Efficient QRPA calculation for two-neutrino double-beta decay nuclear matrix element

Nobuo Hinohara

Center for Computational Sciences, University of Tsukuba, Japan

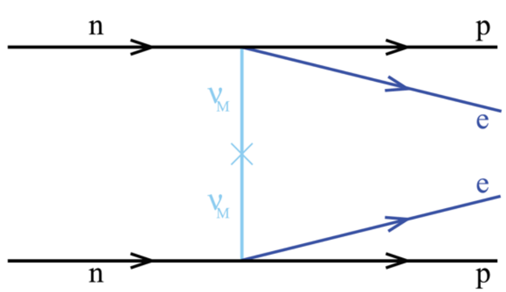
Double-beta decay and nuclear matrix elements

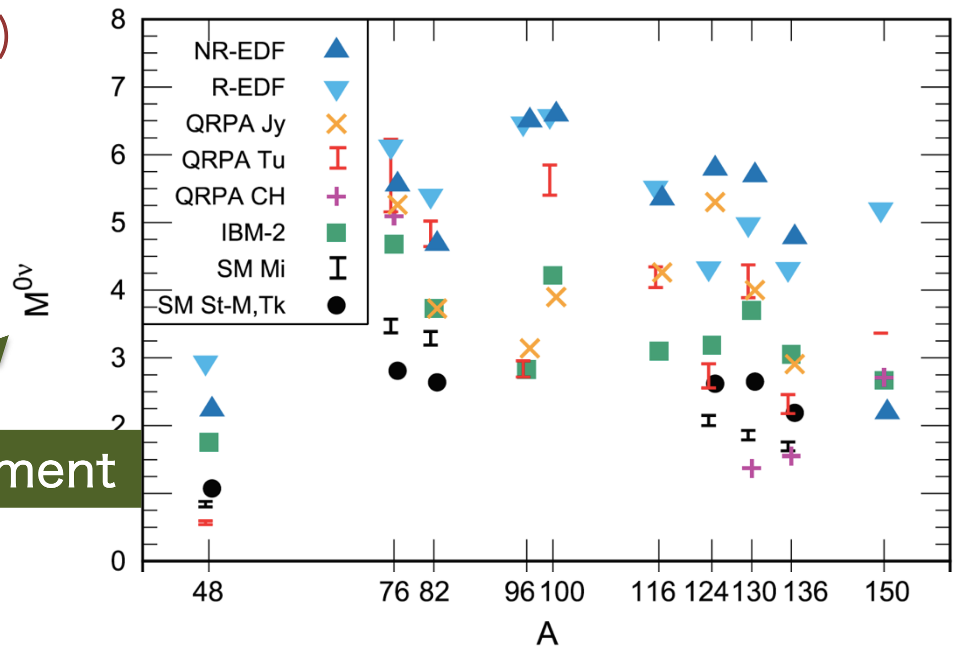
neutrino mass

- neutrino mass is zero within the standard model
- neutrino oscillation measurements clarifies the finite mass of neutrino

Neutrino oscillation cannot determine the neutrino mass
Double-beta decay may determine it if the neutrino is Majorana particle

neutrinoless double-beta decay(0νββ)





Engel and Menéndez, Rep. Prog. Phys. **80**, 046301 (2017)

half life of 0νββ (from experiments)

nuclear matrix element

$$(T^{0\nu}_{1/2})^{-1} = G_{0\nu}(Q_{\beta\beta}, Z)|M_{0\nu}|^2\langle m_{\beta\beta}\rangle^2$$

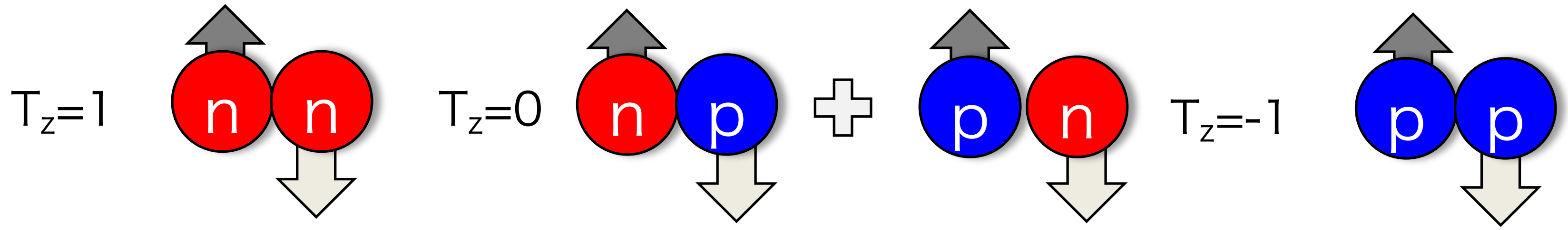
phase space factor   effective mass of electric neutrino

- Precise value of the NME is necessary to determine the neutrino mass
- Theoretical values of NMEs are within a factor of 2-3

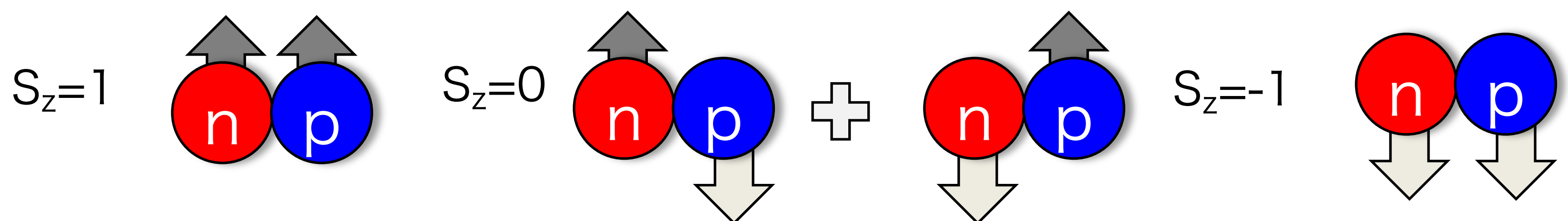
Nuclear matrix element and g_{pp}

neutron-proton pairing: possible source of NME uncertainty

Isovector (T=1, S=0) pairings: isospin symmetric, spin antisymmetric

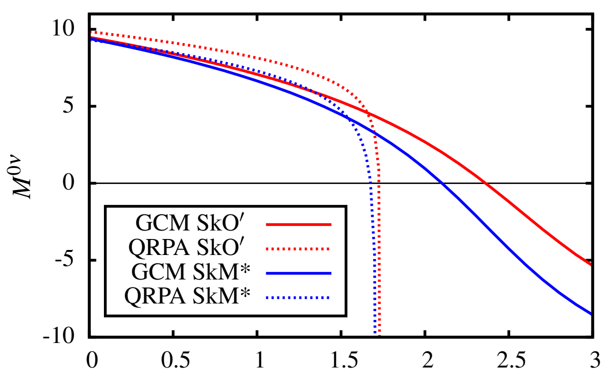


Isoscalar (T=0, S=1) pairings: isospin antisymmetric, spin symmetric



Suppression of the NME by the neutron-proton pairing

NME of ^{76}Ge



isoscalar neutron-proton pairing strength

GCM:
NH and Engel, Phys. Rev. C **90**. 031301 (2014)

