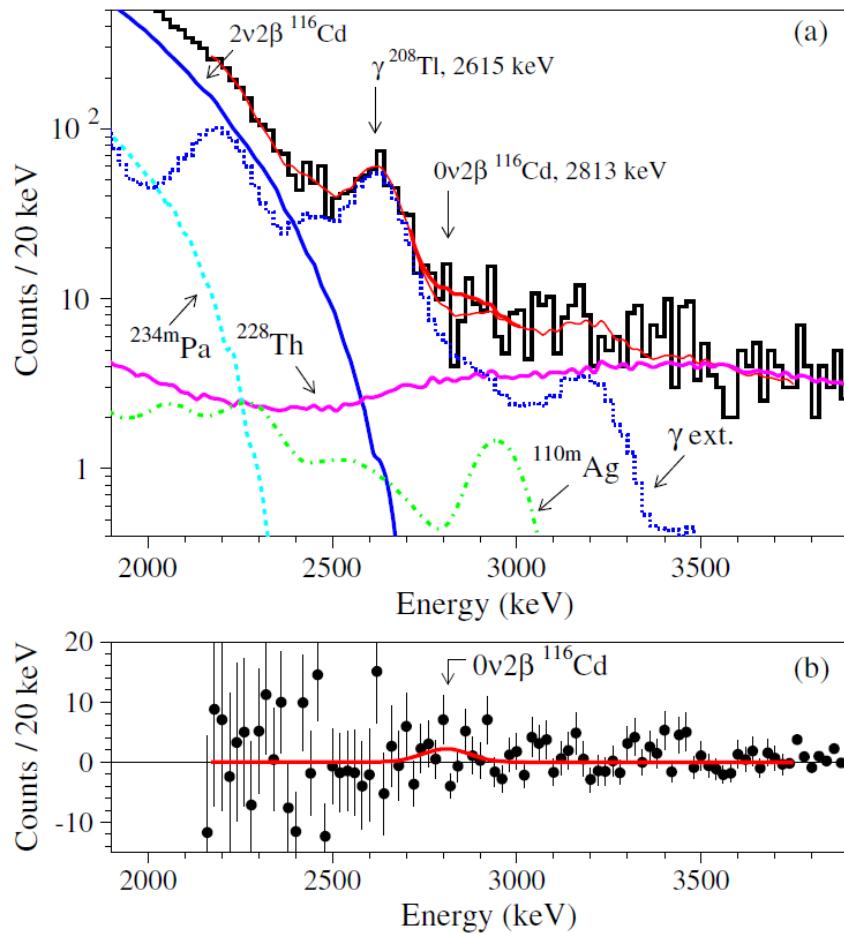
Phase space factor ( $10^{-21} \text{ yr}^{-1}$ ), Reference	Effective nuclear matrix element
2764 [68]	$0.1173^{+0.0027}_{-0.0024}$
3176 [68] (SSD model)	$0.1094^{+0.0025}_{-0.0023}$
2688 [69]	$0.1189^{+0.0027}_{-0.0025}$

[68] J. Kotila and F. Iachello, PRC 85 (2012) 034316

[69] M. Mirea et al., Rom. Rep. Phys. 67 (2015) 872

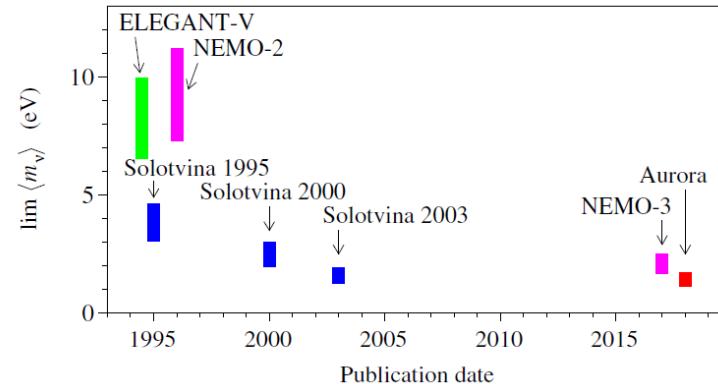
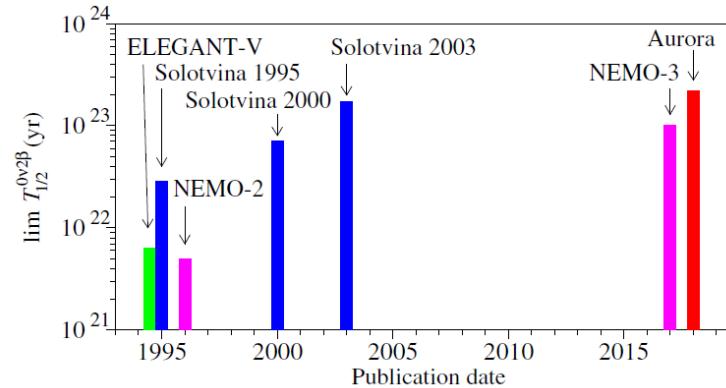
## 2. $2\beta0\nu$ decay of $^{116}\text{Cd}$ (g.s. to g.s.)

**26831 h + 8493 h from previous stage with background rate  
~0.1 counts/(keV kg yr) at 2.7–2.9 MeV = 35324 h**



$\gamma(\beta)$  energy spectrum, CWO-1 and CWO-2, 35324 h together with the main components

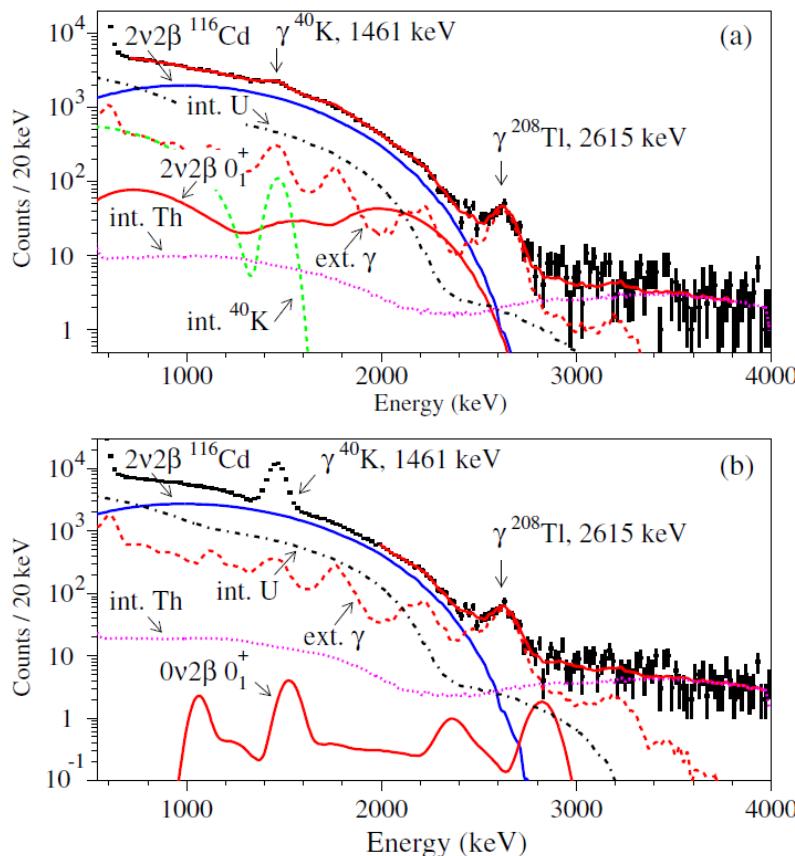
**Best fit: 2160–3740 keV,  $\chi^2/\text{ndf} = 1.01$   
 $S = -4.5 \pm 14.2 \rightarrow S < 19.1$  counts  
 $T_{1/2}(2\beta0\nu) > 2.2 \times 10^{23} \text{ yr } 90\% \text{ C.L.}$**



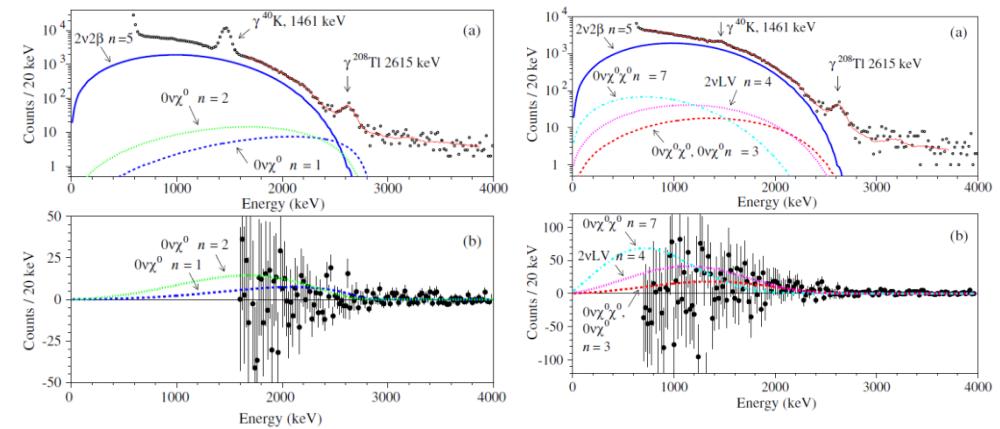
$m_\nu$ - $\lambda$ - $\eta$  ellipsoid: limits on  $m_\nu$ ,  $\lambda$ ,  $\eta$