g_{pp} cannot be determined from the nuclear ground state properties

QRPA for double-beta decay

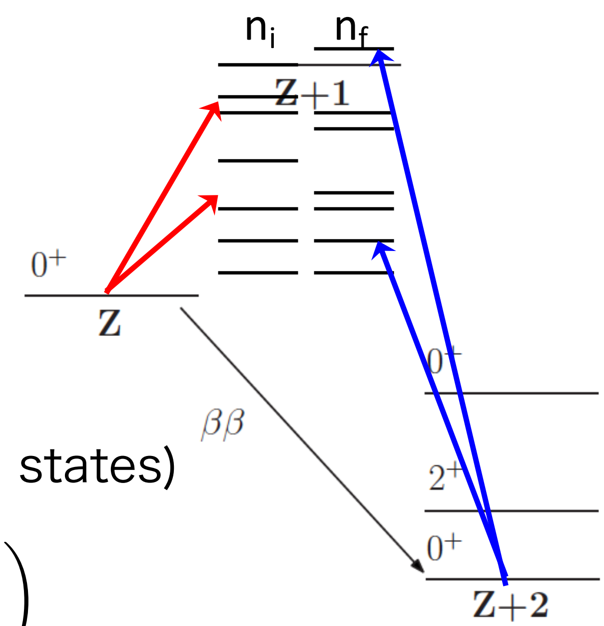
- g_{pp} can be determined from β decay and 2νββ decay

2νββ

$$M^{2\nu} = M^{2\nu}_{\rm GT} - \frac{g_V^2}{g_A^2} M^{2\nu}_{\rm F},$$

$$M^{2\nu}_{\rm F} = \sum_n \frac{\langle 0_f^+ | \sum_a \tau_a^- | n \rangle \langle n | \sum_b \tau_b^- | 0_i^+ \rangle}{E_n - \frac{M_i + M_f}{2}},$$

$$M^{2\nu}_{\rm GT} = \sum_n \frac{\langle 0_f^+ | \sum_a \boldsymbol{\sigma}_a \tau_a^- | n \rangle \cdot \langle n | \sum_b \boldsymbol{\sigma}_b \tau_b^- | 0_i^+ \rangle}{E_n - \frac{M_i + M_f}{2}}$$



two pnQRPA calculations (from initial and final states)

QRPA equation $\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = \Omega_n \begin{pmatrix} X^n \\ -Y^n \end{pmatrix}$

intermediate states $|n_i, K\rangle = \hat{O}^\dagger_{n_i,K} |0_i^+\rangle$ $\quad |n_f, K\rangle = \hat{O}^\dagger_{n_f,K} |0_f^+\rangle$

$$\hat{O}^\dagger_n = \sum_{\mu\nu} X^n_{\mu\nu} a^{n\dagger}_\mu a^{p\dagger}_\nu - Y^n_{\mu\nu} a^p_\nu a^n_\mu$$

Two sets of intermediate states do not agree because the QRPA is an approximation
→ matching of the intermediate states

$$\langle n_f, K | n_i, K \rangle = \sum_{pn} \sum_{p'n'} [X^{f,K*}_{pn} X^{i,K}_{p'n'} - \alpha Y^{f,K*}_{pn} Y^{i,K}_{p'n'}] \mathcal{O}_{p'p}(\alpha) \mathcal{O}_{n'n}(\alpha)$$

O(α) is a overlap quantity from the two HFB states

Matching of the intermediate states

QTDA overlap

Mustonen et al., Phys. Rev. C **87**, 064302 (2013)

$$\langle N|N'\rangle = \mathcal{N}^{-1}\sum_{pn}\sum_{p'n'} X^{N*}_{pn} X^{N'}_{p'n'}\left(\mathcal{R}_{p'p} + \sum_{p''}\mathcal{S}_{p'p''}D_{p''p}\right) \times \left(\mathcal{R}_{n'n} + \sum_{n''}\mathcal{S}_{n'n''}D_{n''n}\right). \qquad \text{(A17)}$$

$$\mathcal{N} = \langle \mathrm{HFB}_f|\mathrm{HFB}_i\rangle^{-1} = \sqrt{\det(1 + D^\dagger D)}. \qquad \text{(A11)}$$

$$a_k^{(f)\dagger} = \sum_n \left(\mathcal{R}_{kn} a_n^{(i)\dagger} + \mathcal{S}_{k,-n} a_{-n}^{(i)}\right),$$

$$\mathcal{R}_{kn} = \langle n|k\rangle(u_k u_n + s_k v_k s_n v_n), \qquad \mathcal{S}_{k,-n} = \langle n|k\rangle(u_k s_n v_n - s_k v_k u_n).$$

QRPA overlap

Simkovic et al., Nucl. Phys. A **733**, 321 (2004)

$$M^{2\nu}_{\mathrm{GT}} = \sum_{m_i m_f}\sum_{K=0,\pm 1} \frac{\langle 0_f^+\|\beta^-\|1(K), m_f\rangle\langle 1(K), m_f|1(K), m_i\rangle\langle 1(K), m_i\|\beta^-\|0_i^+\rangle}{(\omega_K^{m_f} + \omega_K^{m_i})/2}$$

$$\langle 1(K), m_f|1(K), m_i\rangle = \sum_{l_i l_f}\left[X^{m_f}_{l_f K} X^{m_i}_{l_i K} - Y^{m_f}_{l_f K} Y^{m_i}_{l_i K}\right]\mathcal{R}_{l_f l_i}\langle \mathrm{BCS}_f|\mathrm{BCS}_i\rangle.$$

$$\mathcal{R}_{ll'} = \langle p\rho_p|p'\rho_{p'}\rangle\left(u_p^{(i)} u_{p'}^{(f)} + v_p^{(i)} v_{p'}^{(f)}\right)\langle n\rho_n|n'\rho_{n'}\rangle\left(u_n^{(i)} u_{n'}^{(f)} + v_n^{(i)} v_{n'}^{(f)}\right)$$

$$\langle \mathrm{BCS}_f(\Omega)|\mathrm{BCS}_i(\Omega)\rangle = \prod_{k=1}\left(u_k^{(f)} u_k^{(i)} + v_k^{(f)} v_k^{(i)}\right)$$