### 3. $2\beta$ decays to excited levels, $2\beta0\nu$ decays with majoron(s) emission, Lorentz violating $2\beta2\nu$ decay

**Fit of experimental spectrum by background model +  $2\beta2\nu$  distribution + additional distribution for transition to excited state**



**Fit for  $2\beta2\nu$  and  $2\beta0\nu$  decays to the first  $0_1^+$  level of  $^{116}\text{Sn}$  (1757 keV)**



**Fits for majorons with spectral index  
SI = 1, 2 (at higher energies) and  
SI = 3, 4, 7 (at lower energies)**

TABLE VI. Summary of the obtained results on  $2\beta$  processes in  $^{116}\text{Cd}$ . The limits are given at 90% C.L., except of the results of [47], obtained at 68% C.L.

Decay mode	Transition, level of $^{116}\text{Sn}$ (keV)	$T_{1/2}$ (yr)	Best previous limits (yr) Reference
$2\nu$	g.s.	$(2.63^{+0.11}_{-0.12}) \times 10^{19}$ yr	see Table I and Fig. 12
$2\nu$	$2^+$ (1294)	$\geq 9.8 \times 10^{20}$	$\geq 2.3 \times 10^{21}$ [48]
$2\nu$	$0^+$ (1757)	$\geq 5.9 \times 10^{20}$	$\geq 2.0 \times 10^{21}$ [48]
$2\nu$	$0^+$ (2027)	$\geq 1.1 \times 10^{21}$	$\geq 2.0 \times 10^{21}$ [48]
$2\nu$	$2^+$ (2112)	$\geq 2.5 \times 10^{21}$	$\geq 1.7 \times 10^{20}$ [47]
$2\nu$	$2^+$ (2225)	$\geq 7.5 \times 10^{21}$	$\geq 1.0 \times 10^{20}$ [47]
$0\nu$	g.s.	$\geq 2.2 \times 10^{23}$	$\geq 1.7 \times 10^{23}$ [32]
$0\nu$	$2^+$ (1294)	$\geq 7.1 \times 10^{22}$	$\geq 2.9 \times 10^{22}$ [32]
$0\nu$	$0^+$ (1757)	$\geq 4.5 \times 10^{22}$	$\geq 1.4 \times 10^{22}$ [32]
$0\nu$	$0^+$ (2027)	$\geq 3.1 \times 10^{22}$	$\geq 0.6 \times 10^{22}$ [32]
$0\nu$	$2^+$ (2112)	$\geq 3.7 \times 10^{22}$	$\geq 1.7 \times 10^{20}$ [47]
$0\nu$	$2^+$ (2225)	$\geq 3.4 \times 10^{22}$	$\geq 1.0 \times 10^{20}$ [47]
$0\nu\chi^0 n = 1$	g.s.	$\geq 8.2 \times 10^{21}$	$\geq 8.5 \times 10^{21}$ [45]
$0\nu\chi^0 n = 2$	g.s.	$\geq 4.1 \times 10^{21}$	$\geq 1.7 \times 10^{21}$ [32]
$0\nu\chi^0 n = 3$	g.s.	$\geq 2.6 \times 10^{21}$	$\geq 0.8 \times 10^{21}$ [32]
$0\nu\chi^0\chi^0 n = 3$	g.s.	$\geq 2.6 \times 10^{21}$	$\geq 0.8 \times 10^{21}$ [32]
$2\nu LV n = 4$	g.s.	$\geq 1.2 \times 10^{21}$	...
$0\nu\chi^0\chi^0 n = 7$	g.s.	$\geq 8.9 \times 10^{20}$	$\geq 4.1 \times 10^{19}$ [77]

TABLE VII. Limits on lepton-number violating parameters. The limits are given at 90% C.L.