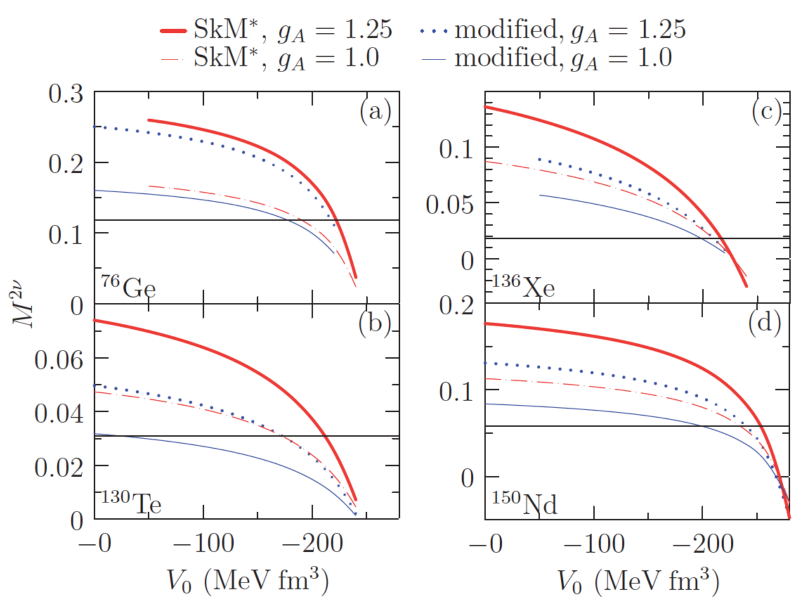
Skyrme QRPA for double-beta decay

Mustonen and Engel, Phys. Rev. C **87**,064302 (2013)

- Skyrme EDF: global parametrization in full sp model space
- QRPA calculation for ^{76}Ge, ^{130}Te, ^{136}Xe, ^{150}Nd
- HFB model space: two-dimensional, 20 fm (0.7 fm mesh)
  - axial deformation allowed
- QRPA

$$\begin{pmatrix} A & B \\ B^* & A^* \end{pmatrix} \begin{pmatrix} X^n \\ Y^n \end{pmatrix} = \Omega_n \begin{pmatrix} X^n \\ -Y^n \end{pmatrix}$$

two-quasiparticle dimension ~ 500,000, truncation to 15,000
(almost 2p or 2h states and states with high 2qp energy removed)
full diagonalization necessary for double-beta decay calculation

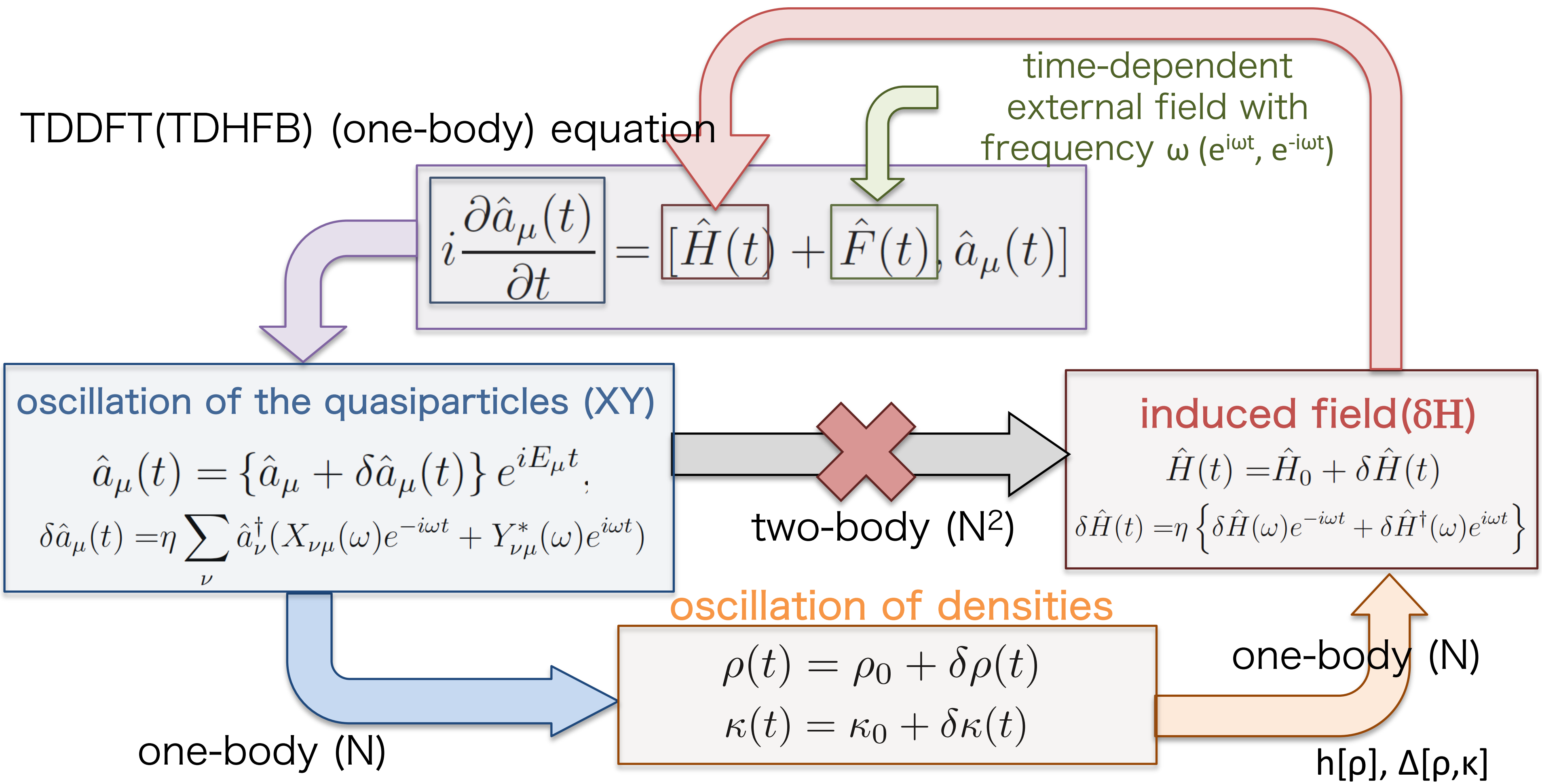


QRPA with large model space is computationally demanding

Finite-amplitude method (FAM)

Nakatsukasa et al., Phys. Rev. C **76**, 024318 (2007)

- QRPA: small-amplitude limit of time-dependent density functional theory
- FAM: linear response formalism of QRPA for DFT
- efficient iterative solution to the QRPA problem
- one-body induced field not through two-body AB matrices
- easy implementation on top of existing mean-field codes

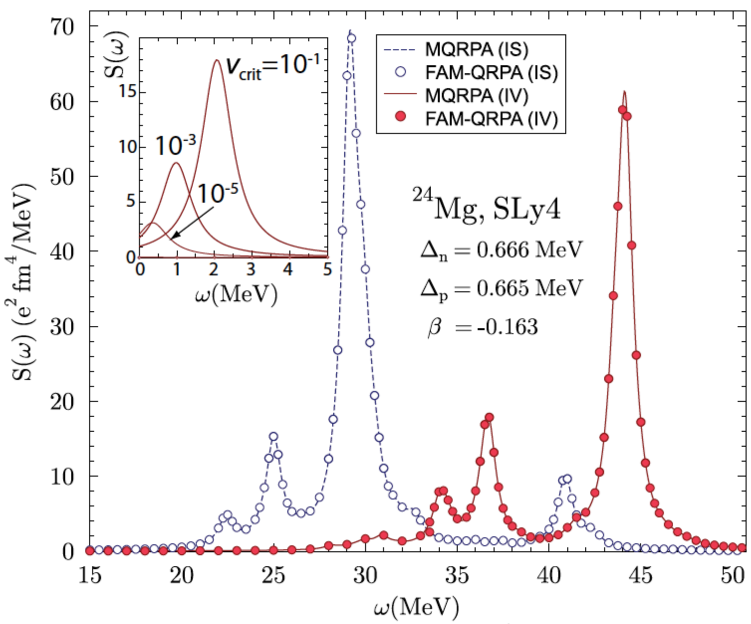


FAM strength function

strength function $$S(\hat{F},\omega) = \sum_{\mu<\nu} F^{20}_{\mu\nu} X_{\mu\nu}(\omega) + F^{02}_{\mu\nu} Y_{\mu\nu}(\omega)$$

$$\frac{dB}{d\omega}(\hat{F},\omega) = -\frac{1}{\pi}\mathrm{Im}S(\hat{F},\omega) = \frac{\gamma}{\pi}\sum_{\nu}\left\{\frac{|\langle\nu|\hat{F}|0\rangle|^2}{(\Omega_i-\omega)^2+\gamma^2} - \frac{|\langle 0|\hat{F}|\nu\rangle|^2}{(\Omega_i+\omega)^2+\gamma^2}\right\}$$

equivalence to QRPA



Stoitsov et al., Phys. Rev. C **84**, 04135(R) (2011)

FAM implementations

Skyrme EDF

- 3D coordinate HF (Nakatsukasa PRC(2007))
- 1D coordinate (HFBRAD) (Avogadro PRC(2011))
- 2D HO HFB (HFBTHO) (Stoitsov PRC(2011))
- 2D coordinate HFB (HFBAX) (Pei PRC(2014))
- 2D HO HFB (HFBTHO, np) (Mustonen, PRC(2014))
- 3D HFB (CR8) (Washiyama, PRC(2017))

RMF

- 1D coordinate RMF (Liang, PRC(2013))
- 2D HO RMF (Niksic, PRC(2013))

- complex energy $\omega + i\gamma$ has to be used (Imaginary part γ: width)
- equivalent to **Lorentzian-smeared QRPA strength function**
- no additional truncation in the QRPA level

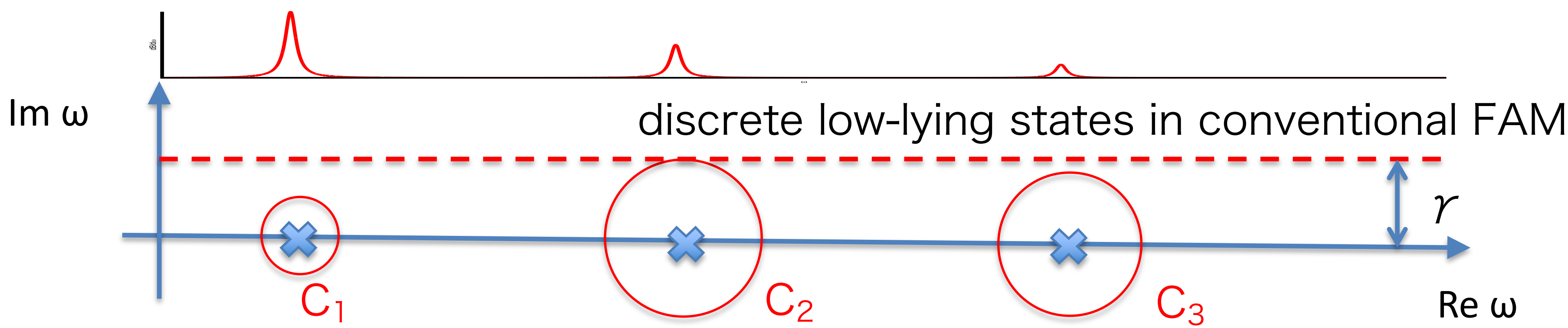
FAM for discrete low-lying states

NH, Kortelainen, Nazarewicz, Phys. Rev. C **87**, 064309 (2013)

strength function $$S(\hat{F},\omega) = \sum_{\mu<\nu} F^{20}_{\mu\nu} X_{\mu\nu}(\omega) + F^{02}_{\mu\nu} Y_{\mu\nu}(\omega)$$

FAM amplitudes and strength functions have first-order poles at QRPA energies

$$X_{\mu\nu}(\omega) = -\sum_i \left\{ \frac{X^i_{\mu\nu}\langle i|\hat{F}|0\rangle}{\Omega_i - \omega} + \frac{Y^{i*}_{\mu\nu}\langle 0|\hat{F}|i\rangle}{\Omega_i + \omega} \right\} \qquad Y_{\mu\nu}(\omega) = -\sum_i \left\{ \frac{Y^i_{\mu\nu}\langle i|\hat{F}|0\rangle}{\Omega_i - \omega} + \frac{X^{i*}_{\mu\nu}\langle 0|\hat{F}|i\rangle}{\Omega_i + \omega} \right\}$$



Contour integration can extract each QRPA solution (X^λ, Y^λ, and Ω_λ)

Ω_i (MeV)		strength(isoscalar monopole)		
MQRPA	FAM	MQRPA	FAM-C	FAM-D
1.3185	1.3183	5.729(−4)	5.776(−4)	5.781(−4)
1.3731	1.3731	1.539(−2)	1.510(−2)	1.511(−2)
2.4582	2.4581	0.1796	0.1784	0.1783
2.5998	2.5975	2.957(−3)	3.060(−3)	3.057(−3)
3.6687	3.6657	0.5776	0.5788	0.5788
5.1185	5.1212	3.539(−4)	4.360(−4)	4.345(−4)
7.4108	7.4084	0.4900	0.4848	0.4848

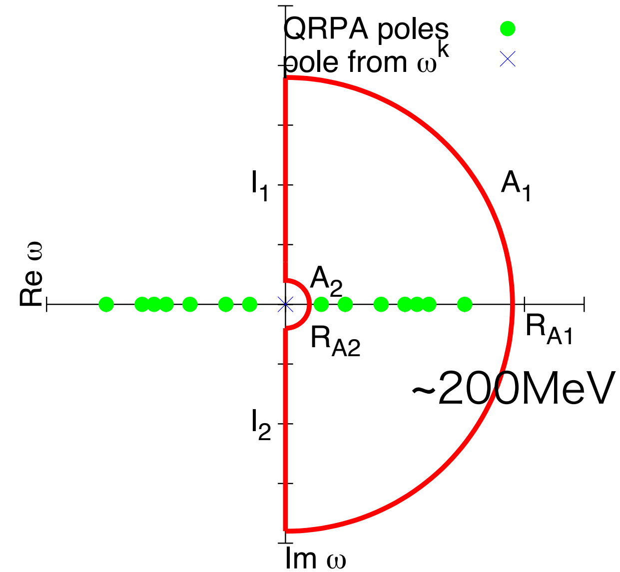
FAM for sum rules

NH, Kortelainen, Nazarewicz, Olsen, Phys. Rev. C **91**,044323 (2015)