Parameter	Limit
Effective light Majorana neutrino mass $\langle m_\nu \rangle$	$\leq (1.0 - 1.7) \text{ eV}$
Effective heavy Majorana neutrino mass $ \langle m_{\nu_h}^{-1} \rangle ^{-1}$	$\geq (10 - 28) \times 10^6 \text{ GeV}$
Right-handed current admixture $\langle \lambda \rangle$	$\leq (1.8 - 22) \times 10^{-6}$
Right-handed current admixture $\langle \eta \rangle$	$\leq (1.6 - 21) \times 10^{-8}$
Coupling constant of neutrino with majoron $\langle g_{ee} \rangle$	
$\chi^0, n = 1$	$\leq (6.1 - 9.3) \times 10^{-5}$
$\chi^0, n = 3$	$\leq 7.7 \times 10^{-2}$
$\chi^0\chi^0, n = 3$	$\leq (0.69 - 6.9)$
$\chi^0\chi^0, n = 7$	$\leq (0.57 - 5.7)$
R-parity violating parameter $\lambda'_{111}$	$\leq 2.5 \times 10^{-4} \times f$ (see text)
Lorentz-violating parameter $a_{\text{of}}^{(3)}$	$\leq 4.0 \times 10^{-6} \text{ GeV}$

NME for  $m_\nu$ :

- J. Barea et al., PRC 91 (2015) 034304 (IBM)
- F. Simkovic et al., PRC 87 (2013) 045501 (QRPA)
- N.L. Vaquero et al., PRL 111 (2013) 142501 (EDFT)
- J. Hyvärinen et al., PRC 91 (2015) 024613 (pnQRPA)
- L.S. Song et al., PRC 95 (2017) 024305 (EDFT)

PSF:

- J. Kotila, F. Iachello, PRC 85 (2012) 034316

## Conclusions

After near 5 yr of data taking at LNGS (3600 m w.e.), the Aurora experiment to investigate  $2\beta$  processes in  $^{116}\text{Cd}$  with 1.162 kg of enriched (82%)  $^{116}\text{CdWO}_4$  scintillators is finished

$T_{1/2}$  for  $2\beta 2\nu$  is precisely measured:  $T_{1/2}(2\beta 2\nu) = 2.63^{+0.11}_{-0.12} \times 10^{19}$  yr

The most stringent limit for  $2\beta 0\nu$  is obtained:  $T_{1/2}(2\beta 0\nu) > 2.2 \times 10^{23}$  yr, equivalent to Majorana  $\nu$  mass limits:  $m_\nu < 1.0 - 1.7$  eV (depending on NME)

Limits on  $2\beta 2\nu$  and  $2\beta 0\nu$  decays to excited levels:  $T_{1/2} > 10^{20} - 10^{22}$  yr

Limits on  $2\beta 0\nu$  decays with different majorons:  $T_{1/2} > 10^{21} - 10^{22}$  yr

Limits on right-handed admixtures in weak interaction, heavy  $\nu$  mass, majoron-neutrino coupling constants, Lorentz-violating  $2\beta 2\nu$  decay

**Děkuji za pozornost!**

## P.S. Lorentz-violating $2\beta 2\nu$ decay

$$d\Gamma/dt_1 dt_2 = C \cdot e_1 p_1 F(t_1, Z) \cdot e_2 p_2 F(t_2, Z) \cdot [(t_0 - t_1 - t_2)^5 + 10 \overset{\circ}{a}_{\text{of}}^{(3)} (t_0 - t_1 - t_2)^4]$$

$$\Gamma = \Gamma_{2\nu} + \Gamma_{2\nu\text{LV}}$$

$$\Gamma_{2\nu} = CI_5, \quad \Gamma_{2\nu\text{LV}} = 10 \overset{\circ}{a}_{\text{of}}^{(3)} \cdot CI_4$$

$$I_5 = \int_0^{t_0} dt_1 e_1 p_1 F(t_1, Z) \times \int_0^{t_0-t_1} dt_2 e_2 p_2 F(t_2, Z) (t_0 - t_1 - t_2)^5$$

$$I_4 = \int_0^{t_0} dt_1 e_1 p_1 F(t_1, Z) \times \int_0^{t_0-t_1} dt_2 e_2 p_2 F(t_2, Z) (t_0 - t_1 - t_2)^4$$

$$10 \overset{\circ}{a}_{\text{of}}^{(3)} = \frac{\Gamma_{2\nu\text{LV}}}{\Gamma_{2\nu}} \cdot \frac{I_5}{I_4} = \frac{T_{1/2}^{2\nu}}{T_{1/2}^{2\nu\text{LV}}} \cdot \frac{I_5}{I_4}$$

In the Primakoff-Rosen approximation  $F(t, Z) \sim e/p$

$$I_5 = t_0^7 (t_0^4 + 22t_0^3 + 220t_0^2 + 990t_0 + 1980)/83160$$

$$I_4 = t_0^6 (t_0^4 + 20t_0^3 + 180t_0^2 + 360t_0 + 1260)/37800$$

# Monument in Kyiv to Vitaly Primakov (revolutioner), grand-uncle of Henry Primakoff



*Henry Primakoff*

Primakoff-Rosen approximation