QRPA sum rule

$$m_k(\hat{F}) = \sum_{\nu>0} \Omega_\nu^k |\langle \nu | \hat{F} | 0 \rangle|^2$$

Contour integration including all the positive energy poles



$$S(\hat{F},\omega) = -\sum_{\nu>0} \left\{ \frac{|\langle \nu | \hat{F} | 0 \rangle|^2}{\Omega_\nu - \omega} + \frac{|\langle 0 | \hat{F} | \nu \rangle|^2}{\Omega_\nu + \omega} \right\}$$

$$m_k(\hat{F}) = \frac{1}{2\pi i} \oint_D \omega^k S(\hat{F},\omega) d\omega = \sum_{\nu>0} \Omega_\nu^k |\langle \nu | \hat{F} | 0 \rangle|^2$$

TABLE V. Sum rules (in MeV^k e^2 fm^4) for the isoscalar and isovector monopole operators calculated with the MQRPA and the FAM. The FAM calculations were performed by using $N_{A_1} = 301$, $N_{A_2} = 101$, and $N_{I_1} = 200$ integration points.

	$k=-4$	$k=-3$	$k=-2$	$k=-1$	$k=0$	$k=1$	$k=2$	$k=3$	$k=4$
MQRPA(ISM)	0.013 077	0.037 185	0.253 118	5.00 072	139.825	4200.82	131 368	4 342 358	157 906 069
FAM(ISM)	0.012 579	0.036 992	0.253 186	5.00 385	139.844	4199.44	131 277	4 336 644	157 058 272
MQRPA(IVM)	0.00 063 616	0.00 273 872	0.07 120 949	2.78 540	113.908	4735.03	199 525	8 527 358	370 625 216
FAM(IVM)	0.00 043 157	0.00 275 227	0.07 133 510	2.78 615	113.908	4734.30	199 510	8 524 830	368 643 941

IEWSR

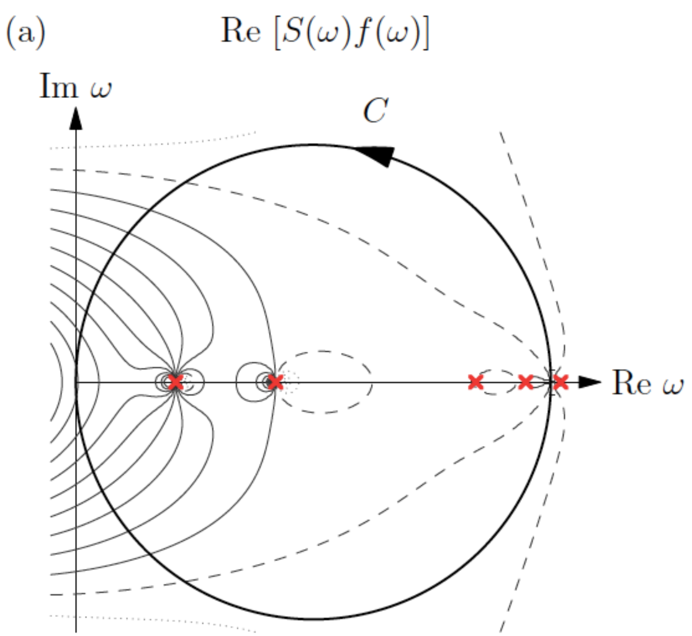
EWSR

FAM for pnQRPA

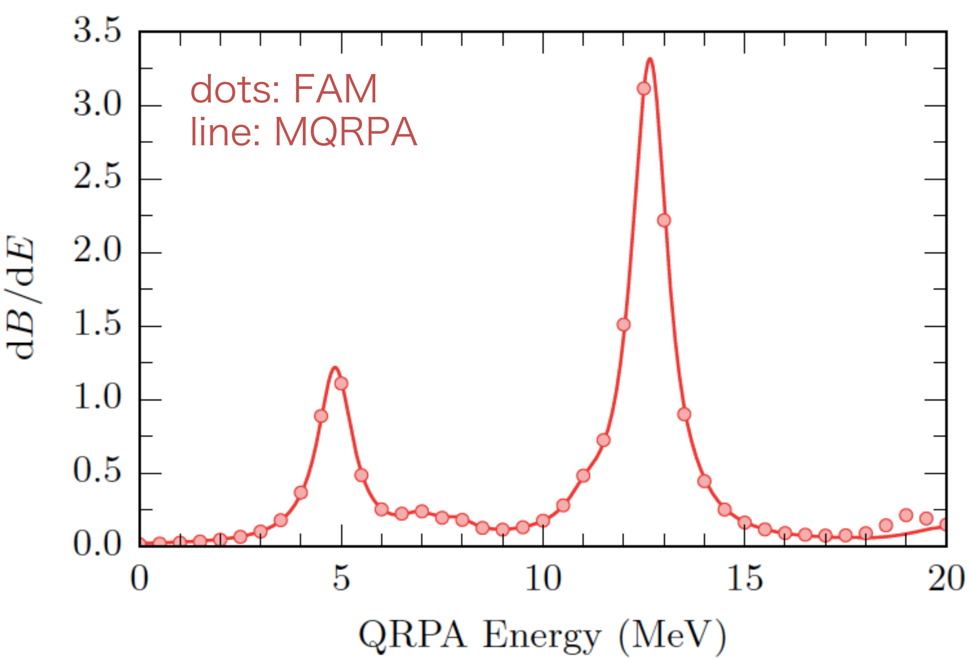
Mustonen Shafer, Zenginerler, Engel, Phys. Rev. C **90**, 024308 (2014)

FAM code development for pnQRPA (Skyrme-EDF HFBTHO)

β decay strength
(contour integration)



Gamow-Teller resonance (^{22}Ne)



Global calculations, new Skyrme parametrizations fitted to β decay and GT
Mustonen and Engel, Phys. Rev. C **93**, 014304 (2016)

2νββ decay NME from the FAM

NH, Engel, in preparation

external fields $\hat{F}^{K=0}_{\mathrm{F}-} = \sum_a \tau^-_a \quad \hat{F}^K_{\mathrm{GT}-} = \sum_a \sigma^K_a \tau^-_a$

FAM amplitude

$$X^i_{pn}(\omega_i, \hat{F}^K_{\mathrm{F/GT}\pm}) = -\sum_{n_i>0} \left\{ \frac{X^{n_i,K}_{pn} \langle n_i, K | \hat{F}^K_{\mathrm{F/GT}\pm} | 0^+_i \rangle}{\Omega_{n_i,K} - \omega_i} + \frac{Y^{n_i,K*}_{pn} \langle 0^+_i | \hat{F}^K_{\mathrm{F/GT}\pm} | n_i, K \rangle}{\Omega_{n_i,K} + \omega_i} \right\}$$

$$Y^i_{pn}(\omega_i, \hat{F}^K_{\mathrm{F/GT}\pm}) = -\sum_{n_i>0} \left\{ \frac{Y^{n_i,K}_{pn} \langle n_i, K | \hat{F}^K_{\mathrm{F/GT}\pm} | 0^+_i \rangle}{\Omega_{n_i,K} - \omega_i} + \frac{X^{n_i,K*}_{pn} \langle 0^+_i | \hat{F}^K_{\mathrm{F/GT}\pm} | n_i, K \rangle}{\Omega_{n_i,K} + \omega_i} \right\}$$

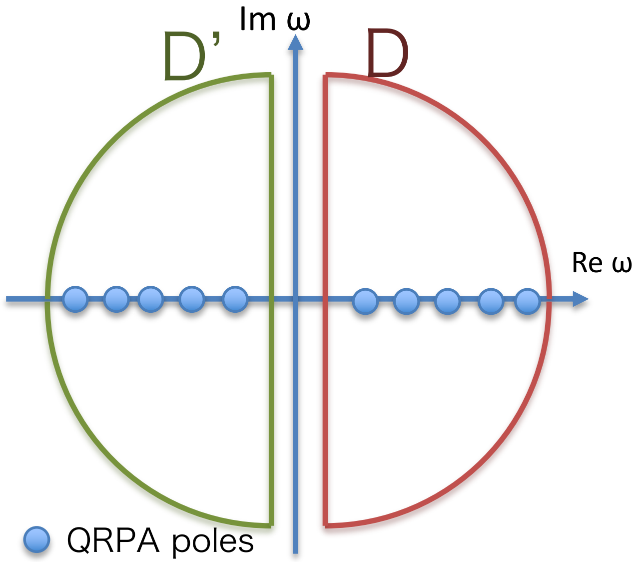
two FAM calculations (from initial state with ω_i, final state with ω_f)

$$\mathcal{T}_{pn,p'n'}(\alpha, \omega, \hat{F}^K_{\mathrm{F/GT}-}, \omega', \hat{F}^{-K}_{\mathrm{F/GT}-}) = Y^f_{pn}(\omega', \hat{F}^{-K}_{\mathrm{F/GT}-}) X^i_{p'n'}(\omega, \hat{F}^K_{\mathrm{F/GT}-}) - \alpha X^f_{pn}(\omega', \hat{F}^{-K}_{\mathrm{F/GT}-}) Y^i_{p'n'}(\omega, \hat{F}^K_{\mathrm{F/GT}-})$$

NME related part

$$\begin{aligned}
\mathcal{T}_{pn,p'n'}(\alpha, \omega, \hat{F}^K_{\mathrm{F/GT}-}, \omega', \hat{F}^{-K}_{\mathrm{F/GT}-}) = \sum_{\nu_i>0,\nu_f>0} & \boxed{\frac{(X^{\nu_f,-K*}_{pn} X^{\nu_i,K}_{p'n'} - \alpha Y^{\nu_f,-K*}_{pn} Y^{\nu_i,K}_{pn}) \langle 0^+_f | \hat{F}^{-K}_{\mathrm{F/GT}-} | \nu_f, K \rangle \langle \nu_i, K | \hat{F}^K_{\mathrm{F/GT}-} | 0^+_i \rangle}{(\Omega_{\nu_f,-K} + \omega')(\Omega_{\nu_i,K} - \omega)}} \\
& + \frac{(Y^{\nu_f,-K*}_{pn} X^{\nu_i,K}_{p'n'} - \alpha X^{\nu_f,-K*}_{pn} Y^{\nu_i,K}_{p'n'}) \langle \nu_f, -K | \hat{F}^{-K}_{\mathrm{F/GT}-} | 0^+_f \rangle \langle \nu_i, K | \hat{F}^K_{\mathrm{F/GT}-} | 0^+_i \rangle}{(\Omega_{\nu_f,-K} - \omega')(\Omega_{\nu_i,K} - \omega)} \\
& + \frac{(X^{\nu_f,-K*}_{pn} Y^{\nu_i,K}_{p'n'} - \alpha Y^{\nu_f,-K*}_{pn} X^{\nu_i,K}_{p'n'}) \langle 0^+_f | \hat{F}^{-K}_{\mathrm{F/GT}-} | \nu_f, K \rangle \langle 0^+_i | \hat{F}^K_{\mathrm{F/GT}-} | \nu_i, -K \rangle}{(\Omega_{\nu_f,-K} + \omega')(\Omega_{\nu_i,K} + \omega)} \\
& + \frac{(Y^{\nu_f,-K*}_{pn} Y^{\nu_i,K}_{p'n'} - \alpha X^{\nu_f,-K*}_{pn} X^{\nu_i,K}_{p'n'}) \langle \nu_f, -K | \hat{F}^{-K}_{\mathrm{F/GT}-} | 0^+_f \rangle \langle 0^+_i | \hat{F}^K_{\mathrm{F/GT}-} | \nu_i, -K \rangle}{(\Omega_{\nu_f,-K} - \omega')(\Omega_{\nu_i,K} + \omega)}.
\end{aligned}$$

2νββ decay NME from the FAM



NH, Engel, in preparation

first order poles at

$$\omega_i = \pm\Omega_{n_i,K} \text{ and } \omega_f = \pm\Omega_{n_f,K}$$

$$\mathcal{M}(\alpha, \hat{F}^{K}_{\mathrm{F/GT}-}, \hat{F}^{-K}_{\mathrm{F/GT}-}; f_{p'n',pn}(\omega,\omega'))$$

$$= \left(\frac{1}{2\pi i}\right)^2 \int_D d\omega \int_{D'} d\omega' \sum_{pnp'n'} \mathcal{T}_{pn,p'n'}(\alpha,\omega,\hat{F}^{K}_{\mathrm{F/GT}-},\omega',\hat{F}^{-K}_{\mathrm{F/GT}-}) f_{p'n',pn}(\omega,\omega')$$

contour D encircles all the positive energy poles
contour D' encircles all the negative energy poles

double GT transition

$$M_{\mathrm{DGT}} = \sum_{K=-1}^{1} \mathcal{M}(\alpha, \hat{F}^{K}_{\mathrm{GT}}, \hat{F}^{-K}_{\mathrm{GT}}; f_{p'n',pn}(\omega,\omega')) = \mathcal{O}_{p'p}(\alpha)\mathcal{O}_{n'n}(\alpha)),$$

2νββ NME

$$M^{2\nu}_{\mathrm{F}} = \mathcal{M}(\alpha, \hat{F}^{K=0}_{\mathrm{F}}, \hat{F}^{K=0}_{\mathrm{F}}; f_{p'n',pn}(\omega,\omega')) = \frac{2\mathcal{O}_{p'p}(\alpha)\mathcal{O}_{n'n}(\alpha)}{\omega-\omega'})$$

$$M^{2\nu}_{\mathrm{GT}} = \sum_{K=-1}^{1} \mathcal{M}(\alpha, \hat{F}^{K}_{\mathrm{GT}}, \hat{F}^{-K}_{\mathrm{GT}}; f_{p'n',pn}(\omega,\omega')) = \frac{2\mathcal{O}_{p'p}(\alpha)\mathcal{O}_{n'n}(\alpha)}{\omega-\omega'})$$

FAM with (σ)τ- from initial state, FAM with (σ)τ- from final state

complex-energy FAM allows us to compute DGT and 2νββ NME very efficiently

Test calculation for sum rule

Fermi sum rule $\sum_i |\langle n_i| \sum_a \tau_a^- |0^+_{\rm gs}\rangle|^2 - |\langle n_i| \sum_a \tau_a^+ |0^+_{\rm gs}\rangle|^2 = N - Z$

Ikeda Sum rule $\sum_{Ki} |\langle n_i| \sum_a \sigma_a^K \tau_a^- |0^+_{\rm gs}\rangle|^2 - |\langle n_i| \sum_a \sigma_a^K \tau_a^+ |0^+_{\rm gs}\rangle|^2 = 3(N - Z)$

Sum rule from double contour integration

Fermi

$$\sum_{\nu>0} |\langle \nu, K=0 | \hat{F}^{K=0}_{\rm F\mp} |0^+_i\rangle|^2 = \left(\frac{1}{2\pi i}\right)^2 \int_D d\omega \int_{D'} d\omega' \sum_{pn} \mathcal{T}^{0^+_i=0^+_f}_{pn,pn}(\alpha=1, \omega, \hat{F}^{K=0}_{\rm F\mp}, \omega', \hat{F}^{K=0}_{\rm F\pm})$$

Gamow-Teller

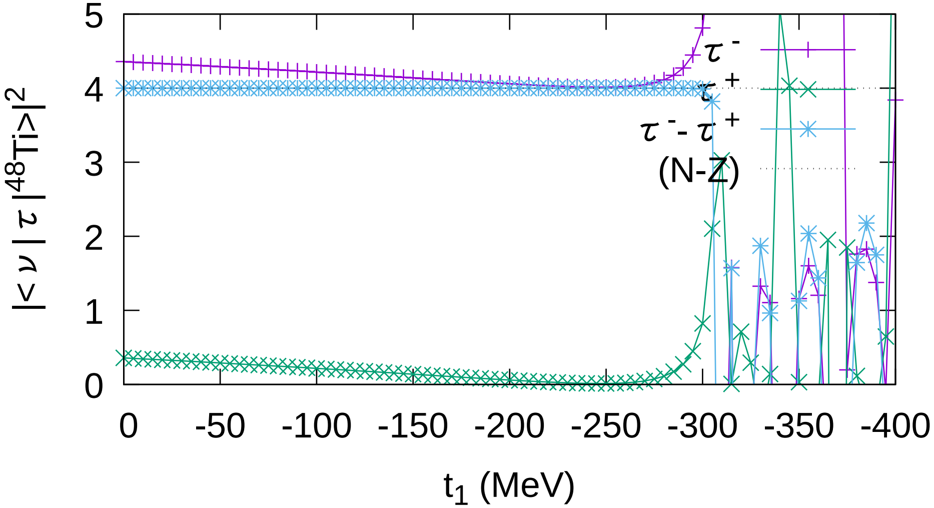
$$\sum_{\nu>0} |\langle \nu, K | \hat{F}^{K}_{\rm GT\mp} |0^+_i\rangle|^2 = \left(\frac{1}{2\pi i}\right)^2 \int_D d\omega \int_{D'} d\omega' \sum_{pn} \mathcal{T}^{0^+_i=0^+_f}_{pn,pn}(\alpha=1, \omega, \hat{F}^{K}_{\rm GT\mp}, \omega', \hat{F}^{-K}_{\rm GT\pm})$$

Test calculation for sum rule

code: HFBTHO, pnFAM(Mustonen et al., Phys. Rev. C **90**, 024308 (2014))

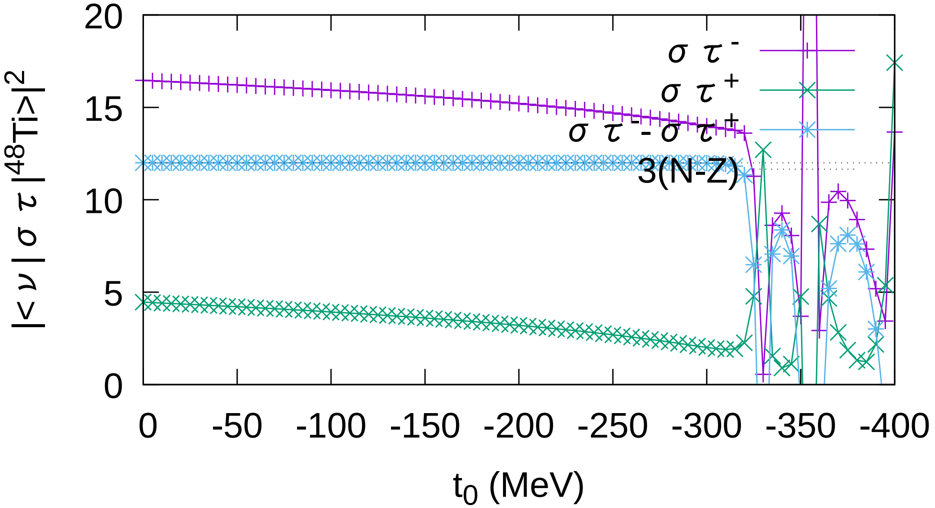
- small calculation without MPI parallelization (^{48}Ti)
- HFBTHO: SLy4, N_{sh}=5, no np pairing in the HFB level
- isovector pairing Vn=-230 MeV fm^3, Vp=-260 MeV fm^3
- t_0: isoscalar pn pairing, t_1: isovector pn pairing (volume-type)
- contour integration: 200 points for the arc, 200 points for the line

Fermi



$t_0=V_{np}$= -240 MeV fm^3

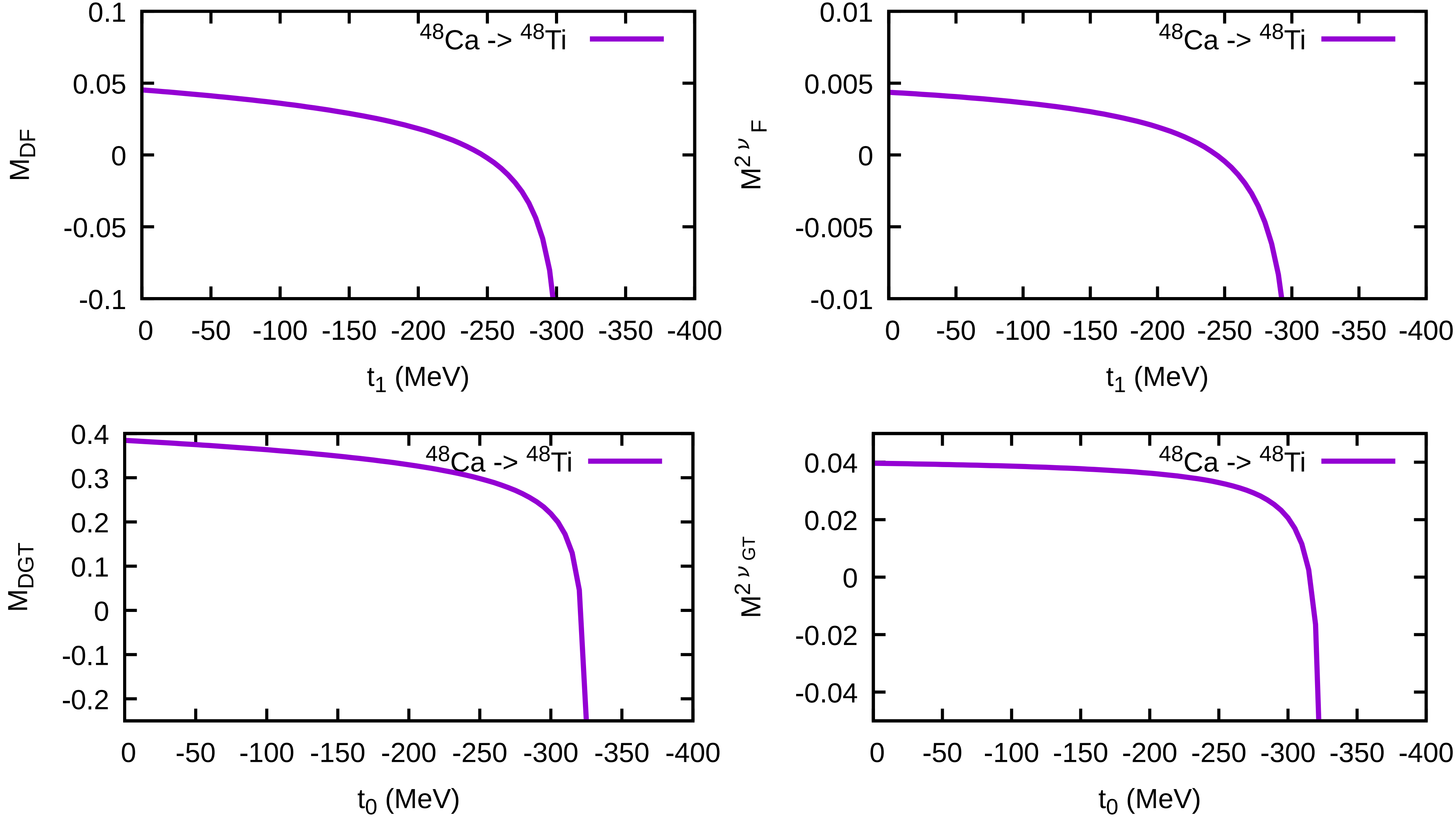
Gamow-Teller



$t_1=V_{np}$=-240 MeV fm^3

phase transition to np-pair condensation at t_1<-300, t_0<-310.
where some of QRPA poles become imaginary.
the integration does not correspond to the sum rule anymore

DGT and 2νββ nuclear matrix element



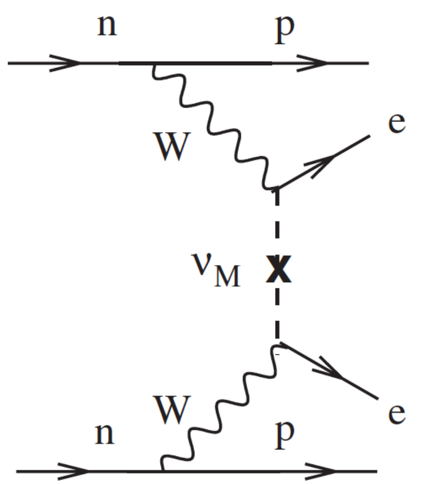
2νββ experimental value：

$M_{2\nu}$ = 0.046±0.004 MeV^{-1} (Barabash, Nucl. Phys. A **935**, 52 (2015), with g_A=1.27)

0νββ decay NME from the FAM

0νββ: not very easy

NH, Engel, in preparation



$$M^{0\nu} = M^{0\nu}_{\mathrm{GT}} - \frac{g_{\mathrm{V}}^2}{g_A^2} M^{0\nu}_{\mathrm{F}} + M^{0\nu}_{\mathrm{T}},$$

$$M^{0\nu}_{\mathrm{F}} = \frac{2R}{\pi g_A^2} \int_0^\infty q dq \frac{h_{\mathrm{F}}(q)}{q + \bar{E} - (E_i + E_f)/2} \langle f | \sum_{ab} j_0(q r_{ab}) \tau_a^- \tau_b^- | i \rangle$$

$$M^{0\nu}_{\mathrm{GT}} = \frac{2R}{\pi g_A^2} \int_0^\infty q dq \frac{h_{\mathrm{GT}}(q)}{q + \bar{E} - (E_i + E_f)/2} \langle f | \sum_{ab} j_0(q r_{ab}) \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b \tau_a^- \tau_b^- | i \rangle$$

$$M^{0\nu}_{\mathrm{T}} = \frac{2R}{\pi g_A^2} \int_0^\infty q dq \frac{h_{\mathrm{T}}(q)}{q + \bar{E} - (E_i + E_f)/2} \langle f | \sum_{ab} j_2(q r_{ab}) [3(\boldsymbol{\sigma}_j \cdot \hat{\boldsymbol{r}}_{ab})(\boldsymbol{\sigma}_k \cdot \hat{\boldsymbol{r}}_{ab}) - \boldsymbol{\sigma}_a \cdot \boldsymbol{\sigma}_b] \tau_a^- \tau_b^- | i \rangle$$

- ☐ neutrino propagator(intermediate state)
- ☐ integral over the neutrino momentum
- ☐ two-body operator is not separable (neutrino potential)

expansion of spherical Bessel function

$$j_0(q r_{ab}) = 4\pi \sum_{l=0}^{\infty} j_l(q r_a) j_l(q r_b) \sum_{m=-l}^{l} Y^*_{lm}(\hat{\boldsymbol{r}}_a) Y_{lm}(\hat{\boldsymbol{r}}_b)$$

q-integral and l,m-summation must be computed separately

Summary

- Finite-amplitude method for QRPA in nuclear density functional theory for $2\nu\beta\beta$ nuclear matrix element has been formulated.
  - Calculation with large two-quasiparticle space ($\sim 10^6$) possible
  - Significant reduction of the computational time is expected for $2\nu\beta\beta$ NME and DGT transition
  - Useful for determining the neutron-proton pairing globally using available $2\nu\beta\beta$ half lives/benchmarking existing EDFs
  - possible extension to $0\nu\beta\beta$
- Implementation based on pnFAM code (HFBTHO)
  - test calculation with N_{sh}=5 completed
  - next step: parallelization for large-scale calculation
- Collaborator: Jon Engel (University of North Carolina, Chapel Hill, USA)